Privacy-Preserving Decentralized Price Coordination for EV Charging Stations

Chenbei Lu
Institute for Interdisciplinary Information Sciences
Tsinghua University, Beijing 100084 China
leb20@mails.tsinghua.edu.cn

Jiaman Wu
Department of Civil and Environmental Engineering
University of California, Berkeley, CA 94720 USA
jmwu@berkeley.edu

Chenye Wu
School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, Guangdong 518172 China, and the Shenzhen Institute of Artificial Intelligence and Robotics for Society, Shenzhen, Guangdong 518129 China
chenyewu@yeah.net

Abstract—Price competition among electric vehicle (EV) charging stations is as fierce as the competition among gas stations. Nash equilibrium (NE) is a solution concept that can characterize a competition’s efficient and stable state. However, calculation of the equilibrium is often time-consuming and requires complete information on the charging stations. Rapidly changing charging stations often hinder reaching equilibrium. In this study, we analyze price competition with service capacity constraints and use an ordinal potential game framework to investigate the structure of the competition. By constructing the ordinal potential function, the equilibrium characterization is converted to identifying the solution through a single-objective optimization. We further propose a decentralized algorithm to enable effective price coordination to achieve equilibrium with maximized social welfare. To preserve the privacy of charging stations from internal collusion and external attacks, an advanced secure multi-party computation technology known as the Paillier Cryptosystem is customized for our proposed decentralized algorithm. Numerical studies based on field data suggest the significance of our framework.

Index Terms—EV Charging; Potential Game; Decentralized Control; Secure Multi-Party Computation

I. INTRODUCTION

Increasing public awareness of global warming has driven a low-carbon transition across industries at an unprecedented rate. In the transportation sector, which accounts for 20% of total carbon emissions worldwide, the electrification of vehicles has become an essential means of achieving low-carbon targets in most countries. The UK government has formally implemented a sales ban on petrol and diesel cars by 2030 [1]; the US government also released an executive order to regulate the market price with certain privacy-preserving abilities and has already been applied to coordinate EV charging stations profits [5], and EV coordination based on mean-field game [6]. However, optimal cooperative pricing is often impractical because charging stations in a given region are often competitors. To understand how charging stations set prices, different classical game theoretical models have been proposed, including the hierarchical game [7], the supermodular game [8], and the Stackelberg game [9], where the existence and uniqueness of the Nash equilibrium (NE) are proven to produce the stable prices. Lu et al. analyzed a storage-aided pricing game, with a concrete characterization of the equilibrium prices in [10]. However, NE is often difficult to reach in practice for two reasons. First, charging stations may not be willing to share all information. Thus, games often have incomplete information. Second, the charging demand changes rapidly with time, in stark contrast to the self-adjustment process of the market price. Decentralized control [11] is an efficient way to regulate the market price with certain privacy-preserving abilities and has already been applied to coordinate EV charging. Contreras-Ocaña et al. proposed a Dantzig-Wolfe decomposition to solve the building-EV demand-scheduling problem in a decentralized fashion in [12]. There are limited studies on the decentralized coordination of charging fees. Knirsch et al. presented reliable, decentralized, and privacy-

Chenye Wu is the corresponding author. This work was supported in part by Shenzhen Institute of Artificial Intelligence and Robotics for Society.
preserving bidding for EV charging supply-demand matching in [13]. This is the first study to consider price competition among EV charging stations in a decentralized fashion to the best of our knowledge. The privacy-preserving properties of decentralized control can be enhanced by several important branches of secure multi-party computation [14], including homomorphic encryption [15], garbled circuit [16], and zero-knowledge proof [17]. Most decentralized control algorithms are designed to handle decoupled constraints, e.g., [18] and [19]. However, our competition is associated with coupled constraints, which makes the naive application of the classical algorithm impractical. In addition, we want to point out that there is a separate body of literature focusing on applying differential privacy in the electricity sector, including non-intrusive load monitoring [20], load profiling [21], etc.

