Solving Combined Optimal Transmission Switching and Optimal Power Flow sequentially as convexified Quadratically Constrained Quadratic Program

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Abstract—A novel approach to efficiently solve a combined optimal transmission switching and optimal power flow problem is derived and evaluated. The approach is based on a sequentially solved quadratically constrained quadratic program with a new way of convexification. This convexification requires adapted modeling and uses two procedural steps: First, nonlinear equality constraints are eliminated by quadratically approximating their inverse system of functions and inserting it into the objective function and into the inequality constraints. The resulting objective function and inequality constraints are approximated quadratically. Second, the remaining nonconvex parts of the objective function and the inequality constraints are identified by eigenvalue analysis of their Hessian matrices and eliminated by using piecewise linear approximations.

Issues of accuracy and convergence are examined and countermeasures are presented. Additionally, the avoidance of islanding and the sequentiality of parallel circuits are considered. Case studies show fast und stable convergence of the approach.

Index Terms—Optimized Transmission Switching, Network configurations, Optimal Power Flow, Quadratically Constrained Quadratic Program, Convexification

I. INTRODUCTION AND STATE OF THE ART

Massive changes, caused by the energy transition and the liberalization of energy trade across several interconnected networks, burden the electric power system closer to or beyond their operational limits. Furthermore, increasing regulatory pressure pushes the network operators to be more efficient. Constrained optimization is the mathematical method to find an efficient operating point, which complies with all operating limits. Moreover, the finding of an optimal operating point is complicated by

- the growth of interconnected networks,
- the consideration of multiple voltage levels and an increasing amount of decentralized generation units,
- the extensions of security constraints and
- further degrees of freedom caused by adjustable and switchable transmission assets.

Although there has been substantial progress in advanced methods for optimizing power systems in the last years [1], the optimal power flow (OPF) [2], [3] as well as its extensions is still challenging to solve today [4] [5] [6]. Basically, the associated power equations, interrelating the active and reactive powers with the nodal voltages, lead to nonlinearity and nonconvexity of the optimization problem. Extensions of the equations and the optimization exacerbate the challenges of solving. Although the aforementioned cited references are several years old, they still reflect the general state of the art and the persisting challenges, according to the best knowledge of the authors. Recent progress in research, worth mentioning according to the author’s conviction, is discussed hereinafter: Convex relaxations of the power equations have attracted a lot of attention and revealed interesting insights and noticeable progress in the last years [7] [8] [9] [10] [11] [12]. For radial networks even an exact convex relaxation has been found [13]. Reference [14] is seen as an up-to-date survey of the state of the art of relaxations of the Power Flow Equations. However, for meshed networks latest research has shown that the relaxation approaches developed so far lack of accuracy and feasibility [15]. Strengthening the relaxations and quickly and reliably tightening the lower and upper bounds during presolve and solve are still subject of research. The continuing necessity for research becomes apparent by the Grid Optimization Competition (GOC) most recently announced by the Advanced Research Projects Agency-Energy (ARPA-E) of the U.S. Department of Energy (DOE) [16]. The aim of the still running GOC is to accelerate the development of methods for solving the most pressing power system problems.

