Network-Secure Aggregator Operating Regions with Flexible Dispatch Envelopes in Unbalanced Systems

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***Abstract*—Expansion of DER capacity in distribution networks increasingly requires DSO coordination to prevent violation of network limits. Dynamic operating envelopes (DOE) facilitate network-secure integration of DER in wholesale markets by applying variable connection-point import/export limits. In unbalanced networks, conventional DOE approaches may lead to network-infeasible outcomes when envelope utilisation is uncertain. The robust dynamic operating envelope (RDOE) approach in literature addresses this limitation, however DER participation in wholesale markets is significantly reduced compared to conventional DOE. We propose robust aggregator operating regions with flexible dispatch envelopes (RAR-DE), combining the strengths of DOE and RDOE approaches. We leverage aggregators' abilities to coordinate customers, enabling the DSO to assign sophisticated operating regions that maximise their market participation. The DSO then facilitates adherence to complex regions, providing parameters to compute market-responsive envelopes for each DER. Simulations demonstrate the combination of favourable market participation opportunities and network-feasible outcomes.**

***Index Terms*—Aggregators, operating envelopes, unbalanced three-phase networks, wholesale markets, robust polytopic projection**

I. Introduction

Widespread uptake of distributed energy resources (DER), such as rooftop solar and batteries at residential and community scales, is causing power systems to decentralise. Flexible market services offered by DER present opportunities to address system-wide challenges, including intermittency of renewable generation and the need for increasing reserves. Aggregators facilitate the provision of network-support services by DER, managing complex market interactions on behalf of large groups of customers. While this presents opportunities for households and wholesale markets, the growing volumes of DER capacity connected to constrained low-voltage distribution networks increasingly threaten to cause voltage constraint violations.

Wholesale markets operating at the transmission network-scale generally lack visibility over the real-time state of distribution networks. This is often due to either 1) impractical complexity of modelling all MV/LV-networks within a wholesale market's domain centrally and within market trading periods, and/or 2) mandated role separations between network operators and market operators in unbundled power systems, as observed in Australia and Europe. As a result, wholesale markets generally cannot clear aggregator bids in a manner that guarantees the *network-security*[1] of distribution networks. Distribution system operators (DSOs) are responsible for the safe operation of their networks, and are increasingly obliged to lead some form of network-secure coordination of DER.

Dynamic operating envelopes (DOE) [1] [2] are an increasingly popular approach to enable network-secure participation of DER aggregators in wholesale markets. This model has attracted significant interest from industry in Australia [3]. Under this model, each connection point is assigned dynamic limits to their import and export capacity by the (DSO) to ensure network-feasible power flows. Envelopes may be viewed as a form of real-time DER hosting capacity over short time periods, adapting to changes in background network utilisation. In a market sense, aggregators participating in wholesale markets must ensure any market-cleared portion of their aggregate bids can be fulfilled while respecting envelopes assigned across their DER portfolio at the customer level [4]. This approach falls under the *centralised market model* in TSO-DSO coordination literature [5], where coordination between the TSO and DSO is not strictly required (although it may help to improve market outcomes), facilitating deployment in existing market structures.

Dynamic operating envelopes can be calculated according to simple rules, or by solving a constrained optimisation problem [6]. A common approach in literature is to solve an optimal power flow problem (OPF), either centrally at the DSO level [7] or using distributed locational marginal price-based methods [8]. These approaches generally assume that network-security of envelopes with respect to voltage constraints can be verified by evaluating the network's state under just two operating points, specifically a) maximum imports across all DER and b) maximum exports across all DER. In *balanced networks*, this two-point verification is generally sufficient to ensure voltage magnitudes throughout the network remain bounded within safe operating limits, even when utilisation of envelopes by DER is uncertain.

In practice, distribution networks are generally unbalanced, and single-phase DER may contribute to unbalanced loading across the network. Recent works have demonstrated that mutual impedances between phases can cause bus injections on a

[1]This paper refers to *network-security* of a distribution network as the condition that network constraints remain satisfied under all potential scenarios of DER behaviour, factoring any DSO-imposed restrictions on DER activity.

given phase to either increase or decrease voltage magnitudes on adjacent phases, depending on network characteristics [9]. Due to the DSO's uncertainty of DER behaviour within their envelopes, we therefore require a more sophisticated approach than validating two edge-cases of DER setpoints to ensure network-feasibility of envelopes.

Robust dynamic operating envelopes (RDOE) [10] recently proposed in literature address this concern. Under this framework, the DSO explicitly derives the collective *feasible region*[2] for DER in unbalanced networks, as would be performed if the DSO was responsible for operating all DER. The DSO then determines DER-level *robust dynamic operating envelopes* such that all combinations of potentially independently-operated DER setpoints remain bounded within the aforementioned network-feasible region. Geometrically, these robust dynamic operating envelopes form a hyper-rectangular *operating region* for DER when plotted in high-dimensional spaces representing collective DER setpoints. The orthogonality of this region is due to the assumption that each DER may be operated independently.

We will demonstrate that this requirement for orthogonality has a restrictive effect on total network capacity available for allocation. To provide a geometric interpretation, non-zero mutual impedances between phases have the effect of distorting an unbalanced network's overall *feasible region* (often defined primarily by voltage constraints) into a slight parallelepiped shape. Network-feasible DER setpoints that result in high feeder-scale imports and exports become inaccessible to orthogonal DER *operating regions*. In practice, large portfolios of DER may be operated by a small number of aggregators, representing opportunities to expand capacity allocation through considered coordination approaches.

