Risk-aware Participation in Day-Ahead and Real-Time Balancing Markets for Energy Storage Systems.

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***Abstract*—The increasing volatility of electricity prices due to increased uncertain renewable energy generation gives rise to interesting short-term arbitrage opportunitites for Energy Storage Systems (ESS) operators. Whereas prior research has shown the possibility to exploit inter-temporal arbitrage opportunities in the real-time balancing market, this paper formulates a two-stage optimization methodology that allows ESS operators to also engage in inter-market arbitrage by participating in both the day-ahead and real-time balancing markets. However, the effectiveness of such a strategy is heavily influenced by expected inter-market price differentials, which is known to be difficult to predict when it involves the imbalance price. To address this issue, we propose a risk-averse approach by incorporating the conditional value at risk in the ESS objective function. The proposed methodology is applied to a case study of the Belgian electricity market, where we demonstrate the effectiveness of (i) the combined market participation compared to an ESS participating in either market, and (ii) the risk-aware methodology by showcasing improved ex-post out-of-sample profit performance of a risk-averse compared to a risk-neutral ESS.**

***Index Terms*—Balancing market, day-ahead market, energy storage, model predictive control, stochastic optimization.**

NOMENCLATURE

Sets and Indices

Ω Set of scenarios, indexed ω
R^+ Set of upward regulation bids, indexed r^+
R^- Set of downward regulation bids, indexed r^-
T Set of time steps, indexed t

Decision Variables

ϕ_ω Total profit for scenario ω
CVaR_β Conditional Value at Risk for risk level β [€]
e_t^+ Energy discharged at time step t [MWh]
e_t^- Energy charged at time step t [MWh]
$e_t^{DA,+}$ Discharging energy offered in day-ahead at time step t [MWh]
$e_t^{DA,-}$ Charging energy offered in day-ahead at time step t [MWh]
$e_t^{imb,+}$ Discharging energy offered in RT at time step t [MWh]
$e_t^{imb,-}$ Charging energy offered in RT at time step t [MWh]
$P_{t,\omega}^{imb}$ Imbalance price for scenario ω at time step t [€/MWh]
$s_{t,\omega}^{r^+}$ Activated energy for upward regulation at for scenario ω at time step t [MWh]
$s_{t,\omega}^{r^-}$ Activated energy for downward regulation at for scenario ω at time step t [MWh]
SoC_t State of Charge at time step t [MWh]
VaR Value at Risk [€]

Parameters

β Risk level of the CVaR_β [-]
Δt Duration of a time step [h]
η_c Charging efficiency [-]
η_d Discharging efficiency [-]
$\overline{P^d}$ Maximum discharging power [MW]
$\overline{SoC}$ Maximum energy content [MWh]
$\overline{P^c}$ Maximum charging power [MW]
$\underline{SoC}$ Minimum energy content [MWh]
P_t^{DA} Day-ahead price at time step t [€/MWh]
S_{r^+} Upper boundary for upward reserve interval r^+ [MWh]
S_{r^-} Upper boundary for downward reserve interval r^- [MWh]
$SI_{t,\omega}$ System Imbalance for scenario ω at time step t [MW]

I. INTRODUCTION

As the share of intermittent renewable energy sources in today's energy mix continues to increase, the need for grid flexibility grows and prices in short-term electricity markets become more volatile. This creates arbitrage opportunities for Energy Storage Systems (ESS). The conventional approach involves inter-temporal arbitrage, a strategy centered around tactically scheduling the ESS's charging and discharging actions while considering price variations of an electricity market over time. When operating in multiple markets, inter-market arbitrage becomes an alternative strategy. The goal of inter-market arbitrage lies in capitalizing on price differences between two parallel markets [1]. This paper focuses on combining Real-Time (RT) balancing actions of an ESS with Day-Ahead (DA) market participation, specifically in European-style markets, and as such capture the possibility of both inter-market and inter-temporal arbitrages.

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Multi-market participation typically involves a combination of the day-ahead (DA) market, reserve market and RT balancing market. A first stream of scientific research focusing on ESS participation in a multi-market setting considers joint participation in the DA and reserve markets. Here, the ESS optimizes its bids placed in the DA and reserve markets, considering possible outcomes of reserve activation. The uncertain character of those activations poses a major challenge in this line or research, as it results in uncertainty on the state of charge at a particular time. Various strategies exist for managing the uncertainty associated with RT activations during the DA decision moment. One approach is to model the ESS so that it can meet its reserve capacity in all scenarios, which corresponds to a worst-case perspective. In [20], Kahliniserobari and Wu ensure with worst-case constraints that the ESS is able to deliver all of its cleared reserve capacity, but do not explicitly model the underlying uncertainty, and therefore cannot capture balancing market revenues. A similar constraint on reserve activations is employed by Schillemans et al. in [2] while considering uncertainty in the revenue from activations through probabilistic intervals of possible System Imbalance (SI) values. In [4], the worst-case reserve activation constraints are combined with scenarios of the SI, allowing them to develop a model for a risk-averse BESS by means of the Conditional Value at Risk (CVaR). Such worst-case scenario approaches are highly restricting in the capacity that is allocated to the reserve market. This sparked Toubeau et al. [3] to introduce probabilistic constraints on the reserve activations leveraging scenarios of the SI, allowing the ESS to bid more aggressively in the reserve market at the expense of not being able to deliver the reserves in all scenarios.