In this study, we analyze price competition considering the service capacity constraints of each station. We prove that it is by nature an ordinal potential game with a unique NE. By constructing the potential function, we transform characterizing the NE into solving a single-objective optimization. Furthermore, we propose a carefully designed decentralized algorithm customized for the game to solve the NE efficiently. We further combine the secure multi-party computation technology known as the Paillier Cryptosystem [22] with the decentralized coordination algorithm to preserve the privacy of charging stations from internal collusion and external attacks. The following diagram in Fig. 1 illustrates the logical flow of this study.

**A. EV User Model**

EV user charging demand $D$ changes with time. However, to simplify the analysis, we assume that the changing demand can be predicted accurately (by the operator) in the previous time period using modern predictive methods.

Different charging stations have different degrees of attraction to users. For example, the queuing time before charging, the charging price, and the geographic location may influence a station’s attraction. This attraction can also be seen as the relative utility that charging stations bring to EV users. We denote the relative utility of choosing station $k$ by $U_k$.

For users with multiple choices, the Logit model [23] is a widely used model that can capture the essence of the stochastic choice process. Given the utility set $U = \{U_1, U_2, ..., U_N\}$, the percentage of EV users who choose station $k$ is dictated by the Logit model as

$$f_k(U_k) = e^{\mu_k} \left( \sum_{i=1}^{N} e^{\mu_i} \right)^{-1}, \forall k,$$  

where $\mu$ reflects the degree of rationality in choosing a charging station. Intuitively, users with a smaller $\mu$ choose the station with the highest utility with a higher probability. Users with a larger $\mu$ choose different charging stations with equal probabilities.

**B. EV Charging Station Model**

Charging stations provide charging services to EV users and collect service fees. The grid electricity price is $\pi$. To maximize their revenues, $N$ charging stations compete to attract EVs by setting proper charging prices, which directly influences the EV utility. We assume that the EV utility is of the following linear form:

$$U_k(p) = L_k - p,$$  

where $p$ denotes the charging price, and $L_k$ denotes station $k$’s intrinsic attraction in addition to the price (e.g., the location of the station, the efficiency of the service, and infrastructures within the station). The attraction factor $L_k$ is assumed to be the privileged information of each charging station.

The competition is constrained by the number of charging piles at the charging stations. The service capacity of station $k$ is denoted as $C_k$; the EV utility for choosing station $k$ can be characterized as follows:

$$U_k(p_k) = (L_k - p_k) \cdot \min \left( 1, C_k \cdot (f_k(U_k))^{-1} \right),$$  

This equation denotes the expected utility for EVs choosing station $k$. Specifically, for any EV, it will be successfully charged with probability $\min \left( 1, C_k \cdot (f_k(U_k))^{-1} \right)$ and gain utility $L_k - p_k$. If not successfully charged, its utility will be 0. The expectation term yields the result.
III. Price Competition

Considering a period with total charging demand $D$, given all the charging prices of the $N$ stations $P = \{p_1, p_2, \ldots, p_N\}$, station $k$'s expected attracted demand $D_k$ is expressed as

$$D_k = D f_k(U_k) \min \left(1, C_k \cdot (f_k(U_k)D)^{-1}\right).$$

(4)

This allows us to characterize the corresponding revenue $R_k$ for charging station $k$:

$$R_k = D(p_k - \pi)f_k(U_k) \min \left(1, C_k \cdot (f_k(U_k)D)^{-1}\right).$$

(5)

This revenue depends on both its price $p_k$ and the decisions of other charging stations. Using the revenue function as the utility function for each charging station naturally leads to a game among the charging stations. We formally introduce this game as follows:

**Price Competition Game (PCG):**

- **Players**: The charging stations;
- **Strategy Space**: The collection $p = [p_1, \ldots, p_N]$ of all stations' charging prices. Specifically, $p_k > \pi, \forall k$;
- **Utility Functions**: The expected revenue $R_k$ of each charging station $k$ defined in Eq.(5).

The difficulty in analyzing the PCG lies in the non-differentiable $\min$ operator in utility functions. To address this challenge, the case without service capacity constraints is analyzed to understand the game structure before characterizing the properties of NE in the service capacity-constrained case.