In this paper, focus is put on progress of research of the authors in the context of optimized transmission switching (OTS) as part of an optimal power flow (OPF). The degrees of freedom provided by OTS are particularly difficult to model and approximate and they furthermore complicate the solution process due to the integer nature of the variables and their significant and nonlinear influence on the state of the network. For these reasons, OTS plays a dominant role in challenge two of the aforementioned GOC. Moreover, islanding of the network needs to be prevented when changing switching states in error-free operation as well as in contingencies. To the best knowledge of the authors a review of transmission switching and network topology optimization that still reflects the general state of the art and the persisting challenges is given in [17]. Some further noteworthy approaches and references are collected in [18]. OTS is important not only for power flow control, but also for voltage maintenance in high and low load situations [19]. The challenges of OTS complicate the solution of the OPF and so linear approximations of the power equations are widely used in OTS, due to the reliability and computational efficiency of Mixed-Integer Linear Programming (MILP). Due
to their simplicity, DC-approximations and Power Transfer Distribution Factors (PTDFs) are often-used linear approximations, despite the disadvantages resulting from their neglects. However, investigations of the accuracy of these linear approximations showed poor performance in several cases, sometimes even leading to increases in the objective function when minimization was sought [20]. Besides approximations, empirical, stochastic or heuristic approaches are suitable for solving the OPF considering OTS and they have the advantage that they can use the exact problem description and may find a locally optimal solution quickly [21]. However, these aforementioned approaches cannot guarantee or proof global optimality. Therefore, an approach based on mathematical programming is preferred in the following. This paper aims to present an approach with adequate accuracy and the property to quickly find a feasible solution with a high probability of global optimality. Therefore, the OPF considering OTS is approximated and sequentially solved as mixed-integer convexified quadratically constrained quadratic program (MI-cQCQP). The approximation-approach differs from other approaches, especially the DC-approximations, by not using predefined neglects and by using a novel way of convexification.

II. PRECEDING WORK OF THE AUTHORS

Preceding versions of the approach of sequentially solving a convex approximation have already been published: the first step in the development of the approach is a Linear Program (LP) in [22]. Linear approaches (varying in the simplifications used) can achieve a relatively fast approximate solution, but approximating nonlinear behavior with linear equations inherently results in inaccuracies and oscillatory behavior of sequential optimization. The second step in the development of the approach is a convex Quadratic Program (cQP) in [23], which reveals better results regarding the accuracy of the approximation of the objective but still shows oscillating behavior. The third step in the development of the approach was a convex Quadratically Constrained Quadratic Program (cQCQP) in [24], which is able to overcome the oscillations problems. A comparison of cQP and cQCQP displays that cQPs can converge in locally optimal solutions compared to the solutions of the cQCQP. In [25] a detailed reasoning and derivation of the cQCQP-approach is given and in [26] an extension of this approach by in-phase and quadrature voltage controlled transformers was published and compared to a particle swarm optimization (PSO). Similar to the extension in [26], investigations of different approaches for the extension by OTS were published in [27], but challenges of accuracy and convergence remained. Enhanced research and development of modelling OTS lead to the results in this paper.

III. TECHNICAL APPROACH IN THIS PAPER

The following modeling and solving approach is based on the aforementioned preceding works of the authors on convexificated quadratic approximations. With respect to the content in the introduction and state of the art of this paper, it must be pointed out that the convexification approach used in the aforementioned and in this paper is not a convex relaxation, but a convex approximation. In [25], the authors presented details on this distinction. Due to the approximations, the approach carries the unavoidable risk of cutting off relevant parts of the solution space, but it has been shown to be accurate in the applications to date. Compared to relaxations, approximations have the advantage of being tight in terms of feasibility.

This chapter is divided into six sections:

A. The Optimal Power Flow formulation without Transmission Switching is presented.

B. Different ways of modelling transmission switching in general are described.

C. Adaptions of the modeling approach are described to enable the accuracy of the approximation approach.

D. The implementation of Transmission Switching into the approximation approach is shown in detail.

E. The prevention isolated nodes and network areas is shown.

F. The modeling of the sequentiality of parallel circuits and some simplifications the solver benefits from are shown.

A. Optimal Power Flow without Transmission Switching

In (1) an exact complex valued formulation of the optimization problem for the Optimal Power Flow (without Transmission Switching) is given. In (1) \( p_n \) and \( q_n \) are vectors of nodal active and reactive powers, \( \Psi_n \) is a vector of the complex valued nodal voltages and \( \mathbf{v}_N \) its diagonal matrix, \( \mathbf{I}_{NT} \) is the complex valued block-diagonal terminal admittance matrix of all transmission assets and \( \mathbf{I}_{NT} \) the incidence-matrix of nodes and terminals. Additional details on the parameters and variables can be found in [26]. The costs for losses and the costs for active power redispatch are kept separate in the objective function to enable the distinction later needed in the approximation approach. Reactive power redispatch is assumed to be without costs.