Another line of research in multi-market participation considers the Day-Ahead Market (DAM) in conjunction with participation in an energy market closer to Real-Time (RT), rather than focusing on ancillary services like the previous research stream. One key benefit of this approach is that it removes the uncertainty tied to reserve activation. This topic has been explored most prominently in the context of US-style RT markets. This typically involves a two-stage approach, with a first optimization in day-ahead, and a re-optimization closer to RT with updated information, e.g. on price forecasts. The second stage is often modeled through Model Predictive Control (MPC), where the ESS optimizes its schedule deploying a receding horizon. There are two distinct approaches: in [8]–[11], the potential outcomes in the RT are not considered in the DA market, which decouples (and as such, simplifies) the problem. However, this does not allow for exploiting arbitrage opportunities in the day-ahead optimization stage. In [12], Arteaga et al. co-optimize DA and RT decisions, deploying a robust optimization approach to hedge against the price uncertainty.

However, in the setting of European electricity markets, there is no direct equivalent to the US RT-market. Indeed, the demand side of European RT balancing markets constitutes the activated reserves, driven by the SI. This makes price profiles much more volatile and uncertain compared to the US RT market which is driven by energy demand. The continuous intraday market differs from the US RT market by its continuous nature with bi-lateral trades, potentially resulting in many prices for a single delivery period, increasing the complexity of modeling this market compared to one with a single clearing. Metz et al. have explored the possbility for ESSs to exploit both intertemporal and intermarket arbitrages between the German 15-minute and 60-minute continuous intra-day market in [5]. On the other hand, Bottieau et al. propose an implicit balancing strategy. This entails deliberate out-of-balance actions that allow market participants to engage in the RT balancing market without day-ahead commitments. Smets et al. further elaborated this approach in [7] by introducing a strategic ESS deploying an MPC algorithm to exploit these implicit balancing opportunities. It is noteworthy that such an implicit balancing strategy resembles, but is distinct from participation in the US RT-market. The demand side of the RT balancing market is dominated by the SI - leading to reserve activations - as opposed to to energy load in the US RT market. This SI is much more difficult to predict [13], leading to more volatile and uncertain prices compared to the US RT market.

As of yet, no research has explored the potential of combining ESS inter-temporal and inter-market arbitrage through re-optimization in the context of European electricity markets. Moreover, no previous work investigated the effect of risk-aversion in such two-stage approaches. To address this gap, we develop a two-stage model where in the first stage, a risk-averse ESS co-optimizes its anticipated profits in both DA and RT balancing market. The second stage entails an MPC algorithm based on [7] re-optimizing the implicit balancing actions in RT. The scientific contribution of this work is threefold:

- We develop a two-stage model allowing ESSs to explore both inter-temporal and inter-market arbitrage opportunities between the day-ahead and RT balancing market, taking into account price uncertainty in the latter through scenarios.
- We propose a risk-averse methodology through the conditional value at risk to manage the balance of engagement in inter-market as opposed to inter-temporal arbitrages.
- We apply the methodology to a realistic case study of the Belgian balancing market using actual price and system imbalance data to show the effectiveness of the proposed method.

II. Methodology

In this section, we construct an optimization framework that represents a strategic ESS, operating in the DA market as well as the RT balancing market through implicit balancing. Section II-A outlines the concept of the model, followed by section II-B that translates the concept into an optimization program. Lastly, section II-C discusses the scenario generation technique used to represent the uncertainty in the RT balancing market.

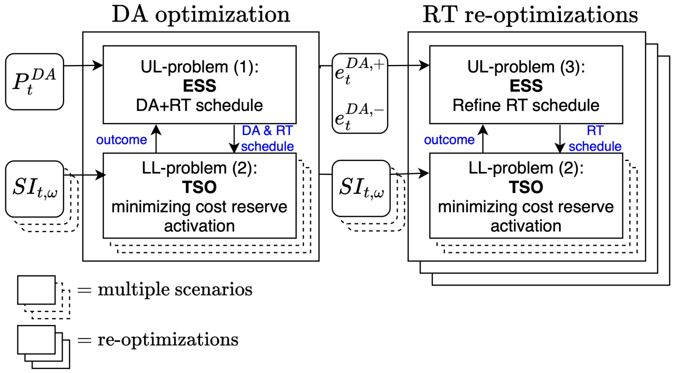

Fig. 1. High-level overview of the model used in this paper.

A. Market dynamics and decision stages

Figure 1 provides a high-level overview of the methodology. The model consists of two bi-level optimization problems, each representing one decision stage: the DA stage and the RT stage.

The upper-level problem of the DA stage determines the ESS's most profitable commitments in the DA market based on exogenous price data, P_t^{DA} , while already considering possible future profits through implicit balancing. The lower-level problem represents the balancing market clearing, which takes scenarios of the SI, $SI_{t,\omega}$, information on bids placed in the balancing market, and the upper-level implicit balancing decisions as input to return the imbalance price. As such, the ESS is modeled as price-maker in the RT balancing market.

