Abstract—Rising market participation of DER through aggregators increases the risk of network constraint violation at the distribution level. Existing approaches to network-secure DER coordination often require centralised resource management/oversight, or involve complex iterative negotiations between prosumers and the distribution service operator (DSO). In response to industry needs for readily-implementable solutions, we propose a straightforward network-secure bid curtailment approach performed by DSOs, allowing for separation of DSO and aggregator roles while factoring aggregator preferences communicated through bids. In extension to previous work, our approach directly maximises expected aggregator benefit in the market by factoring forecast market prices and bid prices into the objective. We mitigate risks of inaccurate market forecasts using a price-probabilistic stochastic program, bringing benefit to within 1% of the perfect information case in our simulations using real data. We demonstrate a 9% improvement in aggregator benefit compared to related approaches, with further gains available through real-reactive power co-optimisation.

Index Terms—Stochastic optimisation, bid curtailment, aggregators, distribution network, energy and reserve markets

I. INTRODUCTION

Accelerating uptake of distributed energy resources (DER) is driving decentralisation of power generation. Rooftop solar, residential batteries and community-scale projects offer sustainable and increasingly cost-effective opportunities to reduce reliance on conventional energy sources. The gradual waning of centralised dispatchable generation requires DER to play a greater role in ensuring consistency of supply and system security, through energy arbitrage and reserve services. This is particularly needed in the context of increased intermittent generation and declining system inertia.

Aggregators are entering electricity markets as intermediaries for customers with DER, enabling provision of market-scale services. Unlike centralised generation, DER services must pass through restrictive low- and medium-voltage networks before accessing transmission networks. Increasing DER market participation therefore poses growing risks of current and voltage limit violations at the distribution level. Wholesale markets lack visibility over the state and operational constraints of distribution networks, and are therefore unable to clear aggregator bids in a distribution-safe way when DER participation is high.

Herein lies the challenge of extracting maximum market benefit from DER, crucial to facilitate transitions to low-carbon decentralised power systems, while simultaneously ensuring the security of distribution networks. This challenge is compounded by the need for readily-implementable solutions which easily integrate into existing market structures, without incurring significant transition costs undermining the market value of DER services. This paper addresses these challenges by proposing a framework within which distribution system operators (DSOs) can curtail aggregator bids for network security in a market-efficient manner, without significant disruption to existing market structures.

Coordination of DER at the distribution network level has been studied in a variety of contexts. We begin our review of literature by broadly categorising existing approaches as follows: central control, distributed locational marginal price approaches and operating envelope calculation.

Central control methods, e.g., [1]–[4], assume the DSO, or an equivalent entity, has full visibility over the state of DER in the network, and full control over their deployment. This allows for a comprehensive optimisation of real and reactive power utilisation, however the requirements for full system visibility and control are incompatible with our chosen case study, in which multiple aggregators bid in competition for market access. This category also often overlooks the preferences of individual aggregators/prosumers.

Distributed locational marginal pricing (DLMP) approaches, e.g., [5]–[7], do not require full network control and do consider the private preferences of aggregators. In this framework, the DSO facilitates private iterative bidding by aggregators for constrained capacity, updating locational marginal pricing with each iteration. This method is capable of achieving optimal solutions; however, the industry are unlikely to embrace this iterative method in practice due to its relative complexity of implementation. The coupling of network and aggregator sub-problems requires significant amounts of fast communication, and dictates to a large extent how aggregators must represent and solve their sub-problems. There is a need to develop simpler alternatives that have a greater chance of being adopted by the power system industry which already has a lot of complexity to manage.

Operating envelope approaches, e.g., [8], [9], are less intrusive than central control methods and simpler to implement than DLMP methods. In this framework, each connection point is assigned a safe operating region in the real-reactive power
plane, calculated either according to simple rules or by solving a constrained optimisation problem. The simplest approach is allocation of permanent envelopes at every connection point; however, this leads to overly-restrictive outcomes based on worst-case scenarios which cannot account for evolving circumstances and preferences. Dynamic operating envelopes have been proposed to generate less restrictive curtailments using information such as aggregator bids [10]. Our approach fits within this definition.

In this paper we propose a framework for DSOs to efficiently curtail aggregator multi-market bids with cross-market dependencies to avoid network constraint violation. We do so in a manner which maximises the value of services provided to the market, while incentivising competitive bidding amongst aggregators. This is achieved by maximising the expected benefit to aggregators through a scenario-based stochastic program. The DSO shapes bids in advance of each trading period, ensuring that only network-secure bid combinations are eventually submitted to the market operator.