In this paper we propose Robust Aggregator operating Regions with flexible Dispatch Envelopes (RAR-DE). The primary distinction of our work compared to the literature is that we leverage aggregators' ability to coordinate their customer dispatch in groups. This coordination allows the DSO to provide structured allocations of network capacity that better span the underlying feasible operating region of unbalanced distribution networks. As a result, aggregators can provide a wider range of capacity to wholesale markets without risking voltage constraint violations in distribution networks. We achieve these structured allocations through robust polytopic projection of the network's feasible region onto aggregator-specific subspaces, within which aggregators can play a coordination role. We note that adherence to complex projected polytopic regions represents additional complexity for aggregator operations. To facilitate aggregator adherence to their robust assigned regions, we propose market-responsive flexible *dispatch envelopes* at the DER-level, that aggregators can compute with trivial complexity when the market clears using simple scalar functions already derived by the DSO.

To summarise, our key contributions are:

- Robust Aggregator operating Regions (RAR), a flexible framework for allocating network capacity to aggregators in a network-safe manner in unbalanced networks. We leverage opportunities for coordination between customer groups to expand feeder-scale capacity allocations when compared to existing robust approaches in unbalanced systems.
- Market-responsive Dispatch Envelopes (DE) for DER, a simple framework for aggregators to adhere to complex robust aggregator regions when satisfying feeder-scale market dispatch outcomes. These DER-level envelopes adjust to market outcomes according to scalar affine functions defined by the DSO. When the market clears, aggregators can compute envelopes immediately through trivial computations, ultimately producing envelopes of the same structure as obtained by conventional dynamic operating envelope approaches.

In Section II we calculate robust aggregator operating regions by applying robust polytopic projection of the network feasible region onto sub-spaces representing groups of DER under an aggregator's control. In Section III we derive scalar functions for aggregators to compute flexible market-responsive dispatch envelopes for DER, meaning that aggregators do not need to model complex polytopic regions. In Section IV we present simulation results, then conclude in Section V.

II. Robust Aggregator Operating Regions (RAR) Ensuring Network-Security

In this section, we first derive the network-feasible operating region for DER in an unbalanced distribution network, as if all DER were hypothetically centrally controlled. We then calculate robust operating regions for each aggregator by applying a robust polytopic projection technique, and demonstrate how this process can be optimised to maximise total aggregator capacity that can be traded in wholesale markets.

A. Power flow model and network feasible region

We apply the linearised power flow model for multi-phase radial networks proposed in [11] to obtain expressions for network state variables as a function of DER setpoints[3]. Let $N^d \in \mathbb{N}$ represent the total number of DER customers in the network, and let $\boldsymbol{p} \in \mathbb{R}^{N^d}$ represent real-power setpoints of all DER in the network.

In this paper, our operational constraints consist of upper and lower bounds applied to voltage magnitudes at customer locations (at node-phase granularity). Let $N^c \geq N^d \in \mathbb{N}$ represent the total number of customers (DER and non-DER), resulting in $2N^c$ total scalar network constraints. We apply fixed background loads across network customers, then

[2]In this paper, we reserve the term *envelope* for an individual DER, and use the term *region* to designate a set of setpoints of multiple DER

[3]The linearised power flow model in [11] introduces small linearisation errors. We evaluate our approach using Monte Carlo simulations in Section IV using a non-convex power flow model. We will show that linearisation errors are small, and can be compensated by applying small buffers to voltage bounds (in our case only ~ 0.002p.u.). Linearisation effects are relatively small compared to the effects of mutual impedances observed in our results

express power flow equations. We compile these scalar constraints in matrix form, then isolate variables representing squared voltage magnitudes. This allows us to express squared voltage magnitudes in the form

$$\boldsymbol{v} = \boldsymbol{V}\boldsymbol{p} + \boldsymbol{v^0} \tag{1}$$

where $\boldsymbol{V} \in \mathbb{R}^{N^c \times N^d}$ is a coefficient matrix, and $\boldsymbol{v^0} \in \mathbb{R}^{N^c}$ represents the squared voltage magnitudes at each customer location under the background-only scenario when dispatchable DER are idle. We impose network operational constraints in the form of the system

$$\boldsymbol{A^v}\boldsymbol{v} + \boldsymbol{b^v} \leq \boldsymbol{0} \tag{2}$$

where $\boldsymbol{A^v} \in \mathbb{R}^{2N^c \times N^c}, \boldsymbol{b^v} \in \mathbb{R}^{2N^c}$. Substituting the expression for squared voltage magnitudes, we obtain

$$(\boldsymbol{A^v}\boldsymbol{V})\boldsymbol{p} + (\boldsymbol{A^v}\boldsymbol{v^0} + \boldsymbol{b^v}) \leq \boldsymbol{0} \tag{3}$$

$$\boldsymbol{A}\boldsymbol{p} + \boldsymbol{b} \leq \boldsymbol{0} \tag{4}$$

where $\boldsymbol{A} \in \mathbb{R}^{2N^c \times N^d}, \boldsymbol{b} \in \mathbb{R}^{2N^c}$. This linear expression defines the network's underlying feasible operating region for DER with respect to voltage constraints under a linear model, which we denote $\boldsymbol{\mathcal{F}} \subset \mathbb{R}^{N^d}$. The process above is also described in more detail in [10].