In contrast to the obtained DA schedule in the first stage, the RT charging schedule can still be re-optimized during the following RT optimization stage. The RT stage is covered by the second part of the model, which is a rolling horizon implementation of a bi-level optimization problem that is similar to the problem of the DA stage. This time, the DA stage outcome ($e_t^{DA,+}$ and $e_t^{DA,-}$) is considered as input, together with updated SI scenarios. The RT charging schedule is re-optimized for every time step. This allows for the integration of up-to-the-minute data regarding the ESS's SoC and the observed SIs, leading to improved predictions and consequently, a more optimal RT charging schedule. The MPC algorithm from this paper adopts a a three-step procedure inspired by [7]. The first step involves the generation of scenarios for a limited amount of future time steps, drawing upon past data. Secondly, these scenarios serve as the basis for re-optimizing the ESS implicit balancing schedule. In the last step, the charging or discharging decision of the first time step will be executed and a new SoC of the ESS will be observed.

B. Model formulation

In this section, we discuss the optimization problems introduced in Section II-A. Problem (1) represents the upper level of the DA stage optimization. Equation (1a) is the objective function of the upper-level problem, consisting of two terms. The first term's purpose is to maximise the total profit ϕ_ω for each scenario ω with a probability π_ω. The total profit is further defined in equation (1b) and is the sum of the profit in the DA market and the anticipated profit from the balancing market. Decision variables $e_t^{DA,+}$ and $e_t^{DA,-}$ express the discharging and the charging energy in the DA market respectively, determining the ESS's DA schedule. Similarly, $e_t^{IMB,+}$ and $e_t^{IMB,-}$ define the foreseen RT balancing schedule. The second term of the objective function focuses on maximizing the CVaR$_\beta$. In doing so, the profit within the lowest β quantiles will be kept as high as possible, making the model Risk-Averse (RA). The CVaR$_\beta$ is defined by equations (1c)-(1d). By adjusting factor γ and risk level β, the ESS owner can modify its risk awareness. Equations (1e)-(1f) define the relation between the discharging and the charging energy on the DA market and the balancing market. Their sum equals the physical discharging and charging energy; e_t^+ and e_t^-. For each time step, the State of Charge (SoC) is calculated based on the charge or discharge actions from the previous time step (1g). The cyclic boundary condition (1h) ensures that the ESS returns to the same SoC every day. Additionally, (1i) implies that the ESS does not surpass the maximum and minimum allowed SoC. Constraints (1j)-(1o) inhibit the ESS from exceeding the charge and discharge limits with its DA and physical (dis)charge actions. Technically, only the latter are subject to the ESS's physical properties, since excess (dis)charging in the DA market can be compensated by implicit balancing actions (counteracting the DA bids in RT). Nonetheless, to avoid extreme DA participation strategies that could limit the flexibility in the RT balancing market, we choose to impose limits on the DA schedule. The binary variables z_t and z_t^{DA} in constraints (1j)-(1o) guarantee that the ESS cannot simultaneously charge and discharge.

$$\max_{\Xi^{DA}} \ \gamma \sum_{\omega \in \Omega} \pi_\omega \phi_\omega + (1-\gamma)\text{CVaR}_\beta \tag{1a}$$

subject to:

$$\phi_\omega = \sum_{t \in T} \left[P_t^{DA} \cdot (e_t^{DA,+} - e_t^{DA,-}) \right.$$
$$\left. + P_{t,\omega}^{imb} \cdot (e_t^{imb,+} - e_t^{imb,-}) \right] \quad \forall \omega \in \Omega \tag{1b}$$
$$\text{CVaR}_\beta = \text{VaR} - \frac{1}{\beta} \sum_{\omega \in \Omega} \pi_\omega \xi_\omega \tag{1c}$$
$$0 \leq \xi_\omega \geq \text{VaR} - \phi_\omega \quad \forall \omega \in \Omega \tag{1d}$$
$$e_t^+ = e_t^{DA,+} + e_t^{imb,+} \quad \forall t \in T \tag{1e}$$
$$e_t^- = e_t^{DA,-} + e_t^{imb,-} \quad \forall t \in T \tag{1f}$$
$$SoC_{t+1} = SoC_t + e_t^- \eta_c - e_t^+ / \eta_d \quad \forall t \in T \tag{1g}$$
$$SoC_1 = SoC_{|T|+1} = SoC_{init} \tag{1h}$$
$$\overline{SoC} \leq SoC_t \leq \underline{SoC} \quad \forall t \in T \tag{1i}$$
$$0 \leq e_t^{DA,+} / \eta_d \leq \overline{P^d} \cdot \Delta t \cdot z_t^{DA} \quad \forall t \in T \tag{1j}$$
$$0 \leq e_t^{DA,-} \eta_c \leq \underline{P^c} \cdot \Delta t \cdot (1 - z_t^{DA}) \quad \forall t \in T \tag{1k}$$
$$0 \leq e_t^+ / \eta_d \leq \overline{P^d} \cdot \Delta t \cdot z_t \quad \forall t \in T \tag{1l}$$
$$0 \leq e_t^- \eta_c \leq \underline{P^c} \cdot \Delta t \cdot (1 - z_t) \quad \forall t \in T \tag{1m}$$

$$z_t^{DA} \in \{0,1\} \qquad \forall t \in T \tag{1n}$$

$$z_t \in \{0,1\} \qquad \forall t \in T, \tag{1o}$$

where $\Xi^{DA} = \{e_t^{DA,+}, e_t^{DA,-}, e_t^{imb,+}, e_t^{imb,-}, z_t^{DA}, z_t | t = 1, \ldots, 96\}$.