Compared to central control and DLMP-based approaches, this approach provides the following advantages:

1) Clear role separation between aggregators, DSO and market operators, and the ability to work with existing wholesale market structures. Aggregators have the freedom to develop bids like conventional participants.

2) Greatly simplified constrained optimisation problem, with DER constraints decoupled from the network constraints, and which avoids iteration back and forth between the DSO and aggregators.

The trade-off is prevention of iterative bid refinement as with DLMP methods, due to the separation of bidding and curtailment processes. This is arguably positive from the industry and market perspectives as it enhances predictability and simplifies implementation compared to dynamic approaches.

The proposed approach is most related to [11] and our previous work [12], in which operating envelopes are determined at the nodal level to minimise the $l_2$-norm of power curtailments. This achieves a greater spread across nodes than under an $l_1$-norm, producing a ‘fairer’ outcome [11]. However, we argue that the primary objective should be maximising the net social welfare of the problem because the DSO can determine transfers correcting for perceived inequalities when social welfare is maximised.

The approach in [12] improved the social welfare outcome and drove competition among aggregators on a node-by-node basis by applying curtailments to the least competitive bids first. However, this was done in a relatively ad-hoc manner as a second step after nodal curtailments were determined. It does not consider relative competitiveness across nodes in a principled way, nor does it factor the likelihood of different market outcomes and different market values. This misses key opportunities to improve efficiency that smarter allocations of the scarce distribution network capacity can achieve.

To improve the market outcomes, in this paper, we obtain curtailments by directly factoring in market and bid prices to an optimisation that maximises expected aggregator benefit. This corresponds to maximising social welfare when there is sufficient aggregator competition and their bids are cost-reflective. This bid-shaping process occurs prior to the market clearing, so we formulate our problem as a stochastic program in order to optimise the allocation of curtailment according to likely market outcomes. Moreover, we expand network throughput capacity through real and reactive power co-optimisation.

A benefit of this more principled representation of market value is our ability to consider additional market services, expanding on [12] to include frequency regulation on top of energy and contingency services. This increases the number of simultaneously-dispatchable markets from two to three. In this expanded setting, we move away from the restrictive two-dimensional trapezium representation of cross-market dispatch constraints used by the Australian National Energy Market (NEM). Instead, we propose a general representation of feasible dispatch using linear inequality constraints, facilitating application across a wider range of markets.

To summarise, our key contributions are:

- A variation of the nodal operating envelopes concept, that allocates capacity to aggregators based on the market value and bid value of services offered. We demonstrate this can achieve an 8% improvement to the market outcome compared to alternative operating envelope approaches that capture this value using heuristics. Importantly, we do not introduce prohibitive complexity into DSO calculations or aggregator bidding decisions.

- A stochastic programming extension which further increases benefit relative to alternatives. This brings the performance to within less than 1% of what the hypothetical perfect information case obtains in our simulations.

In Section II we present our mathematical framework for shaping bids using a single reliable price forecast in each market. In Section III we discuss risks of inaccurate price forecasts and propose a stochastic formulation using estimates of price probabilities. In Section IV we implement our framework on a single-phase MV network using real data and compare to literature. We conclude in Section V.

II. PRICE-DETERMINISTIC SHAPING USING FORECASTS

This section develops an approach to shaping aggregator bids using a confident prediction of market price values $\rho \in \mathbb{R}^{N_m}$, where $N_m \in \mathbb{N}$ is the total number of markets operating concurrently.\(^1\) We call this approach *price-deterministic* in

\(^1\)As a notation guide,

- Bold font is applied to vectors and matrices only. Scalar entries of vectors/matrices are not bolded, but sub-matrices and sub-vectors remain bolded.
- Matrices and vectors: subscripts are used for indexing entries or sub-matrices, and superscripts are used for labelling variables.
- All other mathematical entities (scalars, functions, sets): subscripts and superscripts are used for labelling. In these cases superscripts help to convey physical meaning, and subscripts identify the instance (often indexed) of node or bid characterised by the variable.
the sense it optimises benefit assuming zero uncertainty in market price. Curtailment results are network-secure for any potential market price outcomes, even if the price forecasts are inaccurate.