B. Robust polytopic projection of the network feasible region onto aggregator sub-domains

We now demonstrate how this region can be partitioned favourably between aggregators. Let $N^a \in \mathbb{N}$ represent the number of aggregators in the network, each operating $N^d_a \in \mathbb{N}$ DER customers. We assume that all DER are operated by aggregators, even if $N^d_a = 1$ for some aggregators a. We group DER according to their aggregators, and re-express $\boldsymbol{p}$ as a concatenation of aggregator-specific DER real power dispatch vectors

$$\boldsymbol{p}^\top = \left[\boldsymbol{p^1}^\top \dots \boldsymbol{p^{N^a}}^\top\right] \tag{5}$$

with $\boldsymbol{p^a} \in \mathbb{R}^{N^d_a}$ for $a \leq N^a$. We aim to derive aggregator regions $\boldsymbol{\mathcal{R}^a} \subset \mathbb{R}^{N^d_a}$ for each aggregator's collective DER setpoint $\boldsymbol{p}^a$ such that

$$\forall a \leq N^a : \boldsymbol{p^a} \in \boldsymbol{\mathcal{R}^a} \implies \boldsymbol{p} \in \boldsymbol{\mathcal{F}} \tag{6}$$

or equivalently

$$\boldsymbol{\mathcal{R}^1} \times \boldsymbol{\mathcal{R}^2} \times \cdots \times \boldsymbol{\mathcal{R}^{N^d_a}} \subseteq \boldsymbol{\mathcal{F}} \tag{7}$$

Network capacity allocation among aggregators would ideally achieve equality in (7). In practice, partitioning the network's feasible region between increasing numbers of aggregators may produce latent space within the feasible region $\boldsymbol{\mathcal{F}}$ that becomes inaccessible to DER under robust frameworks.

Begin by defining sub-matrices of $\boldsymbol{A}$ following a similar approach as in (5)

$$\boldsymbol{A} = \left[\boldsymbol{A^1} \dots \boldsymbol{A^{N_a}}\right] \tag{8}$$

where $\boldsymbol{A}^a \in \mathbb{R}^{2N^c \times N^d_a}$ for aggregators $a \leq N^a$. It follows that

$$\boldsymbol{A}\boldsymbol{p} = \sum_{a=1}^{N^a} \boldsymbol{A}^a \boldsymbol{p}^a \tag{9}$$

Each row in (4) represents a network constraint, meaning that rows $\boldsymbol{A}^a_{i,:}$ capture the impacts of aggregator a's DER on each constrained network state variable. Conceptually, we wish to limit each aggregator's contribution to raising the value of each scalar constraint function bounded above in (4). To achieve this, we introduce vectors $\boldsymbol{s^a} \in \mathbb{R}^{2N^c}$ for $a \leq N^a$, and require that

$$\forall a \leq N_a : \boldsymbol{A}^a \boldsymbol{p}^a \leq \boldsymbol{s^a} \tag{10}$$

and additionally require that

$$\sum_{a=1}^{N^a} \boldsymbol{s^a} \leq -\boldsymbol{b} \tag{11}$$

By constraining the cumulative impacts of each aggregator on each scalar constraint function, it then follows that

$$\boldsymbol{A}\boldsymbol{p} + \boldsymbol{b} = \sum_{a=1}^{N^a} (\boldsymbol{A}^a \boldsymbol{p}^a) + \boldsymbol{b} \tag{12}$$

$$\leq \sum_{a=1}^{N^a} (\boldsymbol{s^a}) + \boldsymbol{b} \tag{13}$$

$$\leq -\boldsymbol{b} + \boldsymbol{b} = 0 \tag{14}$$

satisfying (4). We are therefore able to assign network-secure operating regions to aggregators in the form of polytopes $\boldsymbol{\mathcal{R}^a}$, defined by linear systems

$$\boldsymbol{A^a}\boldsymbol{p^a} - \boldsymbol{s^a} \leq \boldsymbol{0} \tag{15}$$

The structure of these aggregator operating regions is inherited from the network's feasible region. Each scalar constraint in the definition of aggregator regions $\boldsymbol{\mathcal{R}}^a \subset \mathbb{R}^{N^d_a}$ is a projection of a scalar constraint defining the network's feasible region $\boldsymbol{\mathcal{F}} \subset \mathbb{R}^{N^d}$ into aggregator-specific domains in $\mathbb{R}^{N^d_a}$. Translation terms s^a_i applied to each constraint ensure robustness with respect to unknown actions of other aggregators.