Analogous to [6], [7], Problem (2) defines the lower-level optimization, depicting the clearing of the balancing market as performed by the Transmission System Operator (TSO). The objective function (2a) minimizes the TSO's cost of activating balancing reserves. Balancing reserves can provide both upward $s_{t,\omega}^{r^+}$ and downward regulation $s_{t,\omega}^{r^-}$. This activation follows the Available Regulation Capacity (ARC) merit order, defined by bids $r^{+/-}$ with specified prices $\Lambda_t^{r^{+/-}}$ and quantities $S_t^{r^{+/-}}$.

The first constraint (2b) represents the energy balance of the grid, where the SI, adjusted with the upper-level implicit balancing decisions, is offset by the activated reserves. This equation assumes the TSO to have perfect knowledge of the SI and that the ESS is the only market player to take on these out-of-balance actions. The dual variable of this constraint, $P_{t,\omega}^{imb}$, is the imbalance price which is used in the upper-level decision making. The uncertainty on the SI at the time of decision-making is represented with scenarios, which is further discussed in Section II-C. Constraints (2c)-(2d) define the boundaries of each level of reserves.

$$\min. \sum_{r^+ \in R^+} \Lambda_t^{r^+} s_{t,\omega}^{r^+} - \sum_{r^- \in R^-} \Lambda_t^{r^-} s_{t,\omega}^{r^-} \tag{2a}$$

subject to:

$$\sum_{r^+ \in R^+} s_{t,\omega}^{r^+} - \sum_{r^- \in R^-} s_{t,\omega}^{r^-}$$

$$+ (e_t^{imb,+} - e_t^{imb,-}) + SI_{t,\omega} \cdot \Delta t = 0 : P_{t,\omega}^{imb} \qquad \forall t, \omega \tag{2b}$$

$$0 \geq -s_{t,\omega}^{r^+} \geq -S_{r^+} : \nu_{t,\omega}^{r^+}, \mu_{t,\omega}^{r^+} \qquad \forall t, \omega \tag{2c}$$

$$0 \geq -s_{t,\omega}^{r^-} \geq -S_{r^-} : \nu_{t,\omega}^{r^-}, \mu_{t,\omega}^{r^-} \qquad \forall t, \omega \tag{2d}$$

The optimization problem used in the RT stage is very similar to that of the DA stage. Since the DA charging schedule is fixed in the RT stage, $e_t^{DA,+}$ and $e_t^{DA,-}$ are no longer decision variables, but they are still used as parameters in equations (3c) and (3d). Furthermore, the RT model does not include the CVaR$_\beta$. Even though risk-aversion in RT might be interesting for further research, previous research showed that a risk-neutral approach provides better overall profits for RT optimization [7]. The upper-level problem of the RT optimization is represented by equations (3a)-(3j). The lower-level problem is identical to the lower-level problem of the DA stage.

$$\max_{\Xi^{RT}} \sum_{\omega \in \Omega} \pi_\omega \phi_\omega \tag{3a}$$

subject to:

$$\phi_\omega = \sum_{t \in T} \left[P_{t,\omega}^{imb} \cdot (e_t^{imb,+} - e_t^{imb,-}) \right] \qquad \forall \omega \in \Omega \tag{3b}$$

$$e_t^{imb,+} = e_t^{+} - e_t^{DA,+} \qquad \forall t \in T \tag{3c}$$

$$e_t^{imb,-} = e_t^{-} - e_t^{DA,-} \qquad \forall t \in T \tag{3d}$$

$$SoC_1 = SoC_{|T|+1} = SoC_{init} \tag{3e}$$

$$\overline{SoC} \leq SoC_t \leq \underline{SoC} \qquad \forall t \in T \tag{3f}$$

$$SoC_{t+1} = SoC_t + e_t^{-} \eta_c - e_t^{+} / \eta_d \qquad \forall t \in T \tag{3g}$$

$$0 \leq e_t^{+} / \eta_d \leq \overline{P^d} \cdot \Delta t \cdot z_t \qquad \forall t \in T \tag{3h}$$

$$0 \leq e_t^{-} \eta_c \leq \underline{P^c} \cdot \Delta t \cdot (1 - z_t) \qquad \forall t \in T \tag{3i}$$

$$z_t \in \{0,1\} \qquad \forall t \in T, \tag{3j}$$

where $\Xi^{RT} = \{e_t^{imb,+}, e_t^{imb,-}, z_t \ t = 1, \ldots, |T|\}$.

A possible way to solve these bi-level problems is to transform them into a single-level problem. Because the lower-level problem is linear, it can be replaced by its Karush-Kuhn-Tucker (KKT) conditions, which turns the problem into a mathematical program with equilibrium constraints (MPEC) [14]. After the substitution of the lower-level problem, there are two sources of non-linearities left. The first non-linearity is the bi-linear term in the objective function, consisting of the prodruct of the imbalance price and the (dis)charging energy offered in RT. It can be eliminated using the KKT conditions, see e.g. [21]. Secondly, the Fortuny-Amat or 'Big-M' approach [22] can be applied to replace the non-linear complementarity constraints. The model now becomes a mixed-integer linear programming problem (MILP), that can be processed by a solver through a branch-and-cut algorithm, given that the amount of integer variables is not too large [14], [15].

Even though the lower-level problem of the RT model computes the imbalance price, it is not used to calculate the actual revenues from the RT balancing market participation. After all, the price from the model is based on estimations of the SI. Instead, the realized imbalance price is determined ex-post, by adding the ESS's charging or discharging action to the realized SI and finding their associated imbalance price through the ARC merit order.