A. Problem Setting

Our approach begins with aggregators submitting provisional bids to the DSO, on a node-by-node basis, in advance of the upcoming market trading period. In the context of \( N^a \in \mathbb{N} \) aggregators operating in a network of \( N^i \in \mathbb{N} \) nodes, there are \( N^a \leq N^i \times N^n \) bids submitted to the DSO. These \( N^a \) bids can be sorted into \( N^i \) subsets, labelled \( B_i \) for \( i \leq N^i \), containing the indices \( n \) of all market bids \( b_n \) at node \( i \) across all aggregators.

Let tuples \( b_n := (\pi^n, \chi^n, c^n_r) \) represent aggregator bids. These contain bid price and capacity data in matrices \( \pi^n, \chi^n \in \mathbb{R}^{N^k \times N^m} \) respectively, where \( N^k \in \mathbb{N} \) is the number of per-market price bands. Let \( \rho \in \mathbb{R}^{N^m} \) be the vector of predicted market-clearing prices in each market. Let \( P \in \mathbb{R}^{N^m \times N^n} \) be the matrix of market-cleared dispatch outcomes in each market for each bid. It follows that column vectors \( P_{., n} \in \mathbb{R}^{N^m} \) represent market-cleared dispatch outcomes across markets for a given bid, and these must satisfy

\[
P_{m,n} \leq \sum_{b=1}^{N^a} \chi_{b,m} \tag{1}
\]
i.e. market-cleared power in a given market is bounded above by the sum of capacities bid across price bands in that market.

Each bid tuple also contains a cross-market dispatch constraint function \( c^n_r \). This constraint allows aggregators to bid their full capacity into energy, regulation and contingency markets individually without risking market dispatch across all simultaneously, which may be infeasible from the aggregator’s perspective. Specifically, \( c^n_r : \mathbb{R}^{N^m} \rightarrow \mathbb{R}^{N^i} \) is a vector-valued affine function that defines the bid’s feasible dispatch region, a convex polytope in \( \mathbb{R}^{N^m} \). Within the context of the Australian NEM, this takes on the form of the trapezium regions that link energy and reserve markets. Dispatch is feasible from the aggregator’s perspective for a bid \( n \) if and only if

\[
c^n_r(P_{.,n}) \leq 0 \tag{2}
\]

B. Objective Maximising Aggregator Benefit

We define benefit to aggregators as the difference between the market value of traded services and the costs to aggregators of providing those services. We assume that bid prices reflect expected costs to aggregators of delivering capacity. In our objective function, we partition the set of all market indices \( M = \{1, \ldots, N^m\} \) according to whether aggregators pay or are paid market clearing prices in the associated market. This distinguishes the energy load market, indexed by \( m^- \in M \), from the energy generation market and all reserve markets, indexed by values in the set \( M^+ = M \setminus \{m^-\} \).

For a single aggregator, at a single node (i.e. a single bid), benefit gained in one trading period is equal to

\[
f_n(P_{.,n}, \rho, b_n) = \left( \sum_{m \in M^+} \left( \rho_m |P_{m,n} - \Pi^m_{m,n}(P_{m,n})| - \frac{\rho_m}{l} |P_{m,n} + \Pi^m_{m,n}(P_{m,n})| \right) \right) \times \Delta t \tag{3}
\]

where

- \( \rho_m \in \mathbb{R} \), the \( m^{th} \) entry of vector \( \rho \in \mathbb{R}^{N^m} \), is the market-clearing price in market \( m \).
- \( \Pi^m_{m,n} : \mathbb{R} \rightarrow \mathbb{R} \) calculates the cost to the aggregator of providing capacity \( P_{m,n} \) in market \( m \) according to bid price and capacity data in \( b_n \). It is convex and piece-wise linear with gradients equal to entries of column-vector \( \pi^m_n \in \mathbb{R}^{N^k} \) (the prices in each bid band) as we assume bid prices reflect aggregator costs.
- \( l \in \mathbb{R}^+ \) is a unitless loss term.
- \( \Delta t \in \mathbb{R} \), factored for readability, serves to convert power values to energy values over trading periods.

The loss term \( l \) accounts for a DSO’s chosen method to allocate the economic costs of line losses, which we leave arbitrary in this general framework. Its derivation can reflect goals including fairness or total loss minimisation. To avoid bilinearity in the objective, the value of \( l \) is pre-processed assuming an expected network operating point. 3

The overall objective is to maximise the sum of aggregator benefit. We can now define an optimisation problem for an ideal, unconstrained network case

\[
\max \sum_{n=1}^{N^a} f_n(P_{.,n}, \rho, b_n) \tag{4}
\]

s.t. (1), (2) \( \forall n \leq N^a \)

We will next build on this model to account for network constraints by shaping the provisional aggregator bids prior to their submission to the market operator.

