Managing Power Balance and Reserve Feasibility in the AC Unit Commitment Problem

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***Abstract*—Incorporating the AC power flow equations into unit commitment models has the potential to avoid costly corrective actions required by less accurate power flow approximations. However, research on unit commitment with AC power flow constraints has been limited to a few relatively small test networks. This work investigates large-scale AC unit commitment problems for the day-ahead market and develops decomposition algorithms capable of obtaining high-quality solutions at industry-relevant scales. The results illustrate that a simple algorithm that only seeks to satisfy unit commitment, reserve, and AC power balance constraints can obtain surprisingly high-quality solutions to this AC unit commitment problem. However, a naive strategy that prioritizes reserve feasibility leads to AC infeasibility, motivating the need to design heuristics that can effectively balance reserve and AC feasibility. Finally, this work explores a parallel decomposition strategy that allows the proposed algorithm to obtain feasible solutions on large cases within the two hour time limit required by typical day-ahead market operations.**

***Index Terms*—AC power flow, Optimization, Reserve products, Unit commitment**

Nomenclature

Sets

T	Set of time periods t
J	Set of dispatchable devices j
$J^{\{\text{cs,pr}\}}$	Set of {consuming, producing} devices
$J_i^{\{\text{cs,pr}\}}$	Set of {consuming, producing} devices at bus i
J^{ac}	Set of AC transmission lines
I	Set of buses i
$J_i^{\{\text{fr,to}\}}$	Set of AC transmission lines {from, to} bus i
N^p	Set of active reserve zones n
$J_n^{\text{pr,cs}}$	Set of producing **and** consuming devices in active reserve zone n

Parameters

d_t	Duration of time period t
p_{jt}^{min}	Minimum online power of device j at t
p_{jt}^{max}	Maximum online power of device j at t
p_{jt}^{ru}	Maximum online ramp-up rate of device j at t
$p_{jt}^{\text{ru,su}}$	Maximum start-up ramp rate of device j at t
p_{jt}^{rd}	Maximum online ramp-down rate of device j at t
$p_{jt}^{\text{rd,sd}}$	Maximum shut-down ramp rate of device j at t
g_j^{sr}	Series conductance of line j
$g_j^{\{\text{fr,to}\}}$	"{From, To}"-side conductance of line j
b_j^{sr}	Series susceptance of line j
$b_j^{\{\text{fr,to}\}}$	"{From, To}"-side susceptance of line j
b_j^{ch}	Charging susceptance of line j
$p_{nt}^{\text{rgu,req}}$	Regulation up reserve requirement in zone n at t
$p_{nt}^{\text{scr,req}}$	Synchronous reserve requirement in zone n at t

Binary variables

u_{jt}^{on}	On status of device j at t
$u_{jt}^{\{\text{su,sd}\}}$	{Start-up, Shut-down} status of device j at t

Continuous variables

p_{jt}	Dispatched real power of device j at t
p_{it}	Real power mismatch at bus i at t
q_{it}	Reactive power mismatch at bus i at t
$p_{jt}^{\{\text{fr,to}\}}$	Real power through the "{from, to}" side of line j
$q_{jt}^{\{\text{fr,to}\}}$	Reactive power through the "{from, to}" side of line j
z_{jt}	Operating cost of device j at t
v_{it}	Voltage magnitude of bus i at t
θ_{it}	Voltage angle of bus i at t
p_{jt}^{rgu}	Regulation up reserve provided by device j at t
p_{jt}^{scr}	Synchronous reserve provided by device j at t
$p_{jt}^{\text{ru,on}}$	Online ramp-up reserve provided by device j at t
$p_{nt}^{\text{rgu,+}}$	Regulation up reserve shortfall in zone n at t
$p_{nt}^{\text{scr,+}}$	Synchronous reserve shortfall in zone n at t

I. Introduction

The unit commitment problem is a power system scheduling task that is solved in day-ahead and real-time power markets to balance supply with demand at low cost and ensure feasibility with respect to physical, reliability, and operational constraints [1]. Due to the size of networks that grid operators consider (thousands to tens of thousands of buses), discrete commitment decisions, nonlinear alternating current (AC) physics that must be respected, and strict time limits within which commitment decisions must be made, solving the unit commitment problem is a challenging optimization task. The current standard practice is for market operators to use a direct current (DC) approximation of the power flow physics, which allows the unit commitment problem to be formulated as a mixed-integer linear program (MIP) and solved with established and

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performant commercial solvers such as CPLEX, Gurobi, and Xpress [2].