\[
\begin{align*}
\text{min} & \quad c_{loss} \cdot P_{loss} + c_N \cdot \Delta P_N \\
\text{subject to} & \quad \mathbf{I}_T = \mathbf{Y}_{NT} \cdot \mathbf{v}_N \\
& \quad \mathbf{v}_N = \mathbf{I}_{NT}^{-1} \cdot \Delta \mathbf{v}_N \\
& \quad (p_{N0} + \Delta p_n) + j(q_{N0} + \Delta q_n) = 3 \cdot \mathbf{I}_{NT} \cdot \mathbf{v}_N \\
& \quad P_{loss} = 3 \cdot \text{Re} \{ \mathbf{v}_N^T \cdot \mathbf{I}_T \} \\
& \quad p_{N,min} \leq p_{N0} + \Delta p_n \leq p_{N,max} \\
& \quad q_{N,min} \leq q_{N0} + \Delta q_n \leq q_{N,max} \\
& \quad v_{N,\text{min}} \leq |\mathbf{v}_N| \leq v_{N,\text{max}} \\
& \quad |\mathbf{I}_T| \leq |\mathbf{I}_{NT}| \max
\end{align*}
\]  

B. Modelling transmission switching in general

A switching variable \( \sigma_i \) for each terminal of each transmission asset is included to model transmission switching. The \( \sigma_i \) of all terminals can be combined in a vector \( \sigma \) and in a diagonal matrix \( \Sigma_{\sigma} \). The switching variables at both terminals
of one transmission asset need to have the same value. Equation (2) guarantees this relation in the later optimization.

\[
\begin{bmatrix}
1 & -1 \\
1 & -1 \\
& & \ddots \\
& & & 1 & -1
\end{bmatrix}
\sigma_t = \mathbf{E}\cdot \sigma_t = 0 \quad (2)
\]

Parallel circuits of a transmission asset can be represented by discrete variables instead of multiple binary variables, which could have advantages in terms of the branch and cut process of the mathematical programming. But to express logical conditions to prevent islanding of the network, a binary representation is advantageous. Furthermore, quadratic products with one or two binary variables can be linearized by bilinear transformation during presolve [28]. Moreover, the approach derived in section C can only be implemented with binary variables, which will be reasoned in that section. Therefore, the binary representation is preferred. Sequentiality of parallel circuits, which would have been an inherent property of discrete variables, can also be modeled based on a binary representation. The necessary logical conditions are presented in section F of this chapter.

The diagonal matrix \( \Sigma_t \) can be included into the power equations at different positions. These positions can have different electrical interpretations. Electrical interpretations can be derived by splitting the power equations into a system with two equations. Equations (3) to (6) display different variants. In an optimization model without approximations, all variants equally enforce that voltage, current and power of a transmission asset are zero, when the corresponding switching variables on both sides of the transmission asset are zero. But using the approximations derived in the next section of this chapter, the variants have different effects.

\[
Y' = \Sigma_t \cdot I' \cdot V_N
\]

\[
(p_{N,0} + \Delta p_a) + j(q_{N,0} + \Delta q_a) = 3 \cdot Y' \cdot I' \cdot V_N \quad (3)
\]

\[
Y_{TT, \Sigma} = \Sigma_t \cdot Y_{TT}
\]

\[
(p_{N,0} + \Delta p_a) + j(q_{N,0} + \Delta q_a) = 3 \cdot Y_{TT} \cdot I' \cdot Y_{TT} \cdot V_N \quad (4)
\]

\[
I_t = \Sigma_t \cdot Y_{TT} \cdot I' \cdot V_N
\]

\[
(p_{N,0} + \Delta p_a) + j(q_{N,0} + \Delta q_a) = 3 \cdot Y_{NT} \cdot I_t \quad (5)
\]

\[
Y_t = \Sigma_t \cdot I'_{NT} \cdot V_N
\]

\[
(p_{N,0} + \Delta p_a) + j(q_{N,0} + \Delta q_a) = 3 \cdot Y_t \cdot I'_{NT} \cdot V_N \quad (6)
\]