C. Operating Envelope Constraints at Bid and Nodal Levels

We now introduce additional cross-market dispatch constraints on \( P_{.,n} \) in order to restrict bid-level power exchange with the network to be within network-feasible operating envelopes, irrespective of how regulation or contingency services may be activated across the feeder.

When considering possible power transfers, markets interact with each other in non-uniform ways. For example, in Australian NEM, only one contingency response will be active at a time per participant. However, regulation and contingency response could occur simultaneously, meaning they are additive when considering possible power transfer. All of these reserve responses are additionally relative to an underlying energy market dispatch.

To account for this, we re-partition the set of market indices \( M \) from Subsection II-B according to a new criteria. Let

\[3\]Note that network feasibility is later verified using a power flow model accurately accounting for losses; \( l \) only serves to capture cost distribution in curtailment.
$M^G \subseteq M$ represent the set of markets resulting in power export from the aggregator perspective (generation and raise reserves), and let $M^L \subseteq M$ represent the set of markets resulting in power import (load and lower reserves). We now introduce two sets, $\Psi \subseteq 2^{M^L}$ and $\overline{\Psi} \subseteq 2^{M^G}$, containing market combinations that can simultaneously provide response, one for aggregator power import and one for export.

Let variables $\underline{p}_m, \overline{p}_m \in \mathbb{R}$, entries of vectors $\underline{p}_m, \overline{p}_m \in \mathbb{R}^{N^i}$, represent bid-level lower and upper bounds for network-feasible power exchange. These are the key decision variables in our optimisation problem. We restrict power exchange at the bid level to envelopes (intervals) $[\underline{p}_m, \overline{p}_m]$ by defining the following constraints for each bid $n$

\begin{align}
\sum_{m \in \psi} - \mathcal{P}_{m,n} & \geq \underline{p}_m \quad \forall \psi \in \Psi \quad (5) \\
\sum_{m \in \psi} \mathcal{P}_{m,n} & \leq \overline{p}_m \quad \forall \psi \in \Psi \quad (6)
\end{align}

We introduce vectors $\underline{p}, \overline{p} \in \mathbb{R}^{N^i}$ with entries $\underline{p}_i, \overline{p}_i \in \mathbb{R}$ representing nodal-level lower and upper bounds for power exchange. The tightest bounds at a nodal level are obtained through the Minkowski sum, or dilution, of intervals at the bid level. In our framework, we have $0 \in [\underline{p}_m, \overline{p}_m]$ by definition. It follows that

\begin{align}
\underline{p}_i & = \sum_{n \in B_i} \underline{p}_m \\
\overline{p}_i & = \sum_{n \in B_i} \overline{p}_m
\end{align}

(7) and (8) allow the definition of network feasibility criteria for bid-level envelopes $[\underline{p}_m, \overline{p}_m]$.

**D. Network Feasibility Constraints & Reactive Power Co-optimisation**

We begin by stating network feasibility criteria for real-power nodal envelopes, and explore the expansion of network capacity through co-optimisation of real and reactive power. We use the DistFlow model for single-phase-equivalent radial distribution networks [13]. Let $p_i, q_i \in \mathbb{R}$, entries of vectors $p, q \in \mathbb{R}^{N^i}$, be the net real and reactive power exchanged at node $i$ across all aggregators. The DistFlow model states that for all nodes $i \leq N^i$ we have

\begin{align}
P_i - r_i I_i + p_i & = \sum_{j \in D_i} P_j \quad (9a) \\
Q_i - x_i I_i + q_i & = \sum_{j \in D_i} Q_j \quad (9b) \\
V_k - 2(r_i P_i + x_i Q_i) + (r_i^2 + x_i^2)I_i & = V_i \quad (9c) \\
P_i^2 + Q_i^2 & = V_i I_i \quad (9d)
\end{align}

where $D_i$ is the set of $i$’s child nodes; $P_i, Q_i$ and $I_i$ are the real, reactive power and squared current magnitude entering node $i$ from its parent node; $r_i$ and $x_i$ are the resistance and reactance of the branch linking node $i$ to its parent node, and finally $V_i$ is the squared voltage magnitude at node $i$. The DistFlow model is presented in more detail in [13]. We also constrain squared voltage at each bus by

\begin{align}
V_i \leq V_i \leq \overline{V}_i \quad (10)
\end{align}

and current along each branch by

\begin{align}
l_i \leq I_i \leq \overline{l}_i \quad (11)
\end{align}

to remain within network-feasible bounds $V_i, \overline{V}_i, \overline{l}_i \in \mathbb{R}$.