C. Optimisation of projected polytopes

In this paper, we choose to maximise total DER network-support capacity of aggregators which can be offered to wholesale markets. Let $\underline{\boldsymbol{p^a}}, \overline{\boldsymbol{p^a}} \in \mathbb{R}^{N^d_a}$ for $a \leq N^a$ represent aggregator setpoints that satisfy regions $\boldsymbol{\mathcal{R}^a}$, meaning that

$$\boldsymbol{A^a}\underline{\boldsymbol{p^a}} - \boldsymbol{s^a} \leq \boldsymbol{0} \tag{16}$$

$$\boldsymbol{A^a}\overline{\boldsymbol{p^a}} - \boldsymbol{s^a} \leq \boldsymbol{0} \tag{17}$$

We define feeder-scale import and export capacities allocated to each aggregator as

$$p^a_{\max} = \sum_{i=1}^{N^d_a} \left(\overline{p^a_i}\right) \qquad p^a_{\min} = \sum_{i=1}^{N^d_a} \left(\underline{p^a_i}\right) \tag{18}$$

We maximise aggregator participation in wholesale markets by maximising instantaneously-available import and export capacity of aggregators. This is achieved by solving the following optimisation problem

$$\max_{\boldsymbol{s^1},\dots,\boldsymbol{s^{N^a}}} \quad \sum_{a=1}^{N^a} (p^a_{\max} - p^a_{\min}) \tag{19}$$
$$\text{s.t.} \quad (16), (17) \qquad \forall a \le N^a$$
$$(11)$$

Although our simulations in Section IV apply this objective, our framework is ultimately agnostic to a DSO's chosen objective. The DSO may also choose to maximise a fairness-inspired metric such as proportional fairness, or may choose to maximise the sum of expected market outcomes for aggregators. Each case requires modelling the total generation and load capacity of each aggregator at the feeder-scale.

III. Calculating Market-Responsive Dispatch Envelopes (DE) for DER

We now propose a simple process for aggregators to calculate DER-level dynamic operating envelopes $[\underline{\epsilon^a_i}(m^a), \overline{\epsilon^a_i}(m^a)] \subset \mathbb{R}$, in response to wholesale market dispatch outcomes $m^a \in \mathbb{R}$, that ensure aggregators satisfy regions $\boldsymbol{\mathcal{R}^a}$. These allow aggregators to benefit from the flexible shape of aggregator regions obtained in Section II, without the complexity of explicitly modelling them in their dispatch operations. Our concept is illustrated in Figure 1, and we make further reference to its elements throughout Subsection III-B.

A. Motivation to reduce complexity for aggregators

In Section II-B we explained that each aggregator's region $\boldsymbol{\mathcal{R}^a}$ inherits the same number of scalar constraints as the network's feasible region. In our case study in Section IV, an aggregator representing 9 DER customers in an LV feeder of only 55 customers is assigned a region $\boldsymbol{\mathcal{R}^a}$ defined by 110 scalar inequality constraints. In larger feeders, with hundreds or potentially thousands of connection points, satisfying high-dimensional polytopic aggregator regions may be cumbersome for aggregators.

To facilitate aggregators' adherence to these complex regions, our framework instead provides aggregators scalar affine formulae to directly compute DER-level market-responsive operating envelopes $[\underline{\epsilon^a_i}(m^a), \overline{\epsilon^a_i}(m^a)] \subset \mathbb{R}$ for each DER customer i.

B. Market-responsive envelopes by homothetic transformation

In order to calculate functional envelopes at the DER level, we begin by observing that within each network-secure operating polytope $\boldsymbol{\mathcal{R}^a}$ (grey area, Fig. 1) there exists a hyper-cube $\boldsymbol{\mathcal{C}^a} \subseteq \boldsymbol{\mathcal{R}^a} \subset \mathbb{R}^{N^d_a}$ (blue area, Fig. 1) with edges of length $l^a \geq 0$. This hyper-cube $\boldsymbol{\mathcal{C}^a}$ represents a collection of DER-level operating envelopes for aggregator a, that independently guarantee network-feasibility such as in the robust dynamic operating envelope framework [10]. We also know that extreme points $\underline{\boldsymbol{p^a}}, \overline{\boldsymbol{p^a}} \in \boldsymbol{\mathcal{R}^a}$, and $\boldsymbol{\mathcal{C}^a} \subseteq \boldsymbol{\mathcal{R}^a}$, and that $\boldsymbol{\mathcal{R}^a}$ is a convex polytope. It follows that all hyper-cubes obtained by continuous affine contractions of $\boldsymbol{\mathcal{C}^a}$ towards homothetic centres $\underline{\boldsymbol{p^a}}, \overline{\boldsymbol{p^a}}$ (red area, Fig. 1) are also subsets of $\boldsymbol{\mathcal{R}^a}$. These represent our DER-level market-responsive dispatch envelopes, and can be tailored for specific market outcomes (red line, Fig. 1).

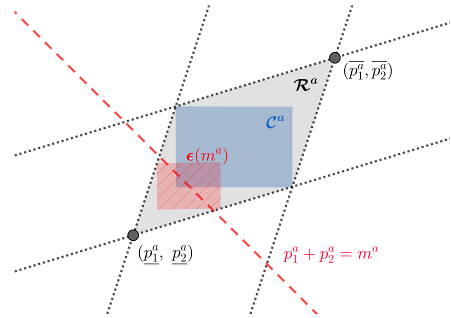


Fig. 1. Illustration of DER-level dispatch envelopes (DE) in response to market dispatch outcome m^a for an aggregator a comprising two DER customers (x-axis representing p^a_1 dispatch, y-axis representing p^a_2 dispatch)