While MIP technology has proven effective in saving hundreds of millions of dollars annually in U.S. markets compared to previously used less accurate Lagrangian relaxation approaches [2], [3], MIP-based unit commitment with a DC approximation requires post-market clearing corrective action to ensure AC feasibility. In a study on the 24-bus IEEE RTS test network, Castillo et al. estimate a 1% cost increase due to these corrections compared to the solution of the unit commitment problem with a full AC power flow model [4]. However, unit commitment with AC power flow constraints (AC Unit Commitment, or AC-UC) on industry-scale networks with practical time limits have not been widely demonstrated.

To investigate whether AC-UC problems can be solved for large-scale networks within realistic time limits, the U.S. Department of Energy's Advanced Research Projects Agency-Energy (ARPA-E) has developed the Grid Optimization Competition (GOC), in which teams compete to develop solvers for the AC-UC problem as specified in the GOC Challenge 3 problem formulation [5]. This problem formulation strives to be representative of modern power market optimization models and includes unit commitment and ramping constraints, AC power flow, variable loads, real and reactive reserve products, startup/shutdown trajectories, line switching, and contingency constraints. The full optimization problem as written in [5] is a large-scale nonconvex, multi-period, multi-scenario mixed-integer nonlinear program (MINLP). To realize the benefits of using this AC-UC problem in real-world electricity markets, high-quality solutions to large-scale instances of this problem must be computed within realistic time limits.

Using the Grid Optimization Competition challenge 3 problem as foundation, this work aims to develop the simplest algorithm capable of obtaining high-quality solutions to the AC-UC problem for the day-ahead market within the two hour time limit prescribed by the competition rules. This "benchmark algorithm" is intended to help inform researchers and market software providers about what features of the AC-UC problem are necessary for a viable solution approach. To this end, the results presented in this paper indicate that: (1) off-the shelf optimization solvers are incapable of solving the full GOC challenge 3 problem within specified time limits; (2) it is possible with current optimization methods (e.g., Gurobi and Ipopt) to develop high-quality heuristics for the AC-UC problem, which can solve industry-scale instances within the reasonable time limits; (3) one of the key challenges in developing such heuristics is to manage the competing requirements of the AC power balance and reserve allocation constraints; (4) problem decomposition and parallelization across multiple cores is essential to achieving the runtime requirements in large datasets with thousands of buses.

The next section begins by introducing the core features of the AC-UC problem that is considered in this work and motivates the aspects of the problem that make it challenging in practice.

II. Problem formulation and scale

The unit commitment formulation considered maximizes market surplus by scheduling dispatchable devices (generators and dispatchable loads) over a 48 hour time horizon (i.e. $T = \{1, \ldots, 48\}$). A simplified version of this unit commitment problem is given by Equation (1).

$$\begin{aligned}
\max_{p_{jt}, u_{jt}} \quad & \sum_{t \in T} d_t \left(\sum_{j \in J^{\text{cs}}} z_{jt} - \sum_{j \in J^{\text{pr}}} z_{jt} \right) \\
\text{s.t.} \quad & \sum_{j \in J^{\text{cs}}} p_{jt} = \sum_{j \in J^{\text{pr}}} p_{jt} && \forall t \in T \\
& \text{Constraints } (2) - (4) \\
& z_{jt} = f_{z_j}(p_{jt}) && \forall t \in T, j \in J \\
& u^{\text{on}}_{jt}, u^{\text{su}}_{jt}, u^{\text{sd}}_{jt} \in \{0,1\}, p_{jt} \in \mathbb{R} && \forall t \in T, j \in J
\end{aligned} \tag{1}$$

This formulation contains a copper-plate real power balance, semi-continuous real power constraints (2), ramping constraints (3) and (4), and a convex piecewise-linear function f_{z_j} for the operating cost of each device. Here, binary variables u^{on}_{jt}, u^{su}_{jt}, and u^{sd}_{jt} encode a device's status as either "on", "starting up", or "shutting down". For example, $u^{\text{on}}_{jt} = 1$ means that device j is scheduled to be online at time period t. In this case, the device's real power p_{jt} must be within its bounds p^{min}_{jt} and p^{max}_{jt}. If $u^{\text{on}}_{jt} = 0$, the device's real power must be zero. Binary variables for start-up and shut-down are used to encode different maximum ramp rates for devices in these states, as implemented in (3) and (4). For more information on unit commitment problems and formulations, see [1], [6]–[8].