- An electrical interpretation of (3) is given by switching voltage sources, shown in Figure 1. Switching off a transmission asset corresponds to switching on the voltage sources with the negative values of the corresponding nodal voltages. Switching on corresponds to zero valued voltage sources.

- An electrical interpretation of (4) is given by switching an admittance, shown in Figure 2. Switching off a transmission asset corresponds to switching on an admittance with the negative value of the transmission asset in parallel. Switching on corresponds to zero valued additional admittance.

- An electrical interpretation of (5) is given by switching a current source, shown in Figure 3. Switching off a transmission asset corresponds to switching on the current sources with the negative values of the corresponding terminal currents. Switching on corresponds to zero valued current sources.

- An electrical interpretation of (6) is given by switching a power source, shown in Figure 4. Switching off a transmission asset corresponds to switching on the power sources with the negative values of the corresponding terminal powers. Switching on corresponds to zero valued power sources.

C. Adoptions of the modeling for the approximation approach

All four aforementioned variants have been implemented into the approximation approach derived in section D for test
purposes. Unfortunately, in all variants massive problems of accuracy and convergence occurred, when switching the binary variables from zero to one or vice versa. It seems that the quadratic approximations are not tight enough to include the massive influence of OTS on the state of the grid. Deeper investigations have suggested the following relations as a cause: When supposing the admittance matrix and the incidence-matrix as constant parameters, the resulting functions in (3) to (6) are functions of third order, because each element is multiplied with two voltages and one switching variable. Figure 5 shows an arbitrary function of third order in solid green, its second-order Taylor-expansions at the point zero in dashed red and its second-order Taylor-expansions at the point one in dotted red.

![Figure 5](image_url)

The approximations can have relevant differences when not using them at the point of their deployment. Furthermore, deeper investigations of the convexified quadratic approximations showed:

- When voltages (3) are switched to their approximate values related to the zero or one values of the switching variable, currents are expected to change in the same direction. But based on the approximations, currents change their value contrary to the expected direction so that the transmitted power stays in the same region as it was before switching;

- the same can be observed with voltages, when currents (4) were switched to their approximate values related to the zero or one values of the switching variable;

- the same can be observed with voltages and currents, when admittances (5) were switched to their exact values related to the zero or one values of the switching variable;

- and the same can be observed with voltages and currents, when powers (6) were switched to their exact values related to the zero or one values of the switching variable.

The possibility to combine the variants in (3) to (6) by multiplying with $\Sigma^T$ multiple times without changing the function values is only given for binary variables. In this particular case, only the function values corresponding to one and zero of the switching variables are of interest and these corresponding values remain unchanged when multiplied by $\Sigma^T$ multiple times. Therefore, the approximation approach does not work for discrete variables, as addressed in section B. Not only the four aforementioned variants but also combinations of them have been implemented for test purposes. Only the combination of all variants shown in (7) delivers accurate approximations for voltage, current and power of a switched transmission asset. Equation (7) contains separate quadratic equations for voltages as well as for currents that are forced to the values related to the zero or one values of the switching variables. So their quadratic product within the power calculation is also forced to the corresponding values.

$$\begin{align*}
\Delta v_T^t &= \sum_{i,T} I_{NT}^t \Delta v_N^t, \\
\Delta i_T^t &= \sum_{i,T} Y_{TT}^t \Delta i_T^t, \\
\Delta p_{N0} + \Delta q_N &= \sum_{i,T} (p_{N0} + \Delta p_N) + j(q_{N0} + \Delta q_N) = 3 I_{NT}^t \Delta v_T^t \Delta i_T^t.
\end{align*}$$

(7)

This observation is counterintuitive, because the order of the resulting functions increases from three to five and so quadratic approximations of them might be expected to become less accurate. However, the calculation formulas of the first order derivatives in the gradients and the second order derivatives in the Hessian matrices change, and it can be observed that the approximation in each sequential step is improved.