Expansion of network-feasible real-power envelopes with reactive power co-optimisation requires different reactive responses during periods of peak generation or peak load. The DSO could enforce a functional relationship between real and reactive power. The objective in (12), for $n \in \mathbb{N}$ defined in (3), represents bid capacities (1) and cross-market dispatch constraint functions $c_i^{\tau}$ in (2), both contained in provisional bids.
• We introduce additional constraints (5), (6) applied to \( P \). These constraints are parametrised by bid-level operating envelope bounds \( p^\beta_n, p^\delta_n \). Their effect is to \textit{shape} or \textit{trim} bid-level feasible regions defined by constraints \( c^\kappa_n \) in order to ensure network feasibility of market dispatch outcomes maximising aggregator benefit.

• We ensure network feasibility of bounds \( p^\beta_n, p^\delta_n \) by relating bid-level envelopes to nodal envelopes in (7), (8), and applying power flow constraints (9) and operational constraints (10), (11) to both extreme operating points of the network, specifically \( p \) and \( \bar{p} \).

• Under the influence of the objective, the values of \( p^\beta_n, p^\delta_n \) in the solution of (12) will maximise aggregator benefit.

• Aggregators are informed of their curtailed dispatch regions defined by the combination of constraints (2), (5), (6). They may submit curtailed bids \( b^\kappa_n = (\pi^a, x^a, c^\kappa_n) \) to the market where

\[
\begin{align*}
    c^\kappa_n(\mathcal{P}_G,n) & \leq 0 \implies (2), (5), (6) \\
\end{align*}
\]

Aggregators would not strictly be required to reduce capacities bid into various bands. This is because new constraint functions \( c^\kappa_n \) satisfying (13) effectively cap dispatchable capacities to within curtailed envelopes.

This concludes the derivation of our framework in the case of a single price forecast. We note that prices are an influential parameter in the objective, and in Section III we present an extension to our framework which accounts for estimates of price uncertainty.

III. STOCHASTIC SHAPING WITH PRICE UNCERTAINTY

This section presents a stochastic programming extension of our approach to account for price uncertainty. We motivate this extension by studying the potential impacts of inaccurate price forecasts on benefit using the approach in Section II.

A. Motivating Case Studies

We consider the extreme scenario in which the projected energy market price \( \rho_E \to \infty \). It follows that projected returns from generation are much larger than supply costs for all \( N^s \) bids across the network, i.e. \( \rho_E \mathcal{P}_{G,n} \gg \Pi^G_n(\mathcal{P}_{G,n}) \) for all \( n \leq N^s \) where \( G \) indicates the energy generation market. This results in all generation capacity becoming approximately equally beneficial to aggregators. In this context, our framework will naturally seek to maximise network throughput without regard to bid prices. Due to restrictive voltage constraints in radial distribution networks, our proposed approach will naturally prioritise market access to nodes close to the transmission network, and consequently constrain market access at far leaf nodes. If this price prediction is inaccurate, and the energy price falls within its usual range, many competitive bids at leaf nodes will be curtailed inefficiently.

This example demonstrates that our framework’s flexibility to adapt to changing prices risks becoming a weakness if price assumptions are inaccurate. We mitigate this risk by proposing a price-probabilistic extension to our framework.

B. Two-Stage Stochastic Programming Formulation

Our stochastic approach is a two-stage stochastic program. Bid operating envelope bounds \( p^\beta_n, p^\delta_n \) are first-stage decision variables optimised under price uncertainty. Expectations of market outcomes are second-stage decisions, maximising aggregator benefit, constrained by shaped cross-market dispatch constraints \( c^\kappa_n \) satisfying envelopes \( [p^\beta_n, p^\delta_n] \), and made in accordance with realised market prices.