$$u^{\text{on}}_{jt} p^{\text{min}}_{jt} \leq p_{jt} \leq u^{\text{on}}_{jt} p^{\text{max}}_{jt} \;\; \forall t \in T, j \in J \tag{2}$$

$$p_{jt} - p_{j,t-1} \leq \begin{array}{l} d_t \left(p^{\text{ru}}_j \left(u^{\text{on}}_{jt} - u^{\text{su}}_{jt} \right) \right. \\ \left. \quad + p^{\text{ru,su}}_j \left(u^{\text{su}}_{jt} + 1 - u^{\text{on}}_{jt} \right) \right) \end{array} \;\; \forall t \in T, j \in J \tag{3}$$

$$p_{jt} - p_{j,t-1} \geq -d_t \left(p^{\text{rd}}_j u^{\text{on}}_{jt} + p^{\text{rd,sd}}_j \left(1 - u^{\text{on}}_{jt} \right) \right) \;\; \forall t \in T, j \in J \tag{4}$$

Binary variables u^{on}_{jt}, u^{su}_{jt}, and u^{sd}_{jt} are the primary source of discrete variables in the GOC AC-UC problem, and ramping constraints (3) and (4) are the primary source of intertemporal linking. The full formulation of this problem can be found in the ARPA-E Grid Optimization Competition Challenge 3 Problem Formulation [5]. Including all specifications on input data, this formulation contains over 300 equations and is omitted here for brevity. The notation in this work is consistent with the problem formulation, which may be consulted for further reference. The remainder of this section summarizes key features of the problem formulation that differentiate it from previous work in the area.

A. AC network constraints

The unit commitment problem in this work considers AC transmission lines and the associated power flow (derived from Ohm's law) and power balance (derived from Kirchhoff's first law) using the polar power-voltage formulation. The power flow equations are defined with i^{fr}_j and i^{to}_j denoting the buses on either side of line j. Real and reactive power on each side of an AC transmission line are given by (5)-(8). For more information on this power flow formulation, see [9]. The power flow equations are nonlinear, nonconvex constraints and add significant complexity when incorporated in the unit commitment problem. The GOC problem formulation also considers DC lines, transformer branches, and control thereof [5]. The algorithms presented in this work but keep transformer operating settings fixed but are free to adjust the power flow along a DC line.

$$p^{\text{fr}}_{jt} = u^{\text{on}}_{jt} \left(\left(g^{\text{sr}}_j + g^{\text{fr}}_j \right) v^2_{it} + \left(-g^{\text{sr}}_j \cos\left(\theta_{it} - \theta_{i't}\right) \; -b^{\text{sr}}_j \sin\left(\theta_{it} - \theta_{i't}\right)\right) v_{it} v_{i't} \right), \quad \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j \tag{5}$$

$$q^{\text{fr}}_{jt} = u^{\text{on}}_{jt} \left(\left(-b^{\text{sr}}_j - b^{\text{fr}}_j - b^{\text{ch}}_j/2 \right) v^2_{it} + \left(b^{\text{sr}}_j \cos\left(\theta_{it} - \theta_{i't}\right) \; -g^{\text{sr}}_j \sin\left(\theta_{it} - \theta_{i't}\right)\right) v_{it} v_{i't} \right), \quad \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j \tag{6}$$

$$p^{\text{to}}_{jt} = u^{\text{on}}_{jt} \left(\left(g^{\text{sr}}_j + g^{\text{to}}_j \right) v^2_{i't} + \left(-g^{\text{sr}}_j \cos\left(\theta_{it} - \theta_{i't}\right) \; -b^{\text{sr}}_j \sin\left(\theta_{it} - \theta_{i't}\right)\right) v_{it} v_{i't} \right), \quad \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j \tag{7}$$

$$q^{\text{to}}_{jt} = u^{\text{on}}_{jt} \left(\left(-b^{\text{sr}}_j - b^{\text{to}}_j - b^{\text{ch}}_j/2 \right) v^2_{i't} + \left(b^{\text{sr}}_j \cos\left(\theta_{it} - \theta_{i't}\right) \; -g_j \sin\left(\theta_{it} - \theta_{i't}\right)\right) v_{it} v_{i't} \right), \quad \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j \tag{8}$$