**D. Implementation of OTS into the approximation approach**

The approach resulting from the preceding works of the authors needs a quadratic system. Equation (7) is already presented in such an form. Furthermore, the total system of quadratic equations must have the same number of variables on the left side and on the right side of the equations for the system to be invertible. Consequently, auxiliary variables and equations need to be inserted. Additional definitions of the parameters and variables are displayed in [26].

The formulation of the optimization problem needs to be real valued to be able to use the convexification approach derived in preceding works of the authors and to use standard solvers for mathematical optimization. Therefore, the complex variables are split into their real and imaginary part. The necessary reformulation is given in (8).

$$\begin{align*}
\left( \begin{array}{c}
\sigma_{t,0} + \Delta \sigma_T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array} \right) &= \left[ \begin{array}{c}
\sigma_T^t \\
\Sigma^T I_{NT}^t - v_{N,i} - v_{T,j} \\
\Sigma^T I_{NT}^t - v_{T,j} \\
\Sigma^T G_{TT}^t - v_{T,j} - v_{T,i} \\
\Sigma^T B_{TT}^t - v_{T,j} - v_{T,i} \\
3 I_{NT}^t (v_{T,i} - v_{T,j}) - 2 i_{T,i} \\
3 I_{NT}^t (v_{T,i} + v_{T,j}) - 2 i_{T,j} \\
0 \\
i_{T,i} \\
i_{T,j} \\
i_{N,i} \\
i_{N,j} \\
P_{loss} \\
P_{loss} \\
\tilde{v}_T^2 \\
\tilde{v}_N^2
\end{array} \right] = f\left( \begin{array}{c}
\sigma_T^t \\
v_{T,j} \\
v_{T,i} \\
i_{T,i} \\
i_{T,j} \\
v_{N,i} \\
v_{N,j} \\
P_{loss} \\
P_{loss} \\
\tilde{v}_T^2 \\
\tilde{v}_N^2
\end{array} \right)
\end{align*}$$

(8)

The system of nonlinear equalities in (8) can now be used to apply the convexification approach. The quadratic approximation of the inverse system of the quadratic equalities...
is needed to insert the equalities into the objective function and into the inequality constraints [25]. To make the system of equations invertible, a distributed slack needs to be added [25]. Subsequently, the nonconvex parts of the functions are identified by eigenvalue analysis and convexified by piecewise linearization, described in [25].

The resulting optimization problem is given in (9) where the gradients $g$ and Hessians $H$ of the approximate functions are indexed with $L$ for the losses, with $V$ for the voltages and with $I$ for the currents. The indices $I$ or $U$ represent a lower or an upper bound.

$$
\begin{align*}
\min & \quad c_{\text{lin}} \left( g_{i} \left[ \Delta p_{N} \right] + \frac{1}{2} H_{i} \left[ \Delta p_{N} \right] \right) \\
\text{s.t.} & \quad \sigma_{i} \in \{0, 1\} \\
& \quad \Delta p_{N,\min} \leq \Delta p_{N} \leq \Delta p_{N,\max} \\
& \quad \Delta q_{N,\min} \leq \Delta q_{N} \leq \Delta q_{N,\max} \\
& \quad g_{v,i} \Delta q_{N} + \frac{1}{2} \sigma_{i} \left[ \Delta p_{N} - \frac{1}{2} \Delta q_{N} - H_{v,i} \Delta q_{N} \right] \leq \left( v_{N,\min} - v_{N,0} \right)^{2} \\
& \quad g_{c,i} \Delta q_{N} + \frac{1}{2} \sigma_{i} \left[ \Delta p_{N} - \frac{1}{2} \Delta q_{N} - H_{c,i} \Delta q_{N} \right] \leq \left( i_{N,\max} - i_{N,0} \right)^{2}
\end{align*}
$$