C. Scenario generation

To capture the uncertainty in the balancing market, we propose to use scenarios of the SI. As the SI exhibits significant auto-correlation, these scenarios ideally capture the inter-temporal dynamics of the SI. To achieve this, we leverage the probabilistic forecasting method proposed in [16], which uses random sampling from a multivariate normal distribution and historical data of the random variable to be forecast to produce scenarios with covariance in line with historical data.

This technique acknowledges that an uncertain variable, SI_i can be transformed into a uniformly distributed variable Y_i by applying its cumulative distribution function (CDF). This variable can then be transformed to a normally distributed variable X_i by applying the inverse probit function. This transformation is applied to historical data of the SI, where every quarter hour of the day is assumed to have a different distribution. The covariance matrix of this data then allows to capture the historical correlations by randomly sampling scenarios of z-scores from a multivariate normal distribution with that covariance matrix. Upon applying the probit function and the inverse cumulative distribution functions, these z-scores are transformed to SI data. We add upon this methodology by taking into account the latest realized SI values through conditional distributions.. Vector $\boldsymbol{X}_{SI} = \{X_{t-J}, ..., X_{t-1}, X_t, X_{t+1}, ..., X_{t+K-1}\}$ consists of the transformed uncertain SI variables of the past J quarter hours and the future K quarter hours. The vector follows a multivariate Gaussian distribution $\boldsymbol{X}_{SI} \sim \mathcal{N}(\boldsymbol{\mu_0}, \boldsymbol{\Sigma_{hist}})$ with:

$$\boldsymbol{X}_{SI} = \begin{pmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \end{pmatrix} \quad \boldsymbol{\mu_0} = \begin{pmatrix} \boldsymbol{\mu_1} \\ \boldsymbol{\mu_2} \end{pmatrix} \quad \boldsymbol{\Sigma_{hist}} = \begin{pmatrix} \boldsymbol{\Sigma_{11}} & \boldsymbol{\Sigma_{12}} \\ \boldsymbol{\Sigma_{21}} & \boldsymbol{\Sigma_{22}} \end{pmatrix}, \tag{4}$$

where 1 refers to the future quarter hours ($i = t...t+K-1$) and 2 refers to the quarter hours in the past ($i = t-J...t-1$). The conditional multivariate normal distribution ($\boldsymbol{X}_{SI,1}|\boldsymbol{X}_{SI,2} = \boldsymbol{x}_{SI,2}$) can be constructed by applying the conditional distribution properties of normal distributions [17] on the multivariate Gaussian distribution $\boldsymbol{X}_{SI}$. From this distribution, n random vectors $\boldsymbol{x}_{SI,1}$ with length K can be generated, where n is the desired number of scenarios. In doing this, the SI scenarios for the next K quarter hours leverage the latest J SI realizations through the historical covariance.

III. Case Study

The proposed methodology is applied to a case study of the Belgian electricity market. In section III-A, we detail the case study design by outlining the data, parameters and the different model variants. Section III-B gives a comprehensive analysis of the outcomes for the risk-neutral and risk-averse models. Section III-C offers a high-level comparison of the different models.

A. Case study design

The optimization model considers a time step size of 15 minutes, equal to the granularity or the settlement period of the Belgian balancing market. While the ESS is assumed to have perfect price foresight in the day-ahead market[1], the uncertainty on the imbalance price is captured through scenarios of the SI, which from the lower level optimization yield scenarios of the imbalance price. We require three types of input: the DA prices, the ARC merit order of each quarter hour and the realized SI per quarter hour. The Belgian TSO (Elia [18]) and the European Network of Transmission System Operators for Electricity (ENTSO-e [19]) publish these sets of data on their online platforms. The ARC merit order has power activation intervals of 100MW. The CDFs, used for scenario generation, are made up of the SI data from January 2015 until February 2023. There are 96 CDFs in total, one for every quarter hour of the day. The number of scenarios is taken to be 25, which is a trade-off between result consistency and computational efficiency. In Table I, we show the computation times of (i) the single DA optimization and (ii) the total of the 96 RT re-optimizations as function of the amount of considered SI scenarios. We conclude that the computation time increases quasi-linearly with the amount of considered scenarios, and hence the proposed model is scalable. In this case study, we model a 20MW/20MWh ESS with a charging and discharging efficiency of 90% each. The maximum and minimum SoC are set to 20MWh and 0MWh respectively, with an initial SoC of 10MWh.

[1]We make this assumption based on the well-known result that state-of-the art forecasters can make much more accurate predictions of the day-ahead price than the imbalance price. As such, we single out the effect of uncertainty in the balancing market in our analysis.

TABLE I
Computation time of DA and RT optimization for varying number of scenarios by a RA ESS on day 3.