**A. No-Capacity Constraints Case**

Assuming that each charging station has infinite service capacity ($C_k \to \infty$), we can infer that:

$$\min \left(1, C_k \cdot (f_k(U_k)D)^{-1}\right) = 1, \forall k. \quad (6)$$

This implies that each station utility function $U_k(p_k)$ becomes continuous in $p_k$. We can show the existence, uniqueness and other properties of the NE:

**Theorem 1**: The general PCG without service capacity constraints has a unique NE [10]. At the equilibrium, the charging stations’ price strategies $P^* = \{p^*_1, \ldots, p^*_n\}$ are expressed as

$$p^*_k = \pi + \mu(1 - \hat{f}_k(p^*_k))^{-1}, \forall k. \quad (7)$$

The proof is provided in Appendix A. $\hat{f}_k(p^*_k)$ and $f_k(U_k)$ are identical in this case, we use the former to directly characterize the influence of $p^*_k$ on the market share $f_k$. It indicates the equilibrium charging price $p^*_k$ has a uniform lower bound $\pi + \mu$. Furthermore, we observe that $p^*_k$ increases linearly in $(1 - \hat{f}_k(p^*_k))^{-1}$, indicating that a charging station with high attractiveness will actually set a higher price in the equilibrium. Despite with a higher price, it still attracts more users.

The intrinsic attraction $L$ is an inherent factor that influences equilibrium prices. Specifically, it holds that:

$$p^*_k = \pi + \mu \left(1 + \mathcal{W}_n \left(e^{\frac{L_k - \pi - \mu}{\mu}} \cdot q^*_k\right)\right), \forall k. \quad (8)$$

where

$$q^*_k = \left(\sum_{i \neq k} e^{\frac{L_i - \pi - \mu}{\mu}}\right)^{-1}, \quad (9)$$

where $\mathcal{W}_n$ denotes the Lambert W function [24]. According to its property, $p^*_k$ is nearly linear in $L_k$, which reveals how investment in intrinsic attraction is attached to the price.

From the perspective of total revenue in the NE, we have the following corollary:

**Corollary 1**: The expected revenue of station $k$ can be expressed as:

$$R_k = D \cdot (p^*_k - \pi - \mu), \forall k. \quad (10)$$

This is a direct result of Theorem 1. It reveals that to ensure a positive revenue during the competition, the surcharge must be set to at least $\mu$. There is a strict linear relationship between revenue and price. Together with Eq. (8), we can infer that $L_k$ affects the revenue $R_k$ almost linearly.

Combining Theorem 1 and Corollary 1, we can obtain the following relationship between revenue $R_k$ and market share $f_k(p^*_k)$:

$$R_k = D \cdot \mu \hat{f}_k(p^*_k)(1 - \hat{f}_k(p^*_k))^{-1}, \forall k. \quad (11)$$

Eq.(11) indicates a superlinear speed of increase in revenue with market share, and directly produces the incentive for charging stations to expand their market share.

**B. Capacity-Constrained Case**

In this subsection, we characterize the NE considering service capacity constraints. The non-differentiable term introduced by service capacity constraints significantly increases the difficulty of characterizing the NE.

However, this difficulty can be partially reduced by a simple and intuitive observation; at the NE, no charging station will set a price to attract demand exceeding its service capacity. Mathematically,

$$f_k(U_k) \cdot D \leq C_k, \forall k. \quad (12)$$

We can also observe that under condition (12), the non-differentiable term automatically disappears, yielding the following new game:

**Price Competition Game+ (PCG+)**

- **Players**: The charging stations;
- **Strategy Space**: The collection $p = [p_1, \ldots, p_N]$ of all stations’ charging prices. Specifically, $p_k > \pi, f_k(U_k) \cdot D \leq C_k, \forall k$;
- **Utility Functions**: The expected revenue $R_k$ of each charging station $k$ satisfies:

$$R_k = D \cdot (p_k - \pi)f_k(U_k), \forall k. \quad (13)$$

The equivalence of PCG and PCG+ is straightforward; PCG+ transforms the non-differentiable term into the constraints of the game, which makes it easier for us to analyze the NE.