In the inverted model, power from the incorporated distributed slack mentioned above establishes the active power balance, similar to power from control reserve. But balancing active-power redispatch with power from control reserve is contrary to regulation. Furthermore, balancing-power is not part of the objective function; only the costs for losses and active-power redispatch are considered (refer to section A). Therefore, an additional linear equality constraint is needed, to ensure balance-neutral active power redispatch. Equation (10) sums up all active power changes $\Delta p_{N}$ to zero and is added to the optimization problem as linear equality constraint.

$$I_{N}^{T} \cdot \Delta p_{N} = 0$$

A balance-neutral reactive power redispatch does not need to be ensured by an additional constraint, because no regulation requires that and reactive series und shunt elements of the network ensure the balance.

The approximate functions are convex and a MI-cCQCP can be solved with appropriate standard solvers in each sequential step of the optimization, e.g. [28].

### E. Prevention of isolated nodes and network areas

An important aspect of transmission switching is the need to avoid isolated nodes and network areas. Explicit constraints are required for mathematical programming, because it is not sufficient to prove the connectivity of a newly found solution subsequently, since the solver might get stuck in an invalid region of the solution space. The solver needs constraints, which prevent from searching and finding solutions in an inadmissible area of the solution space. In [29] a linear mixed-integer description to guarantee connectivity of a communication network based on the theory of graphs is presented. With small adoptions it can be added to the MI-CQCQP in (9). The basic approach is described in the following: An additional dimensionless flow variable $f_t$ that is not related to electric power flows or current flows is defined for both terminals of each transmission asset. The $f_t$ of all terminals can be combined in a vector $f$. Both dimensionless flow variables of one transmission asset must have the negative value of the corresponding other flow variable, so the transmission flow is lossless. Equation (11) guarantees this relation.

$$
\begin{bmatrix}
1 & 1 & \cdots & 1 & 0
\end{bmatrix} f = \mathbf{E} \cdot f = 0 \tag{11}
$$

With $n$ as the number of nodes, the dimensionless flow injection at one arbitrarily selected node (the “source node”) is set to $n-1$ and for all other nodes (the “load nodes”) the dimensionless flow absorption is set to 1. In the following, the arbitrarily selected node is the first node. The incidence-matrix of nodes and terminals $I_{NT}$ is used in (12) to sum up all flow variables at each node and to consider the network topology. Equation (13) makes all $f_t$ semi-continuous variables depending on the switching variables: The flows can only be unequal to zero if the corresponding switching variable is one and are continuously limited between the lower and upper bound $n$.

$$
I_{NT} \cdot f_{t} = n-1 -1 \cdots -1 \tag{12}
$$

$$
\begin{align*}
& f_{t} - n \cdot \sigma_{t} \leq 0 \tag{13}
\end{align*}
$$

Equations (11) to (13) are added to the optimization problem in (9) to guarantee connectivity of all nodes.

### F. Sequentiality of parallel circuits

An optional modelling aspect is the sequential order of parallel circuits. It is not necessary to model such an order, but it helps the solver to distinguish equal solutions and it tightens the solution space. By adding the relation of two binary switching variables of two parallel circuits $\sigma_t$ and $\sigma_{t2}$ in (14), sequentiality of this two parallel circuits can be modeled. It expresses that $\sigma_{t2}$ can only be nonzero, if $\sigma_t$ is nonzero.

$$
\sigma_{t2} - \sigma_{t} \leq 0 \tag{14}
$$
This relation can also be added for more than two parallel circuits. For simplification of the prevention of islanding, it is sufficient to only consider a dimensionless flow variable \( f_i \) for the first of multiple parallel circuits, if sequentiality is considered. Moreover, it is sufficient to only consider a current constraint \( i \leq i_{\text{max}} \) for the first of multiple parallel circuits.