Number of scenarios	**1**	**5**	**10**	**25**	**50**
Optimization time DA [s]	1	2	13	128	640
Optimization time RT [s]	18	70	288	1482	5659

To analyze the effect of risk-related features of our proposed model, two different model variants will be applied to each day. For the first variant, γ is set to zero in (1a), resulting in Risk-Neutral (RN) decisions in the DA stage. The second variant assumes $\gamma = 0.5$ and $\beta = 0.1$ in (1), making it a RA implementation. The re-optimizations in RT are risk-neutral, both for the RN and the RA model variant. To compare the performance against other potential strategies, three other variants are tested: one where the ESS only operates in the day-ahead market (DAM), one where the ESS only operates in the balancing market (IMB) and one variant that operates in both markets, but with additional perfect foresight of the imbalance price (PF). The RN, RA and IMB variants are run with the same set of scenarios that vary for three different days. February 13th, 2023 (day 1), September 19th, 2022 (day 2) and June 29th 2022 (day 3). These days are selected for having different values of correctly predicted signs of the Day-ahead and Real-time Price Difference (DRPD). This DRPD is defined as

$$DRPD_t = \text{imbalance price}_t - \text{day-ahead price}_t. \tag{5}$$

Correctly predicting the $DRPD$ sign is expected to strongly affect the efficacy of inter-market arbitrage actions. As shown in Figure 4, the SI scenarios used for the three selected days result in values of correct DRPD sign prediction covering multiple values across the spectrum of possible outcomes.

B. Results

Table II compares the results for all five different model variants for the selected days. The total daily energy (dis)charged represents the sum of all (dis)charge actions. The total expected profit and CVaR$_\beta$ are obtained during the optimization in the DA stage and are the results of equation (1b) and (1c) respectively. Furthermore, the actual (ex-post, out-of-sample) profit is given for each stage. Figures 2 and 3 present the results of day 3, for the RN and the RA case respectively.

TABLE II
(DIS)CHARGING BEHAVIOUR, CVaR$_\beta$ AND PROFITS OF A 20MW/20MWH ESS.

	RN		RA		DAM		IMB		PF	
DAY 1	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**
E ch. [MWh]	126	312	161	239	33	-	-	133	366	233
E dis. [MWh]	260	157	185	193	41	-	-	108	111	461
CVaR [€]	-2 348	-	7 481	-	-	-	-	-	-	-
profit [€]	18 628	1 158	3 375	20 417	2 624	-	-	11 776	-37 199	93 329
tot. profit [€]	19 785		23792		2 624		11 776		56 130	
DAY 2	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**
E ch. [MWh]	87	343	155	235	56	-	-	139	205	408
E dis. [MWh]	302	102	204	160	59	-	-	113	247	329
CVaR [€]	-3526	-	24527	-	-	-	-	-	-	-
profit [€]	71 642	-21 987	30 489	4 180	12 869	-	-	20 251	11 121	130 026
tot. profit [€]	49 655		34 669		12 869		20 251		141 147	
DAY 3	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**	**DA**	**RT**
E ch. [MWh]	41	365	119	242	33	-	-	116	395	164
E dis. [MWh]	351	36	236	103	41	-	-	94	84	454
CVaR [€]	-27 350	-	13 512	-	-	-	-	-	-	-
profit [€]	95 745	-98 740	42 713	-32 667	6 312	-	-	8 126	-95 635	184 022
tot. profit [€]	-2 995		10 046		6 312		8 126		88 386	

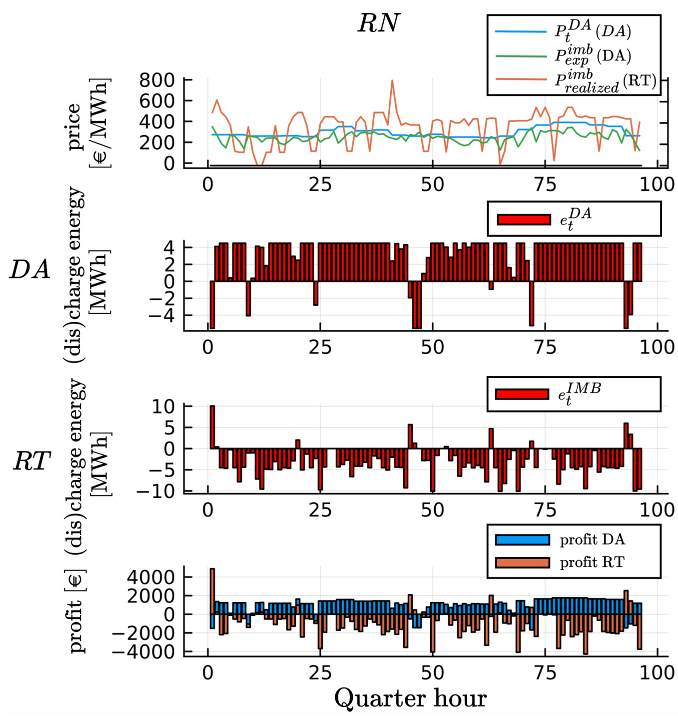

Fig. 2. Results on day 3 by the RN model. The first graph compares the day-ahead price with the actual and expected imbalance price. The second and third figure show the discharge actions in the DA and RT balancing market respectively. The bottom figure depicts the ex-post out-of-sample profits attained in both markets.

1) Risk-neutral case: Figure 2 shows that the ESS is not always operating at maximum power. This can be explained by the price-maker behaviour of the ESS in the balancing market resulting from bi-level model formulation. When offering balancing energy, the SI may fall within a different interval of the ARC merit order. This causes prices to possibly become less favourable. The ESS can strategically choose to withhold energy right to the point where the SI does not cause an undesirable price change, in order to avoid the effect of 'cannibalisation' of its revenues.