C. Potential Game-based Analysis

With the more tractable PCG+, analyzing and calculating the NE is still difficult because all decision variables \( p_i \) are coupled in the utility function and the constraints. Solving the NE is a complex multi-objective optimization of charging station utility functions. The ordinal potential game [25], [26] is a useful concept; a preliminary introduction is presented in Appendix B.

By analyzing the form of each charging station utility function \( R_k \), we construct the following ordinal potential function for PCG+.

**Theorem 3**: The general PCG+ is an ordinal potential game with an ordinal potential function \( \Phi(p) \):

\[
\Phi(p) = D \cdot \left( \prod_{i=1}^{N} (p_i - \pi) e^{\frac{L_i - p_i}{\mu}} \right) \cdot \left( \sum_{i=1}^{N} e^{\frac{L_i - p_i}{\mu}} \right)^{-1}.
\] (14)

Theorem 3 can be verified by confirming the definition of the ordinal potential function. When \( p_k \) changes, all the other \( p_i \) and \( e^{\frac{L_i - p_i}{\mu}} \) do not change in \( \Phi(\cdot) \). The only changed term in \( \Phi(\cdot) \) is the utility function \( R_k \) of the charging station \( k \). Thus, \( \Phi(\cdot) \) changes proportionally to \( R_k \) when \( p_k \) changes. We can deduce that the ordinal potential function \( \Phi(\cdot) \) combines multiple user utility functions into a single optimization objective. According to the properties of the potential game, the local minimum of the potential function conforms to the NE. This is crucial for deriving the following theorem to characterize the NE:

**Theorem 4**: The PCG+ game has a unique NE, which is the solution to the following quasi-convex optimization problem.

\[
\begin{align*}
\max_p & \quad D \cdot \left( \prod_{i=1}^{N} (p_i - \pi) e^{\frac{L_i - p_i}{\mu}} \right) \cdot \left( \sum_{i=1}^{N} e^{\frac{L_i - p_i}{\mu}} \right)^{-1} \\
\text{s.t.} & \quad\hat{f}_k(p_k) D \leq C_k, \forall k, \quad (15) \\
& \quad p_k \geq \pi, \forall k. \quad (16)
\end{align*}
\]

The proof is provided in Appendix C. This theorem provides an explanation of NE in terms of optimization, and also advises us on how to characterize it. We can deduce the properties of this unique NE as follows.

**Theorem 5**: At equilibrium, the charging station pricing strategies \( P^* = \{p^*_1, ..., p^*_N\} \) and the average market price \( \bar{p}^* \) satisfy:

\[
\begin{align*}
& p^*_k = \pi + \mu \left( 1 - \hat{f}_k(p^*_k) \right)^{-1}, \forall k, \quad (18) \\
& \bar{p}^* \geq \pi + \sum_{k=1}^{N} \frac{\hat{f}_k(p^*_k) \mu}{1 - \hat{f}_k(p^*_k)} \geq \pi + \frac{N \mu}{N - 1}. \quad (19)
\end{align*}
\]

This theorem can be proved by checking equation (7) and the first-order optimality condition of \( R_k \) when service capacity constraints are binding or not. The inequality become equality in Eq. (18) if the capacity constraint of station \( k \) is not binding. The inequality become equality in Eq. (19) if all station capacity constraints are not binding. The second equality holds if the market is symmetric, i.e., all stations share the same attraction \( L \). Theorem 5 suggests that an asymmetric market structure and the service capacity limitation drive the market prices up.

D. Centralized Price Coordination

The NE indicates a steady and efficient state of competition. However, it is difficult for the market to efficiently stabilize by itself. These difficulties are twofold. First, gathering information from other charging stations (price \( p_k \), relative attraction \( L_k \), and capacity \( C_k \)) is difficult, especially attempting to preserve privacy. Second, even with all information gathered, the best response dynamics are often slow.