We consider the case of \( N^s \in \mathbb{N} \) discrete possible scenarios for market prices, each with probability \( \mu_s \in \mathbb{R} \) stored in vector \( \mu \in \mathbb{R}^{N^s} \). It follows that \( \sum_{s=1}^{N^s} \mu_s = 1 \). We construct the matrix \( \mathcal{P} \in \mathbb{R}^{N^s \times N^s} \) consisting of the anticipated prices for each market in each scenario, i.e. \( \rho_{m,s} \) is the price in market \( m \leq N^m \) in scenario \( s \leq N^s \). The deterministic equivalent program of our two-stage stochastic program is given by

\[
\begin{align*}
    \max_{s} \mu_s \sum_{n=1}^{N^s} f_n(\mathcal{P}, \rho_{n,s}, b_n) \\
    \text{s.t.} \quad (1), (2), (5), (6), \forall s \leq N^s, \forall n \leq N^n \\
    \quad (7), (8), (9), (10), (11), \forall i \leq N^i
\end{align*}
\]

This stochastic approach models different outcomes for market-cleared power \( \mathcal{P}^* \) for each price scenario \( s \leq N^s \). Each instance of \( \mathcal{P}^* \) across all \( N^s \) scenarios must satisfy the same curtailed cross-market dispatch constraints, i.e. (2) and (5)-(6). It follows that only two network operating points, \( \mathcal{P} \) and \( \mathcal{P}^\prime \) need to be verified for network security for any number of scenarios \( N^s \).

Scenario generation in stochastic programming is commonly performed using Monte Carlo sampling from continuous probability distributions. Although this is efficient in the context of linear problems, our power flow model is non-linear and we seek solutions over short time periods. Our framework therefore requires the DSO to generate a considered selection of scenarios. In Subsection IV-A3 we briefly discuss our chosen approach in the context of the Australian NEM.

This completes the derivation of our stochastic approach to shaping aggregator bids for maximum expected benefit.

IV. SIMULATIONS

In this section we demonstrate the advantages of our benefit-maximisation framework over methods in existing literature through simulations on a standard test network. Data for market prices and forecasts, customer bids and inflexible load/PV are sourced from real datasets. We demonstrate increases in aggregator benefit available through real-reactive power co-optimisation. We present our approach to price scenario generation in the Australian NEM context allowing our stochastic implementation to achieve increased benefit for aggregators.

A. Implementation Details

1) Setup and Data: We use the IEEE 69-bus medium-voltage (12.66kV) network in its standard single-phase radial
configuration. Voltages are constrained to within 5% and currents are also constrained. Branch data is available in [14].

A total of 1,910 customers populate the network, distributed proportionally to the real-power load configuration studied in [14]. We model significant uptake of DER with 50% prosumers operating 5kW or 10kW inverters (40% and 10% respectively)\(^4\). All customer inflexible load and prosumer PV generation are sampled from a dataset of 30 households over 5 days obtained during a trial in Tasmania, Australia [15].

Aggregators bid according to historical data from the Hornsdale Power Reserve (HPR), Lake Bonney Battery (LBB) and Ballarat Battery (BAL) grid-scale batteries as of 12 March 2021. Capacities and trapeziums (bid feasible dispatch regions) are scaled proportionally to summed aggregator inverter capacity at each node. As a result, we do not model battery state of charge. We use contemporary market price and forecast data from the South Australia 1 region of the NEM. The market clears every 5 minutes, and we use forecast data spanning a half-hour window, updated every 30 minutes. We choose 12 March 2021 as this date presents a wide range of market events. This includes a phase of usual price stability (before 17:00) and significant price volatility (after 17:00), which we analyse separately.

2) Loss Factor Approach: Our simulations assume the DSO distributes the economic costs of line losses proportionally to total aggregator exchange in the energy market. Market clearing prices reflect power value at the feeder top. Let \( L, G \) and \( X \) represent projected network-wide total load, generation and net import from the transmission network. We determine \( l \) by solving the quadratic relation \( Gl + \frac{L}{2} + X = 0 \). We pre-process the value of \( l \) using modelled network-feasible \( L, G, X \) on the basis of expected market prices. This requires a precursor iteration of shaping initially using \( l = 1 \).

3) Scenario Generation: We generate scenarios for energy prices using data published by AEMO every half hour, and use forecast reserve market prices across all scenarios. To obtain energy prices, we multiply through 1) pre-dispatch price forecasts, 2) NEM pre-dispatch price sensitivity with respect to changes in demand across regions, and 3) probabilistic forecasts of demand across regions. We condense the total number of scenarios obtained through this method to 8, tailored to capture central tendency and potential extreme events. We add market cap and market floor scenarios, each with probability \( \mu = 10^{-5} \), to account for extreme market events unrelated to demand. We display confidence intervals for energy price using this method in Figure 1. Data used for scenario generation is of same age and same source as forecasts used in the single forecast case, providing a fair comparison of approaches.