To calculate $\boldsymbol{\mathcal{C}^a}$, define matrices $\underline{\boldsymbol{A^a}}, \overline{\boldsymbol{A^a}} \in \mathbb{R}^{2N^c \times N^d_a}$ containing negative and positive elements of $\boldsymbol{A^a}$ respectively. Entry-wise definitions are given by

$$\underline{A^a}_{ij} = \min({A^a}_{ij}, 0) \tag{20}$$
$$\overline{A^a}_{ij} = \max({A^a}_{ij}, 0) \tag{21}$$

such that

$$\underline{\boldsymbol{A^a}} + \overline{\boldsymbol{A^a}} = \boldsymbol{A^a} \tag{22}$$

We calculate these hyper-cubes $\boldsymbol{\mathcal{C}^a} = \prod_{i=1}^{N^d_a} [\underline{c^a_i}, \overline{c^a_i}]$ for all aggregators in a second-stage optimisation problem, maximising length l^a. This is because we seek to maximise the flexibility with which aggregators can disaggregate their wholesale market dispatch outcomes between DER devices. Calculate $\boldsymbol{\mathcal{C}^a}$ using values of $\boldsymbol{s^a}$ solving

$$\max \quad \sum_{a=1}^{N^a} (l^a) \tag{23}$$
$$\text{s.t.} \quad \overline{\boldsymbol{A^a}}\,\overline{\boldsymbol{c^a}} + \underline{\boldsymbol{A^a}}\,\underline{\boldsymbol{c^a}} - \boldsymbol{s^a} \le \boldsymbol{0} \qquad \forall a \le N^a$$
$$\overline{c^a_i} - \underline{c^a_i} = l^a \qquad \forall i \le N^d_a, \forall a \le N^a$$
$$\underline{c^a_i} \le 0 \le \overline{c^a_i} \qquad \forall i \le N^d_a, \forall a \le N^a$$

We calculate hyper-cubic operating regions $\boldsymbol{\epsilon^a_1}(k), \boldsymbol{\epsilon^a_2}(k) \subset \boldsymbol{\mathcal{R}^a}$, representing sets of DER-level operating envelopes, by applying homothetic transformations to $\boldsymbol{\mathcal{C}^a}$ towards focal points $\underline{\boldsymbol{p^a}}$ and $\overline{\boldsymbol{p^a}}$ respectively. Applying scaling factor $k \in [0, 1]$, we either transform $\boldsymbol{\mathcal{C}^a}$ towards $\overline{\boldsymbol{p^a}}$ by calculating

$$\boldsymbol{\epsilon^a_1}(k) = k\boldsymbol{\mathcal{C}^a} + (1-k)\overline{\boldsymbol{p^a}} \tag{24}$$
$$= \prod_{i=1}^{N^d_a} \left[k\underline{c^a_i} + (1-k)\overline{p^a_i},\ k\overline{c^a_i} + (1-k)\overline{p^a_i}\right] \tag{25}$$
$$\subset \boldsymbol{\mathcal{R}^a} \tag{26}$$

or we transform $\boldsymbol{\mathcal{C}^a}$ towards $\underline{\boldsymbol{p^a}}$ by calculating

$$\boldsymbol{\epsilon^a_2}(k) = k\boldsymbol{\mathcal{C}^a} + (1-k)\underline{\boldsymbol{p^a}} \tag{27}$$

$$= \prod_{i=1}^{N_a^d} \left[k\underline{c_i^a} + (1-k)\underline{p_i^a},\, k\overline{c_i^a} + (1-k)\underline{p_i^a} \right] \tag{28}$$

$$\subset \boldsymbol{\mathcal{R}^a} \tag{29}$$

For a given market dispatch outcome m^a, an aggregator must determine whether to apply transformation (24) or (27), and the value of coefficient k. We aim to provide market-responsive dispatch envelopes at the DER level such that the aggregator's wholesale market dispatch requirement is satisfied when DER operate at the centre of envelopes $[\underline{\epsilon_i^a}(m^a), \overline{\epsilon_i^a}(m^a)]$. This affords aggregators the greatest flexibility to change individual DER setpoints while still satisfying wholesale market obligations.

To achieve this, we first calculate the total aggregator dispatch at the centrepoint of the aggregator's central hypercubic region $\boldsymbol{\mathcal{C}^a}$

$$\widehat{c^a} = \sum_{i=1}^{N_a^d} \frac{\underline{c_i^a} + \overline{c_i^a}}{2}$$

Aggregators can then compute regions $\boldsymbol{\epsilon^a}(m^a)$ such that

- if $m^a \geq \widehat{c^a}$, then

$$\boldsymbol{\epsilon^a}(m^a) = \boldsymbol{\epsilon^a_1}(k) \quad \text{for} \quad k = \frac{p^a_{\text{max}} - m^a}{p^a_{\text{max}} - \widehat{c^a}} \tag{30}$$

- if $m^a < \widehat{c^a}$, then

$$\boldsymbol{\epsilon^a}(m^a) = \boldsymbol{\epsilon^a_2}(k) \quad \text{for} \quad k = \frac{m^a - p^a_{\text{min}}}{\widehat{c^a} - p^a_{\text{min}}} \tag{31}$$

The DSO is able to directly compute $\widehat{c^a}$, and two pairs of scalars for each DER representing parameters for affine functions defining upper and lower envelope bounds as functions of m^a. Values for p^a_{max} and p^a_{min} and envelope function parameters are communicated to aggregators before they submit bids to wholesale markets. After the market clears, aggregators can directly compute market-responsive envelopes $[\underline{\epsilon_i^a}, \overline{\epsilon_i^a}]$ in the event that either $m^a \geq \widehat{c^a}$ or $m^a \leq \widehat{c^a}$. As a result, the total computational complexity for aggregators is limited to evaluating two scalar affine expressions of the form $\alpha_i^a m^a + \beta_i^a$ for each of its DER customers i. This final calculation is trivial, and aggregators obtain DER-level envelopes with the same structure as existing envelope approaches.