To promote the competition to quickly reach a stable state while protecting the private information of the charging stations, a trusted central operator is used to collect and store charging station information, and then coordinate prices to reach the NE whenever the charging demand \( D \) changes.

The ordinal potential function transforms the original multi-objective optimization into a single-objective optimization that is more tractable and computationally efficient. The designed centralized price coordination is presented in Algorithm 2 in Appendix D.

IV. Secure Decentralized Price Coordination

In practice, the trusted third-party assumption is often too strong; thus, we design a decentralized algorithm with a privacy-preserving guarantee. By reasonably allocating computation tasks to each charging station and transmitting necessary information in ciphertext, carefully designed decentralized coordination efficiently achieves high security. Fig. 2 illustrates the decentralized coordination process. In the following subsections, the main process of the decentralized algorithm is introduced and the multi-party secure computation technology used in the algorithm is carefully explained.

---

Fig. 2: Illustration of Decentralized Coordination

A. Price Coordination Algorithm

We first linearize the constraints for subsequent design. We denote \( e^{\frac{L_k - p_k}{\mu}} \) as \( y_k, y = [y_1, ..., y_N] \). The equivalent optimization problem is expressed as

\[
\begin{align*}
\max_y & \quad H(y) = \left( \sum_{i=1}^{N} y_i \right)^{-1} \prod_{i=1}^{N} (L_i - \pi - \mu \ln y_i) y_i \\
\text{s.t.} & \quad D \cdot y_k \leq C_k \sum_{i=1}^{N} y_i, \forall k, \quad (21) \\
& \quad y_k > 0, \forall k. \quad (22)
\end{align*}
\]
The feasible region is denoted by $Y_f$; its convexity can be easily confirmed. Using a method similar to that for the proof of Theorem 4, we conclude that the objective function in Eq. (20) is quasi-concave. Thus, the equivalent optimization problem is quasi-convex. We can also verify that the objective function is Lipschitz continuous, which is crucial for the decentralized design. With the Lipschitz constant represented by $K$,

$$
H_k(y)|_{y_k=z} - H_k(y)|_{y_k=z} \leq K|x-z|, \forall x, z, k.
$$

where $H_k(y) = \frac{\partial H(y)}{\partial y}$; $\mu$ is assumed to be the public information that can be inferred from historical data. Decentralized price coordination is detailed in Algorithm 1.

Before iteration, charging stations construct a public key $g$ to encrypt information exchanged with other charging stations. The operator collects service capacity $C_k$’s, initializes $y_k$’s and broadcasts the step size $\gamma$.

In each iteration $t$, the operator broadcasts the feasible variables $y_k$’s to each charging station. Each charging station $k$ decrypts $y_k$ by $g$ in step 6. Next, charging stations conduct secure multi-party computation to calculate two important signals, $G_{1,k}$ and $G_{2,k}$, in steps 9 and 10 to help calculate the gradient while protecting the privacy of each charging station. After that, charging stations use the calculated signals to conduct the gradient descent and encrypt the result by $g$ in steps 12 and 13. As the gradient descent results may not satisfy the constraints, the operator makes the solution feasible according to step 14 and determines whether the solution converges.

It is observed that the operator has no access to $L_k$ because the transmitted $y_k$ is encrypted by $g$. The feasibility of such encryption comes from the homogeneity in the optimization problem in step 14 and the linearity of the constraints. Thus, this design prevents information leakage when the operator is attacked. Furthermore, a secure algorithm can also be internal collusion-free. This property is guaranteed by the secure multi-party computation technology in step 10, which is discussed in detail in the next subsection. In addition to the security, we prove that the decentralized price coordination algorithm converges to the NE. We first prove an important lemma that assists in the proof:

**Lemma 1:** For all $t \geq 1$, the objective function $H(y^t)$ decreases monotonically, and satisfies the following inequality:

$$
H(y^{t+1}) - H(y^t) \leq -\left(\frac{1}{\gamma} - \frac{K}{2}\right)||y^{t+1} - y^t||_2^2. \quad (24)
$$

The proof can be found in Appendix E. It states that the decreasing rate of the objective function is lower bounded, which is sufficient to prove the convergence of Algorithm 1.