Real and reactive power balance equations are given by (9) and (10). Bus power variables p_{it} and q_{it} are mismatch slack variables, or power balance violations, that are penalized in the objective function. In this way, the power balance equations are soft constraints that always have a feasible solution.

$$\sum_{j \in J^{\text{cs}}_i} p_{jt} + \sum_{j \in J^{\text{fr}}_i} p^{\text{fr}}_{jt} + \sum_{j \in J^{\text{to}}_i} p^{\text{to}}_{jt} = \sum_{j \in J^{\text{pr}}_i} p_{jt} + p_{it} \quad \forall t \in T, i \in I \tag{9}$$

$$\sum_{j \in J^{\text{cs}}_i} q_{jt} + \sum_{j \in J^{\text{fr}}_i} q^{\text{fr}}_{jt} + \sum_{j \in J^{\text{to}}_i} q^{\text{to}}_{jt} = \sum_{j \in J^{\text{pr}}_i} q_{jt} + q_{it} \quad \forall t \in T, i \in I \tag{10}$$

In the GOC problem formulation [5] the AC power flow constraints also include shunt devices which are controlled via a collection of discrete "steps". However, shunt steps are considered fixed by all algorithms in this work, so these terms are omitted for brevity.

B. Reserve products

To ensure a power network has sufficient capacity to balance small deviations in supply and demand and to provide backup in case of an unexpected disturbance or shut-down, dispatchable devices provide reserve products in addition to actual (i.e., planned) generation or consumption [10], [11]. The unit commitment formulation considered in this work contains reserve products for both real and reactive power capacity of all dispatchable devices, which includes both generators and loads. Reserve products are categorized as "up" or "down" according to whether they are provided by a device's capacity to add or remove net power from the system. Reserves must be satisfied within active and reactive reserve zones, each of which partition the dispatchable devices based on which network buses they are connected to. The reserve requirements for each zone are implemented as soft constraints with a slack variable that is penalized in the objective if the requirement is not satisfied. The reserve balance equations for regulation up reserve and synchronous reserve are given as examples in (11) and (12). Reserves for each device are constrained by the "headroom" to the device's upper or lower bounds depending on the type of reserve and the type of device. For example, regulation up, synchronous, and online regulation ramp-up reserves are constrained by the headroom to a producer's upper bound, as in (13), or the headroom to a consumer's lower bound, as in (14). In all, the GOC problem formulation considers eight active reserve products and two reactive reserve products. In this work the vectors p^{RES} and q^{RES} are used to denote an assignment of all active and reactive reserve products for all dispatchable devices across all time periods. Higher-quality reserve products, such as regulation up, can be used simultaneously for lower-quality products such as synchronized reserve, as implemented in (11) and (12).

$$\sum_{j \in J^{\text{pr,cs}}_n} p^{\text{rgu}}_{jt} + p^{\text{rgu},+}_{nt} \geq p^{\text{rgu,req}}_{nt}, \quad \forall t \in T, n \in N^p \tag{11}$$

$$\sum_{j \in J^{\text{pr,cs}}_n} \left(p^{\text{rgu}}_{jt} + p^{\text{scr}}_{jt} \right) + p^{\text{scr},+}_{jt} \geq p^{\text{rgu,req}}_{nt} + p^{\text{scr,req}}_{nt}, \quad \forall t \in T, n \in N^p \tag{12}$$

$$p + p^{\text{rgu}}_{jt} + p^{\text{scr}}_{jt} + p^{\text{rru,on}}_{jt} \leq p^{\max}_{jt} u^{\text{on}}_{jt}, \quad \forall t \in T, j \in J^{\text{pr}} \tag{13}$$

$$p - p^{\text{rgu}}_{jt} - p^{\text{scr}}_{jt} - p^{\text{rru,on}}_{jt} \geq p^{\min}_{jt} u^{\text{on}}_{jt}, \quad \forall t \in T, j \in J^{\text{cs}} \tag{14}$$

In addition to penalties incurred if reserve requirements are not satisfied, devices may incur additional cost by providing reserves. As reserve constraints are linear and reserve variables are continuous, the problem of balancing reserve shortfall penalties with device reserve costs may be modeled as a linear program (LP) for fixed power levels and commitment decisions p, q, and u. While these constraints are not complicated individually, the sheer number of constraints and variables required to encode the full reserve model (five headroom constraints and ten reserve variables per device) adds significant complexity to the AC-UC problem.