IV. Simulations

We compare our approach to approaches in literature on the IEEE 906-bus European Low-Voltage Feeder, requiring that customer voltages remain within $[0.94, 1.1]$ per unit. We assign DER to 27 of 55 customers (49% uptake), and their wholesale market participation is managed by three aggregators. The breakdown of each aggregator's portfolio is outlined in Table I, and customer locations are shown in Figure 2. Each DER customer is assigned a 5kW home battery, and a 5kW solar PV system. Background load and solar generation are modelled

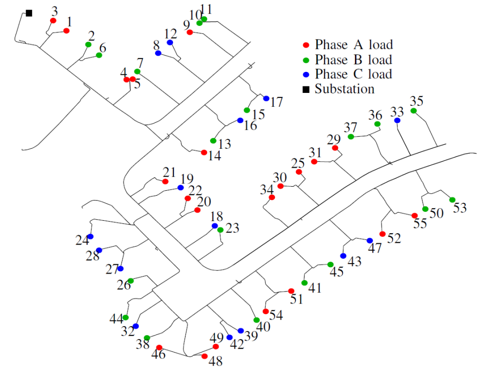


Fig. 2. IEEE 906-bus European low-voltage (LV) test feeder (figure sourced from [12])

TABLE I
Aggregator DER Customers

	Agg. 1	Agg. 2	Agg. 3	
Phase A	1, 5, 48	3, 9, 54	14, 49, 55	(9 of 21 customers)
Phase B	10, 15, 38	23, 36, 40	7, 26, 50	(9 of 19 customers)
Phase C	8, 17, 32	12, 27, 43	16, 19, 39	(9 of 15 customers)

using anonymised smart meter data from Canberra ACT, Australia, shown in Figure 3.

We compare network-security and total market participation outcomes under the following approaches:

- **Dynamic Operating Envelopes (DOE e.g. [1]):** Calculated per DER-customer, satisfying network-security constraints in scenarios of 1) maximum DER imports and 2) maximum DER exports. We model the following policies for aggregators to disaggregate their dispatch:

  (a) **Randomised:** Aggregators disaggregate their wholesale market dispatch outcomes randomly between DER customers, ensuring each satisfies individual envelopes. Under this policy aggregators may only deploy a subset of their DER if their wholesale market dispatch result is small.

  (b) **Merit order:** Aggregators privately compute DER-specific dispatch costs (capturing projected needs of customers with self-consumption preferences, or distribution of battery cycling). These variable costs produce a dynamic merit order hidden from the DSO. Aggregators minimise their dispatch costs by maximising dispatch of their cheapest DER first.

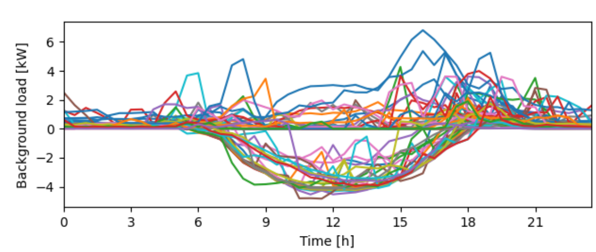


Fig. 3. Real residential background load and solar PV data for 55 customers at half-hour granularity from smart-meter data in Canberra ACT, Australia

- **Robust Dynamic Operating Envelopes (RDOE [10]):** Envelopes are calculated at the DER-level such that network voltage constraints are satisfied under all potential disaggregation scenarios. We apply network-security criteria for envelopes as in [10], and we apply our objective from Section II. We model dispatch disaggregation as under **DOE**.
- **Proposed - Robust Aggregator Regions with flexible Dispatch Envelopes (RAR-DE):** Capacity is allocated at the aggregator-level according to method in Section II, and market-responsive DER-level envelopes are calculated according to method in Section III. To provide a robust comparison, we will model disaggregation of wholesale market dispatch outcomes according to the merit-order approach, as this was observed to pose the greatest risk of constraint violation in results.

We apply the COIN-OR solver (default solver for linear programming using PuLP) on an Intel®Core™ i7-9700 3.00 GHz processor with 16 GB RAM. The average solver time required to generate aggregator feasible regions (Section II) was 0.109 seconds for each market period, and deriving parameters for DER-level operating envelopes (Section III) required an additional 0.037 seconds per aggregator.