**Theorem 6:** Algorithm 1 converges to the NE, i.e., $\lim_{t \to \infty} \sum_{i=1}^{N}(y_{i}^{t+1} - y_{i}^{t})^2 \to 0$.

The proof can be found in Appendix F.

This algorithm is a variant of the gradient projection method [27]. We customize a decentralized algorithm for price coordination by transferring the gradient calculation portion to each charging station, with two major benefits: computation tasks are evenly distributed to all charging stations, which improves the overall computational efficiency, and the privacy-preserving design is improved.

### B. Secure Multi-Party Computation

In addition to protection from external attacks, preventing internal collusion is highly desirable. The multi-party computation task in step 10 of Algorithm 1 is an addition task for charging stations. Specifically, each station $k$ has two secret numbers $G_{1,k}$ and $G_{2,k}$. It is desired to calculate the summations $\sum_{k=1}^{N}G_{1,k}$ and $\sum_{k=1}^{N}G_{2,k}$ without revealing any secret numbers.

This is a classical secure addition task that has been thoroughly studied in cryptography. Additive homomorphic encryption is the classical procedure; we adopt a famous addictive homomorphic encryption algorithm known as the Paillier Cryptosystem. The details of the algorithm are provided in Algorithm 3 in Appendix G.
Similar to the well-known RSA encryption [28], the security of the Paillier Cryptosystem is guaranteed by the complexity of prime factorization. In our case, the collusion of up to $N-1$ charging stations can be resisted. This is the best possible result; when there is a collusion of $N-1$ charging stations, the secret number of the remaining charging station can be calculated by simply subtracting the secret number of each station from the summation. The correctness of the decryption originates from the additive homomorphic properties. Formally, the following theorem holds.

**Theorem 7**: The product of two ciphertexts $\mathcal{E}(m_1), \mathcal{E}(m_2)$ is decrypted as the summation of their corresponding plaintexts $m_1 + m_2$ [22]:

$$D(\mathcal{E}(m_1) \cdot \mathcal{E}(m_2) \mod n^2) = (m_1 + m_2) \mod n^2.$$ (25)

Thus, we can conduct $N-1$ times of homomorphic additions using the Paillier Cryptosystem to achieve secure multi-party computation, ensuring security against internal collusion. The whole algorithm can protect user privacy from external attacks and internal collusion.

V. **Numerical Studies**

In this section, we first seek to understand how the equilibrium market price is influenced by the service capacity $C_k$ and the time-varying demand $D$. We then verify that the decentralized coordination algorithm achieves fast convergence, and illustrate the security of the Paillier Cryptosystem.

A. **Experimental Settings**

We consider ten charging stations located in San Francisco, CA. Their geographical distributions are shown in Fig. 3. The intrinsic attractions $L_k$ of the EV stations are estimated based on their location values, indicated by the house prices in their location. The charging capacity is estimated based on the number of charging piles.

The total charging demand is based on the average demand profile of EVs across the US [29], one type of which is characterized in Fig. 4. The electricity price is from the time-of-use (ToU) pricing scheme of Pacific Gas and Electric Company [30], which is illustrated in Fig. 5.

![Fig. 3: EV Charging Station Distribution](image3)

![Fig. 4: EV Demand during a Day (Work-Home)](image4)

![Fig. 5: ToU Electricity Price](image5)

B. **Effect of Supply-Demand on Equilibrium Price**

In this subsection, we investigate how the ToU price and time-varying demand affect the equilibrium prices. We consider both the work-home demand case and the uncontrolled demand case. Furthermore, we investigate how the station charging capacity influences the equilibrium prices.

Fig. 6 illustrates the equilibrium market prices with work-home charging demand. We can observe that the overall market price trend is similar to that of ToU price. Dominant charging stations with higher $L_k$ set higher charging prices throughout the day. When the EV charging load is low, the equilibrium price changes minimally with demand. For example, from 1 am to 6 am, the market prices change little despite the great increase in charging demand. During high charging load periods from 7 am to 9 am, market prices fluctuate wildly, and the pricing differences between charging stations widen because the overall charging service capacity is nearly saturated at peak times. Charging stations with demand exceeding supply set higher prices, while those with excess service capacity tend to set lower prices to attract more EV users. This result is consistent with our theory.