Another observation can be made when looking at the choice of either charging or discharging in the DA stage of the RN model. The ESS chooses to charge when the average of the scenarios' imbalance prices exceeds the DA price. When the imbalance price is lower, energy will be discharged. This affirms the significant role of the expected DRPD (5). The RN model thus favors inter-market arbitrage over inter-temporal arbitrage. This further becomes clear from observing in Table II the (im)balance of offered charge and discharge energy within a specific market. On day 3, the expected DRPD is mostly negative throughout the day. This triggers the ESS to place more consecutive discharge offers in the DA market than it can physically deliver, which we will refer to as over-offering. This is possible as combined participation in the DA market and the RT market allows such over-offering to be offset in the other market. On day 3, the ESS sells ($350MWh$) more than eight times the amount of energy that it buys ($41MWh$), meaning that the DA profit will naturally be high (€95 744). This does imply that the ESS must counteract some of these offers in RT, leading to an opposite imbalance of offered charge and discharge energy in the RT balancing market. Interestingly, the effect of discharging more energy in the DA market is observed for all the selected days, which can be explained through the observation that the ARC merit order tends to exhibit very low price values for the activation of downward reserves.

The strategy of over-offering and operating at full capacity can make for great profits, but at the same time limits the ESS's flexibility in the RT balancing market, possibly leading to high risks when imbalance price predictions are very different from the actual prices. The $CVaR_\beta$ confirms that in the worst 10% of cases, the average profit will be negative (€−27 350). Due to the balancing market's volatile prices and the low number of instances with correct DRPD sign predictions, the ESS still made a considerable loss (€98 740). For day 1 and day 2, the RN model does manage to make a profit, which is substantially larger than the profit obtained by the models that operate in the two markets separately, but significantly lower than the attainable profit (under perfect price foresight). The latter is the result from the highly volatile and uncertain imbalance prices: whereas our proposed method tends to favor profits in the DA market, the PF model make most profit in the RT balancing market.

In case there is no DA participation, the RT out-of-balance positions are limited to the physical properties of the ESS and inter-temporal arbitrage. The maximum charging energy would then only be $5.56 MWh$. However, pay attention to the different scales for the (dis)charge energy per quarter hour in figure 2: the maximally possible imbalance position now lies at $11.12 MWh$. Take for example quarter hour 1. In the DA market, the ESS decides to place a charging bid, anticipating that the imbalance price for that quarter hour will - on average - be higher. That prediction turns out to be correct. Maximally charging in DA means that it is possible to take an out-of-balance position in the opposite (discharging) direction that is twice as large as the case without DA participation, leading to increased profit compared to the attainable profit without DA market participation.

2) Risk-averse case: In contrast to the RN variant, the RA variant makes a trade-off between maximizing the average profit of the worst 10% of scenarios and maximizing the profit for all scenarios. Figure 3 shows the outcome of the RA model on day 3. We observe that the RA model is more careful in placing DA bids. Indeed, fewer of the bids in the DA market are at full capacity. Additionally, the periods of over-offering are less frequent and shorter.

The ESS decision-making is still strongly influenced by the expected DRPD. However, when taking the CVaR$_\beta$ into account in the objective, there are some quarter hours at which the charging schedule does not strictly follow the expected DRPD. For example, in quarter hour 2 in figure 3, the ESS now decides to charge in the DA stage, leading to a higher profit. For all three days, the negative CVaR$_\beta$'s of the RN variants have turned positive and are significantly higher, which suggest that there is a smaller chance of a very low resulting profit.

Overall, the contrast between the amount of energy that the ESS charges and discharges in DA is now less pronounced. On day 2 for example, the difference between the total charged and discharged energy in the DA market

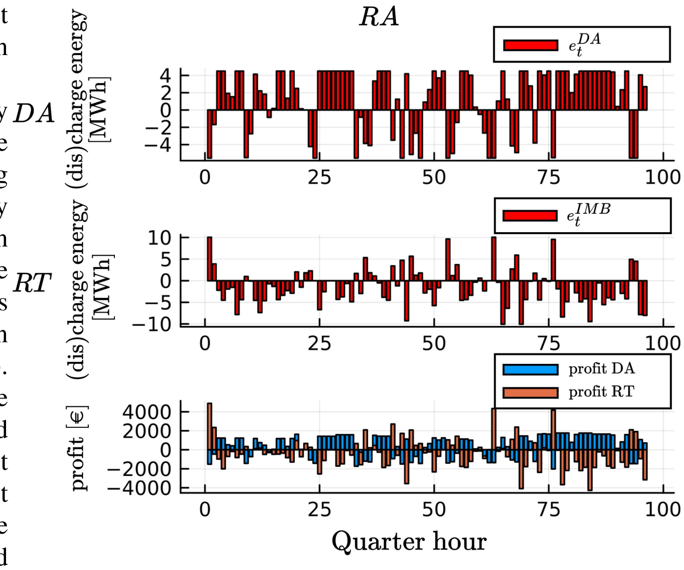

Fig. 3. Results on day 3 by the RA model. The first and second graph show the discharge actions in the DA and RT balancing market respectively. The bottom figure depicts the ex-post out-of-sample profits attained in both markets.

shows $48.79 MWh$, as opposed to $214.93 MWh$ for the RN case. Rather than discharging a lot more than physically possible and having to rely on the RT balancing market for compensation, the RA case opts for an approach with more inter-temporal arbitrage, resulting in more flexibility in the RT balancing market, which is especially useful when the DRPD predictions are inaccurate. Less over-offering in the DA stage results in lower profits in the DA markets, but leads to higher profits in the balancing market.