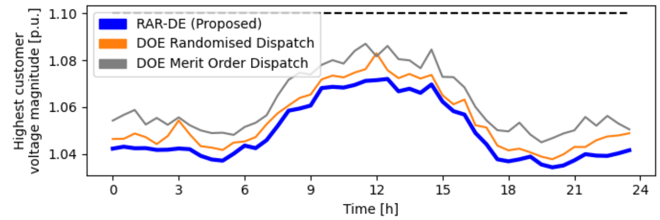
A. Network-security outcomes of Monte-Carlo simulations

For each capacity allocation approach, and for each disaggregation policy considered, repeat the following Monte Carlo simulation protocol:

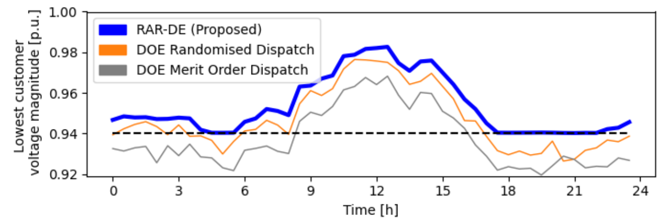
1) Every half-hour (48 time instants), allocate capacity to aggregators in the form of DER-level operating envelopes or aggregator-level operating regions. Validate that voltage constraints are satisfied in cases of maximum DER imports/exports using a non-convex power flow model.
2) Generate 1000 scenarios of wholesale market dispatch outcomes in which aggregators export power to the transmission network, i.e. $0 \leq m^a \leq p^a_{\text{max}}$ for each aggregator.
3) For each scenario, disaggregate wholesale dispatch outcomes to individual DER customers according to a specified disaggregation policy, and evaluate the network state using a non-convex power flow model.
4) Repeat points 2) to 4) modelling wholesale market outcomes in which aggregators import from the transmission network, i.e $p^a_{\text{min}} \leq m^a \leq 0$.

Results demonstrate that our **RAR-DE** approach ensures network constraints are satisfied in all randomised market outcome scenarios[4]. In contrast, the conventional **DOE** method fails to provide this robustness in unbalanced networks.

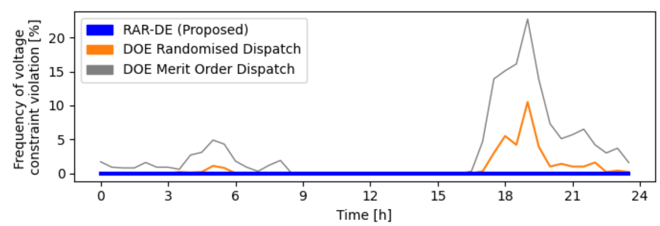
Maximum and minimum voltage magnitudes observed across customer locations are plotted in Figures 4a and 4b. Using the **DOE** approach, customers experience voltage magnitudes as low as 0.926p.u. under randomised disaggregation, and as low as 0.919p.u. under merit-order disaggregation.



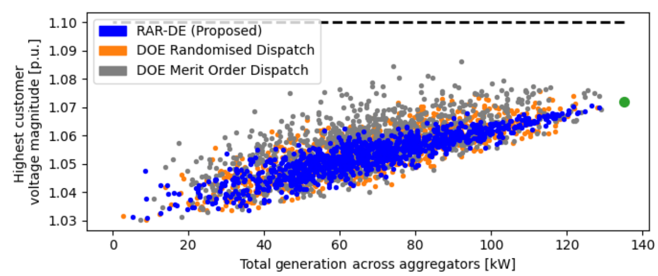
(a) Maximum customer voltages observed for each time instant



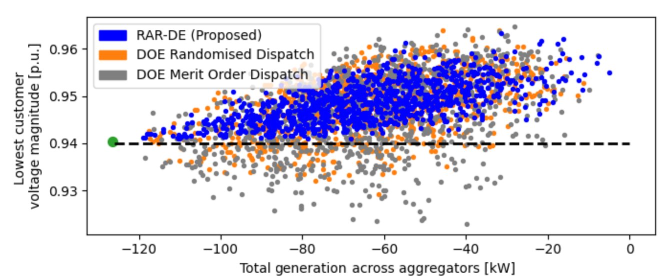
(b) Minimum customer voltages observed for each time instant



(c) Frequency of voltage constraint violation in Monte Carlo analysis



(d) Highest voltage magnitudes in simulations at 12:30pm



(e) Lowest voltage magnitudes in simulations at 7:00pm

Fig. 4. Monte Carlo simulation results

Figure 4c shows voltage constraint violations were observed in morning and evening using the **DOE** approach, with frequency of constraint violation in simulations at 7:00pm reaching 10.5% under randomised disaggregation, and 22.7% under

[4]The **RDOE** approach also ensured network constraints are satisfied in all scenarios. This was an expected result due to inherent robustness in the method. For brevity, we reserve comparisons to **RDOE** until Subsection IV-B.

merit-order disaggregation. While these do not strictly represent probabilities due to our randomised modelling of market dispatch outcomes, these results demonstrate the potential for voltage constraint violation without deliberate mitigation. In contrast, voltage constraints are satisfied in all scenarios when applying the **RAR-DE** approach, demonstrating its robustness with respect to uncertain DER capacity utilisation in unbalanced networks.

Figures 4d and 4e present the distribution of highest and lowest customer voltage magnitudes as a function of total DER dispatch in the network, studying cases of aggregators providing feeder-scale exports at 12:30pm (studying highest voltages) and feeder-scale imports at 7:00pm (studying lowest voltages). Both **RAR-DE** and **DOE** approaches produce similar distributions of extreme customer voltages when overall feeder dispatch is low. As feeder-level dispatch increases towards aggregators' maximum capacities (green dots, common to all approaches as discussed in the sequel), the spread of extreme voltages remains large under **DOE**, resulting in voltage constraint violations. These violations are avoided under **RAR-DE**, and we observe that extreme voltages are bounded by the case of full capacity utilisation (green dot), demonstrating the robustness of our approach.