Fig. 7 illustrates the equilibrium prices in uncontrolled demand cases. A similar pricing fluctuation occurs in the peak hours around 8 pm, but it is much smaller because the peak demand is smaller than in the work-home demand case.

Market price fluctuations occur during peak periods. We investigate how the charging station service capacity affects price fluctuations. Fig. 8 illustrates the impact of station service capacity on equilibrium prices during peak demand. When the station service capacity decreases, a significant increase in the scale of market price fluctuations is observed, indicating the significance of improving charging infrastructure to stabilize market prices.

C. **Convergence of Decentralized Algorithm**

In this subsection, we simulate the decentralized price coordination algorithm and evaluate its convergence efficiency and anti-attack capability. The stopping criteria $\epsilon$ is set as $10^{-5}$.
EV users. Specifically, we plan to employ chance constraints among charging stations by ordinal potential game theory, and for fast market regulation and privacy protection, we use a Paillier Cryptosystem-based privacy-preserving decentralized price coordination algorithm that guarantees convergence while resisting internal collusion and external attacks.

Our work can be extended in various directions. For example, it is interesting to consider a more general price competition game among charging stations by ordinal potential game theory, and deduce critical properties of the associated NE. To achieve fast market regulation and privacy protection, we use a Paillier Cryptosystem-based privacy-preserving decentralized price coordination algorithm that guarantees convergence while resisting internal collusion and external attacks.

VI. CONCLUDING REMARKS

In this paper, we analyze the price competition game among charging stations by ordinal potential game theory, and deduce critical properties of the associated NE. To achieve fast market regulation and privacy protection, we use a Paillier Cryptosystem-based privacy-preserving decentralized price coordination algorithm that guarantees convergence while resisting internal collusion and external attacks.

Our work can be extended in various directions. For example, it is interesting to consider a more general price competition game with dynamic grid prices and heterogeneous EV users. Specifically, we plan to employ chance constraints or robust optimization to incorporate uncertainties of dynamic prices into our design framework. It is also meaningful to consider more services that EVs could contribute to the grid, e.g., frequency regulation and voltage regulation.

REFERENCES


[13] Fabian Knirsch, Andreas Unterweger, and Dominik Engel. Privacy-preserving blockchain-based electric vehicle charging with dynamic tar-


C. Proof for Theorem 4

First, we prove the problem is quasi-convex. We define

\[ x_i = \frac{p_i - \pi}{\mu}, \forall 1 \leq i \leq N, \]

\[ a_i = e^{\pi}, \forall 1 \leq i \leq N, \]

\[ \beta = \mu D^2 \cdot e^{\sum_{i=1}^{N} L_i - (N-1)x_s}. \]

The objective function is transformed into:

\[ F(x) = \sum_{i=1}^{N} a_i e^{-x_i} \]

We prove that the simplified function is a quasi-concave function by checking that its s-upper-level set is convex:

\[ S = \{x \mid F(x) \geq s\}, \forall s \geq \min F(x). \]

Consider two solutions \( x_1 = [x_1, x_{12}, ..., x_{1N}] \) and \( x_2 = [x_2, x_{22}, ..., x_{2N}] \) satisfying:

\[ \beta \prod_{i=1}^{N} x_i e^{-x_i} \geq s, \]

\[ \sum_{i=1}^{N} a_i e^{-x_i} \geq s. \]

Taking the logarithm on both sides yields:

\[ \sum_{i=1}^{N} \ln x_i - \sum_{i=1}^{N} x_i \ln \left( \sum_{i=1}^{N} a_i e^{-x_i} \right) \geq \ln \left( \frac{s}{\beta} \right), \]

\[ \sum_{i=1}^{N} \ln x_i - \sum_{i=1}^{N} \ln \left( \sum_{i=1}^{N} a_i e^{-x_i} \right) \geq \ln \left( \frac{s}{\beta} \right). \]