Our generous modelling of DER uptake compensates for our use of a medium-voltage network, requiring greater throughput to incur constraint violations than low-voltage equivalents. Our formulation in this paper is suitable for single-phase-equivalent networks. We plan to extend to unbalanced three-phase networks in future work, where increased computational challenges may require modifications to our stochastic approach.

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approach using true and forecast prices, and our stochastic approach using generated price scenarios, in each case with and without reactive power co-optimisation. We also simulate aggregator benefit in the no-curtailment case (Ideal), an adaptation of closely related work [11] and [12] for this problem setting (Literature) and optimally calculated fixed real-power envelopes (Basecase). We present results for the whole day, before 17:00 and after 17:00 to compare performance in both usual and extreme market conditions. This partitions 288 market trading periods into 204 and 83 periods.

Each result is benchmarked in rows entitled “Rel. to PI” to the perfect information (PI) case using our framework, either with or without reactive power co-optimisation. This choice of benchmark removes the effects of implementation-specific price forecast inaccuracies. We also compare the ideal case to the real-power perfect information case.

Comparisons reveal that our framework can attain a significantly greater portion of the no-curtailment aggregator benefit than [11] and our previous work [12]. This demonstrates the gains to aggregator benefit available when directly including price information into the objective, rather than curtailing for power at nodes then assigning curtailments to aggregators using prices in consecutive steps. Our approach demonstrates advantage in calm and volatile price conditions.

D. Advantage of Stochastic Approach

Our stochastic approach significantly closes the optimality gap between the perfect information case and the use of forecast prices. The stochastic approach was advantageous throughout the 24-hour period, particularly during the period of high price volatility in which benefit is similar to the perfect information case. Further analysis revealed that trading intervals of greatest stochastic advantage were characterised by the combination of extreme energy market prices, forecasts in the usual range, and probabilistic price scenarios indicating a wider than usual spread. This was observed for a low energy market price at 9:40 and a high energy market price at 19:30.

Our relatively simple implementation of price scenario generation in the NEM context does not use iterative parameter tuning, nor does it use any historical data prior to the most recent forecast. Its strong performance demonstrates the clear benefits in accounting for price uncertainty.

E. Advantage of Real-Reactive Power Co-optimisation

Reactive power co-optimisation allows our framework to reach almost 100% of the ideal no-constraint benefit in the cases of perfect information and our stochastic approach. Improvements reported in Table I may be limited by proximity to the unconstrained case. In any case, this demonstrates tangible improvements are available to aggregator benefit by employing real-reactive power co-optimisation. Greater benefits may be attainable with more sophisticated co-optimisation approaches.

F. Computational Aspects

Average solve time ratio between the stochastic approach with 8 scenarios and the price-deterministic approach was 7.63, with average solve times of 37.3 seconds and 6.8 seconds using each approach. As discussed in Subsection III-B, additional scenarios add cross-market linear inequality constraint instances but to not add further non-linear power flow constraints. An approximately linear increase in solve time is consistent with expectation. The real-reactive power co-optimisation extended solve time in the stochastic case by a factor of 1.5 on average, to 53.2 seconds on average. Solve time values are consistently within 5 minute settlement periods used in the NEM.

Simulations were performed in Julia using IPOPT solver on an Intel® Core™ i7-9700 3.00GHz processor with 16GB RAM.

V. CONCLUSION

We propose a novel bid curtailment approach, suitable for energy and reserve bids with price bands and feasible dispatch regions, which aims to directly maximise benefit gained by aggregators through market participation. Our results demonstrate that directly factoring bid prices and expected market prices into the curtailment objective function can produce advantageous results for aggregators. In this context, market prices are influential on curtailment outcomes, and we demonstrate in simulations that incorporating knowledge of expected price probability distributions through a two-stage
stochastic programming approach can yield greater aggregator benefit than using a single estimate of market prices. We demonstrate that reactive power co-optimisation can attain greater aggregator benefit. In future work we intend to apply this framework to three-phase unbalanced networks, study the potential for curtailment manipulation through dishonest bidding practices, and further explore the co-optimisation of real and reactive power.

REFERENCES