IV. CASE STUDIES

For the case studies, an adaptation of the IEEE 118-bus transmission grid regarding German transmission grid characteristics is used [30]. A MathWorks Matlab [31] dataset of the grid model is available at [32]. The limits of the control variables are the same as in [26]. The costs for grid losses \( c_{\text{loss}} \) and for active power redispatch \( c_i \) are set to 1. The costs for reactive power redispatch and for switching measures are set to 0, but could easily be set to values unequal to zero. The aforementioned choice of costs shifts the focus of these case studies from power plant dispatch to switching state optimization. Bus number 63 is arbitrarily selected as reference node and its voltage phase is set to 0°. The initial grid losses with the voltage control from [30] are \( P_{\text{loss},0} = 189.90 \text{MW} \). The simulations are performed on computers with a 2.7 GHz QuadCore and 16 GB RAM. The modeling language used is Matlab [31] and the solver used is Gurobi [28].

To enhance the speed of the sequential approach, the relative MIP-gap-limit is dynamically adjusted in each sequential step: In the first step, for which a heightened forecast error is to be expected due to the high state changeings by the optimizer and the used approximations, it is set to a value of 100%. In the following steps it is set to the tenth of the MIP-gap-limit of the respective preceding step. Furthermore, a dynamically adjusted time-limit is set for the optimizer that starts with 60 s in the first step and is multiplied with the step-number in each step. The total time for the duration of one step given in the results is a little bit longer, due to the convexification and further precalculations.

Four scenarios are elaborated as case studies:

A. The possible contributions of power plants to active and reactive power redispatch are considered as flexibility. It is the same scenario as scenario 1 in [26] and it serves here as a base case, which is repeated here for ease of comparison. The problem is modeled as cQCQP and solved sequentially.

B. Additional flexibilities are provided by the possibility to switch on or off two parallel systems of each transmission asset. The problem is modeled as cQCQP and solved sequentially.

C. The problem from A is modeled as cQP by deleting the quadratic part of the quadratic constraints and solved sequentially to elaborate a comparison of linear and quadratic approximations.

D. The problem from B is modeled as cQP by deleting the quadratic part of the quadratic constraints and solved sequentially to elaborate a comparison of linear and quadratic approximations.

For each scenario a bar chart is given: the orange bars show the predicted values of the approximation in each sequential step and the blue bars show the actual values, determined with a load flow calculation after each sequential step.

In the optimization, all constraints are modeled as hard constraints, so that apart from a numerical solver tolerance, no constraint-violations can remain if a solution is found to be feasible. But of course, feasibility applies only to the approximated values. If a noticeable difference between the predicted values and the actual values remains in the last step, constraint-violations also remain.

A. PQ-flexibility | sequential cQCQP

Figure 6 shows the results of the basic scenario that only contains continuous variables. The optimization converges after six sequential steps and only takes a few seconds.

In step two (after the first optimization), a noticeable deviation between the predicted value and the actual value can be observed, which can be traced back to the massive changes of the active power flows, leading to inaccuracies of the approximations. However, the missing accuracy in the first step does not influence the accuracy and performance of the complete multistep optimization. Since the changes between two steps fall below a limit of 10^-9 the optimization is terminated after six steps and since only continuous variables are used as flexibility, no MIP-gap is given in the results.

The aforementioned deviation is not observed in that magnitude, if only reactive power redispatch is considered as flexibility. For that subscenario only no graphical evaluation is shown here, because only a small improvement of the objective function on \( P_{\text{loss}} = 188.25 \text{MW} \) is achieved and no differences of the bars would be identifiable.