B. Total market participation outcomes for aggregators

Figure 5 compares total capacity allocations p^a_{min}, p^a_{max} for aggregators under our proposed **RAR-DE** approach and under **RDOE**[5] throughout the day (48 time instants). Our **RAR-DE** approach achieves 29% greater total capacity allocations than **RDOE** on average in this case study, and in general is limited by assumed 5kW DER inverter limits. The **RAR-DE** approach achieved exactly the same capacity allocation outcomes as under the **DOE** approach throughout the day.

This demonstrates our approach successfully combines the favourable feeder-scale capacity allocations of the **DOE** approach with network-feasibility assurances of the **RDOE** approach in unbalanced networks, and this is achieved by leveraging the coordination abilities of aggregators.

C. Illustrations of hyper-cubic regions $\boldsymbol{\mathcal{C}^a}$

We briefly illustrate hyper-cubic inner regions $\boldsymbol{\mathcal{C}^a}$ used to calculate flexible market-responsive dispatch envelopes $\boldsymbol{\epsilon^a}(m^a)$ in Figure 6. In this example, Aggregator 3 (green) will be afforded larger market-responsive envelopes due its larger inner-region $\boldsymbol{\mathcal{C}^3}$. In contrast, Aggregator 2 (blue) will be constrained to rather proportional dispatch. While this may suggest an unfair allocation of capacity between aggregators, it indicates that Aggregator 2 would be more likely to experience curtailment without our flexible framework.

In this example, we have constrained hyper-cubic regions $\boldsymbol{\mathcal{C}^a}$ to be centred around the setpoints where all DER are idle. Aggregators may wish for this constraint to be relaxed, or even

[5]We compare our approach to the instance of **RDOE** in [10] that does not assume knowledge of the operational status of individual DER. This is because DER may be dispatched as imports or exports, depending on the wholesale market outcomes for aggregators.

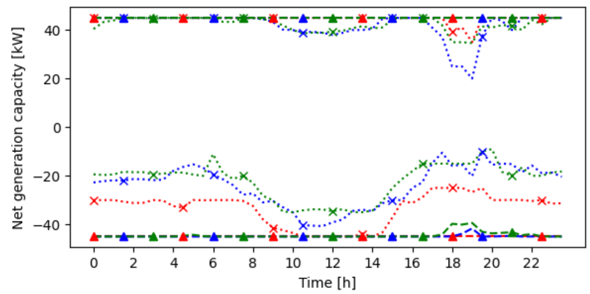


Fig. 5. Total capacity $p^a_{\text{min}}, p^a_{\text{max}}$ allocated to aggregators (identified by colour) under **RAR-DE** and **DOE** (dashed, triangles) and **RDOE** (dotted, crosses) approaches.

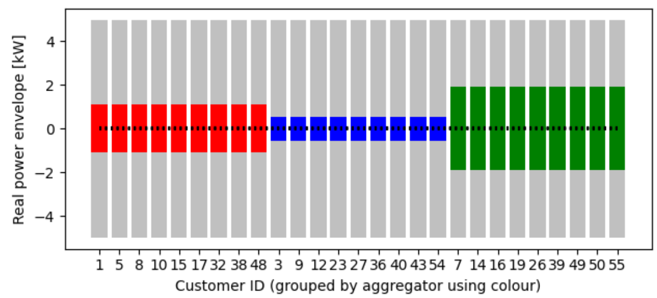


Fig. 6. Example of total capacity allocations $\underline{p^a_i}, \overline{p^a_i}$ under **RDOE** and **RAR-DE** approaches at the DER-level (grey), and inner hyper-cubic regions $\boldsymbol{\mathcal{C}^a}$ (coloured) used as the basis for calculating flexible DER-level dispatch envelopes $\boldsymbol{\epsilon^a}$

consider hyper-*rectangular* regions instead of hyper-*cubic* regions to increase overall flexibility afforded by envelopes $\boldsymbol{\epsilon}^a$. In future work we will investigate the impacts of ensuring minimum dispatch envelope sizes to account for aggregators' dispatch uncertainty.

V. Conclusion

We have proposed a flexible approach to allocate constrained distribution network capacity to aggregators in unbalanced systems. Results demonstrate that our **RAR-DE** approach combines the benefits of **DOE** and **RDOE** approaches, achieving favourable market participation outcomes for aggregators while also ensuring satisfaction of network voltage constraints in unbalanced networks. This is achieved by leveraging aggregators' ability to coordinate the dispatch of their customers in response to outcomes in wholesale markets. This enables the DSO to assign aggregators more complex operating regions, collectively achieving greater coverage of the network's underlying feasible region than existing robust approaches. Our proposed market-responsive dispatch envelopes defined at the DER-level eliminate the need for aggregators to model these complex versatile regions in their operations, instead requiring only trivial computation after markets clear to generate conventionally-structured operating envelopes. In future work we will investigate the effects of requiring minimum dispatch envelope sizes for aggregators, explore the potential for real-reactive power co-optimisation and incorporate uncertainty of background load.

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