C. High-level overview

Of the five model variants, the DAM model variant is the least profitable (with the exception of day 3, where it is the second to least profitable) followed by the IMB variant that yields higher profits due to the volatile imbalance prices and higher clearing frequency compared to the DA market [7]. Both of these model variants are limited to inter-temporal arbitrage. Comparing their performance to the RN and RA variants leads to the conclusion that there is significant merit in inter-market arbitrage between the DA and RT balancing market. The PF model provides the maximum achievable profit for each day. The charging behaviour in the DA stage and the RT stage again demonstrates the application of inter-market arbitrage. When comparing the RN and RA models to their PF counterpart on the loss-making day 3, it is clear that more correct predictions would have lead to more charging in the DA market and discharging in the balancing market, i.e. the opposite of what happened in the RN case.

As was shown, the profit performance of both the RN variant and the RA variant are highly dependent on the accuracy of the price forecasts in the DA stage. Since the DRPD is estimated based on scenarios in the DA stage and

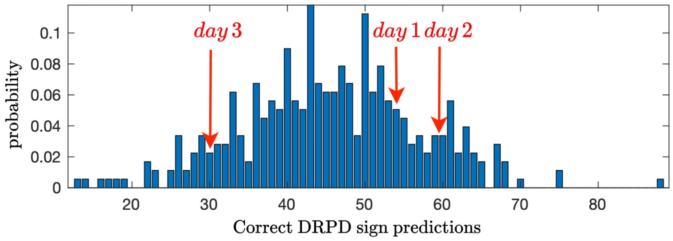

Fig. 4. Experimental probability distribution of the number of correctly predicted DRPD signs. The distribution is obtained by generating 25 scenarios for each day of one year. Then, the sign of the predicted DRPD (which is the average of the 25 values of each scenario) is compared to the sign of the realized DRPD. The number of quarter hours where both of these signs are equal is counted for each day.

since the sign of that DRPD estimation determines the ESS's (dis)charging actions, the number of correctly predicted DRPD signs is a good indicator for the expected performance of the model. Figure 4 displays them on a distribution for an entire year (April 2022 - March 2023). The case of day 2 has the most correct predictions (59), followed by day 1 (54) and day 3 has the least accurate predictions (30). As expected, the ESS is most profitable in day 3 when the RA model is applied, yielding a profit of €10 046. The case of day 2 shows the best result under the RN model with a profit of €49 655. Interestingly, even though for day 1 the amount of correctly predicted DRPD signs is 54 - or 56 % of the instances that day - the RA model performs best with a profit of €23 792, albeit with a relatively small improvement compared to the RN model. Thus, the general trend is that the RA variant is more profitable when DRPD predictions are less accurate. Provided that it is notoriously difficult to correctly predict imbalance prices [13], we can conclude that the proposed risk-averse strategy is beneficial for profit maximization in this setting of inter-market arbitrage between the DA and RT balancing markets.

D. Sensitivity of risk level

While the results in Table II show the actions of a RA ESS, the level of risk-aversion has to be pre-defined by the ESS operator and a priori it is unclear what the optimal risk level is. In this section, we illustrate the impact on the profitability of the ESS when the level of risk-aversion is modulated via the parameter β in Eq. (1c), which is the percentile of worst-case scenarios considered in the objective function through the CVaR. $\beta = 1$ corresponds to the risk-neutral case, whereas $\beta = 0.1$ is the level of risk-aversion that was assumed in Table II. Table III shows the profits obtained in the DA and RT balancing market, as well as the combined profit, assuming varying values of β for day 2. As a general trend, the more risk-averse the ESS behaves (i.e. the lower the value of β), the lower the profit from the DA market, and the higher the profit from the RT market. This is in line with the observation from Table II, where we concluded that a RN operator is more inclined to exploit expected inter-market price differentials in the day-ahead optimization. For the specific case of day 2, where the expected imbalance price tends to be below the DA price, this leads to more discharge decisions and hence more profit in the DA market.

TABLE III
ESS PROFITABILITY IN THE DA AND RT BALANCING MARKET AS A FUNCTION OF RISK LEVEL β ON DAY 2.

	β					
	0.01	**0.05**	**0.1**	**0.25**	**0.5**	**1**
Profit DA [k€]	30.3	30.3	30.5	36.7	51.5	71.6
Profit RT [k€]	4.6	4.3	4.2	2.2	-9.5	-22.0
Profit tot [k€]	34.9	34.7	34.7	34.5	41.9	49.7

IV. Conclusion

With increasing amounts of uncertain and intermittent electricity production from renewable energy sources, electricity prices become more volatile. This constitutes a profit opportunity for storage operators. Whereas it has been shown that it is possible to participate in real-time balancing markets with implicit balancing actions, we have explored the interaction of this strategy with participation in the day-ahead energy market. Our case study of the Belgian balancing market demonstrates that such a combined approach consistently yields higher ex-post out-of-sample profits than participating in either market separately. We also show that the difficult task of correctly predicting the difference between the day-ahead and imbalance price strongly affects the profit performance of our proposed model. Due to the imbalance market's highly volatile nature, adopting a risk-averse attitude in the day-ahead optimization stage is shown to result in superior profit performance compared to a risk-neutral approach under many circumstances.

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