For any \( x^* = \alpha x_1 + (1-\alpha) x_2, \forall 0 < \alpha < 1, \) \( F(x^*) \) satisfies:

\[ \ln \left( \frac{F(x^*)}{\beta} \right) = \alpha \ln \left( \frac{F(x_1)}{\beta} \right) + (1-\alpha) \ln \left( \frac{F(x_2)}{\beta} \right) + H_1(\alpha) + H_2(\alpha), \]
D. Centralized Price Coordination Algorithm

Algorithm 2 Centralized Price Coordination
1: The operator collects each charging station $k$’s relative utility $L_k$ and service capacity $C_k$;
2: Initialize a price collection $p^{(0)}$ satisfying constraints (16)-(17);
3: Solve the quasi-convex optimization problem (15)-(17) from the given initial point $p^{(0)}$;
4: Return the solution $p^*$ and broadcast it to each charging station;

E. Proof for Lemma 1

Step 14 indicates that $y_{t+1}^k$ minimizes $\sum_{i=1}^{N}(y_i - y_i^t + \gamma H_i^t(y_i^t))^2$ over all $y \in \mathcal{Y}_f$. The quasi-convexity of $H(y)$ together with the convexity of the feasible region yields that for all $y \in \mathcal{Y}_f$, it holds:

$$\sum_{i=1}^{N}(y_i - y_i^t + \gamma H_i^t(y_i^t))^2 \geq 0, \quad (41)$$

$$\sum_{i=1}^{N}(y_i^{t+1} - y_i^t + \gamma H_i^t(y_i^t))(y_i - y_i^t) \geq 0. \quad (42)$$

Let $y_i$ be $y_i^t$, we have:

$$\sum_{i=1}^{N}(y_i^{t+1} - y_i^t + \gamma H_i^t(y_i^t))(y_i - y_i^t) \geq 0. \quad (43)$$

Denoting $g(z) = (1 - z)y^t + zy^{t+1}$ yields:

$$H(y^{t+1}) = H(y^t) + \int_0^1 \sum_{i=1}^{N}(H_i^t(q_i(z)))(y_i^{t+1} - y_i^t)dz \quad (44)$$

Combined with Eq.(43), the lemma immediately follows. ■

F. Proof for Theorem 6

As $H(y^t)$ decreases with increasing $t$, we know that $H(y^t)$ converges, and hence $y^*$ will also converge. Using the optimality condition of the optimization, if $y_t = y_e^t$. Eq.(41) holds, indicating that $y^*$ is a local optimum. As a property of quasi-convex optimization, the unique local optimum is the global optimum. Thus, Algorithm 1 converges to the NE. ■

G. Paillier Cryptosystem

The Paillier Cryptosystem is described in Algorithm 3.

Algorithm 3 Paillier Cryptosystem

**Key Generation:**
1: Randomly choose two large primes $p$ and $q$ with equal bit-length; compute $n = pq$;
2: Calculate $g = n + 1$, $\lambda = \phi(n)$, where $\phi(n) = (p - 1)(q - 1)$;
3: Calculate $\mu = \phi(n)^{-1} \mod n$, where $(\cdot)^{-1}$ denotes the modular multiplicative inverse;
4: The public key (for encryption) is $(n, g)$;
5: The private key (for decryption) is $(\lambda, \mu)$;

**Encryption** $(c = E(m))$:
1: Let $m$ be a message waiting for encryption, where $m \in \mathbb{Z}$, $0 \leq m < n$;
2: Randomly choose $r$ where $r \in \mathbb{Z}$, $0 \leq m < n$ and $gcd(z, n) = 1$;
3: The ciphertext $c$ is calculated by $c = g^m \cdot r^n$;

**Decryption** $(m = D(c))$:
1: The plain text $m$ is calculated by $m = L(c^\lambda \mod n^2) \cdot \mu \mod n$, where $L(u) = (u - 1)/n$;

Submitted to the 22nd Power Systems Computation Conference (PSCC 2022).