B. PQΣ-flexibility | sequential cQCQP

Figure 7 shows results of scenario B that allows two parallel systems per transmission asset. The optimization converges after a few steps and finds optimized solutions. As in scenario A, a noticeable deviation between the predicted value and the actual value can be observed in step two (after the first optimization), but it is unexpectedly smaller than in scenario A. Presumably, the cause is the main axis transformation and the subsequent piecewise linearization, which can change due to
the additional switching variables. In the following steps, the optimization shows only small improvements.

As expected with further degrees of freedom, a significantly better solution than in scenario A is found: The losses can additionally be reduced by ca. 20% from 45.6 MW to 36.24 MW by considering transmission switching. Detailed investigations of the solver log revealed that most of the time in each step is spent on mostly unsuccessful attempts to increase the lower bound, but no new or only slightly better solutions are found; thus, the MIP-gap stays more or less constant after the first four steps. Based on this insight, the time limit per step as well as the number of steps could be reduced. Furthermore, only switching on additional systems was observed, but not switching off.

C. PQ-flexibility | sequential eQP

In scenario A a linear approach has been used to optimize only active and reactive power analogously to scenario B. The results are shown in Figure 8.

The optimization converges faster compared to scenario A, but only reaches a suboptimal result compared to the quadratically constrained approach. Furthermore, a noticeably worse forecast accuracy can be observed in the first steps. Since only continuous variables are used as flexibility, no MIP-gap is given in the results.

D. PΩ∑-flexibility | sequential eQP

Figure 9 shows results of scenario D that has the same input data as scenario C but is linearly constrained. The optimization fails after a few steps, due to non-convergent load flow calculations. The deviation between the predicted value and the actual value is massive and can be taken as the reason for the non-convergence.

V. SUMMARY

In this paper, a new method for efficiently solving a combination of Optimal Transmission Switching and Optimal Power Flow is derived, explained and classified in the state of the art. The approach is a sequentially solved approximating quadratically constrained quadratic program, which is convexified in each sequential step. The novel way of approximation and convexification in each step necessitates an adaption of the modeling of Transmission Switching.

In the case studies, the quadratically constrained approach in scenario A and B show high accuracy and convergent behavior in acceptable time. Attempts with linearly constrained quadratic programs in scenario C and D converge into a local optimum for the optimization of active and reactive power only. When adding transmission switching as additional degree of freedom, high accuracy deviations are revealed. Furthermore, the optimization with linear constraints becomes instable and non-convergent after a few steps. These comparisons are further proofs for the superiority of the quadratically constrained approach.

VI. OUTLOOK

The case studies undertaken yet only consider a base case. The consideration of contingencies and their according security constraints will be part of future publications. In addition, test networks of larger size, multiple voltage levels and different topology will be used to verify the approach. The datasets of [16] will be used to achieve this aim and thus a benchmark with existing models and tools already available will be made. This makes a proof of global optimality or at least an estimation of the proximity to global optimality possible.

The combination of the OPF by OTS, showed in this paper, can be broadened by further details: the switching of busbars in substation will be one of the next steps. Busbars and their couplers require special consideration, because in contrast to nodes, busbars are allowed to get isolated and couplers modeled as extremely short lines lead to extremely high conductivities and therefore potentially to numerical problems.

In principle, many OPF-methods are criticized for changing more or less all possible variables, even if their change has only
a small added value for the system, but involves a high implementation effort on the side of the network operator [33]. Further development of the approach presented in this paper will have to take this reasonable criticism into account.

Furthermore, detailed cost models for redispatch are to be considered by linking the approaches to a market simulation. In this context, the single-period problem investigated yet will be extended to multi-period problems.

The gained abilities can also be used to model and optimize network expansion measures.

VII. REFERENCES


[31] MATLAB, 2019, version 9.7.0.1586710 (R2019b) Update 8, Natick, Massachusetts: The MathWorks Inc.