C. Contingency constraints

Security constraints are an important feature of electric power markets, and may be incorporated into Unit Commitment problems [12] or Optimal Power Flow problems [13]. The GOC AC-UC problem considers security constraints that penalize branch flow thermal limit violations in a defined set of contingencies, where each contingency is defined as the loss of a single specified branch in the network. While solutions computed in this work are evaluated in the context of these contingency constraints, the algorithms presented do not account for them as they did not contribute to significant penalties in the objective function value.

D. Problem scale

Early approaches to AC Unit Commitment applied Lagrangian relaxation [14] and Benders decomposition [15] to problems defined on the IEEE 118-bus network. Fu et al. have considered a security-constrained version of this problem on the same network [16] using an augmented Lagrangian and Benders decomposition approach. Castillo et al. apply an outer approximation method to solve AC-UC problems on networks of up to 118 buses and compare with a method that employs DC unit commitment followed by an AC feasibility solve [4], while Tejada-Arango et al. benchmark a direct mixed integer nonlinear programming (MINLP) formulation, an approach based on sequential linear programming, and a second-order conic programming formulation [17]. These previous works on the AC-UC problem have applied exact local MINLP algorithms to this problem and have only demonstrated scalability up to power networks of on the order of 100 buses and 24 time points, which is far from the size desired by industrial practitioners. Section IV-A investigates the scalability of the Knitro [18] MINLP solver on AC-UC problems and confirms that large-scale instances are out of reach. This work considers AC-UC problems on the Grid Optimization Competition ranging from 73 buses to more than 8,000 buses with 48 time points, and approaches solving such large problems by decomposing the full AC-UC problem into several subroutines and parallelizing these tasks across multiple cores.

III. Methods

The decomposition algorithms that are developed in this work rely on several subroutines, each focusing on different aspects of the complete AC-UC problem. This section introduces these subroutines then describes how they are combined into four distinct heuristic solution approaches.

A. Subroutines

1) Copper-plate scheduling: The unit commitment approach taken is to model the full-horizon scheduling problem as a MIP. For tractability, the scheduling problem considers only copper-plate real and reactive power balances. That is, no network model is considered. Only total demand and generation must match, as if all devices are connected to a single conductive plate. The full zonal reserve requirement model is considered in this subproblem as well. The outputs of this subproblem are discrete commitment decisions u, real and reactive power estimates p and q, and real and reactive reserve commitments p^{RES} and q^{RES}.

2) AC optimal power flow (AC-OPF): This work achieves AC feasibility by solving an AC optimal power flow (AC-OPF) problem at individual time periods t. Each subproblem has the discrete commitment decisions fixed and only considers devices that are online in time period t. Device costs are convex piecewise-linear cost functions modeled with the Δ formulation described by [19]. In this work, shunts and transformer taps are fixed to their operating conditions at the initial point. To keep AC-OPF problem sizes manageable, reserve constraints are not considered by these subproblems. Incorporating these constraints would add ten additional variables and five additional inequality constraints per dispatchable device and several additional variables and inequality constraints per reserve zone, nearly doubling the size of the model.

3) Reserve allocation: While reserve products and zonal reserve requirements are incorporated in the unit commitment model, they are not explicitly considered by the AC-OPF model. For this reason, care is taken to reallocate feasible reserves after the commitment schedule and AC-feasible power dispatch is generated. The algorithms discussed in Section III-B differ primarily in how they allocate reserve products based on the methods discussed here.

Greedy reserve allocation, the first method considered for allocating reserves, is a simple greedy procedure that, for every dispatchable device, attempts to allocate the headroom to the device's upper and lower bounds to the appropriate reserve products, starting with the highest value products. In this approach, the cost of providing a reserve and the zonal reserve requirements are not considered. That is, reserve is assigned for every device possible even if doing so is not necessary to satisfy the requirements. This is motivated by the observation that, in most cases, the penalty for failing to meet a reserve requirement is much larger than the cost of providing the reserve, which is often zero.

Tighten device bounds, the second method considered, fixes real reserve products computed by the copper-plate unit commitment schedule on all devices where doing so does not lead to a guaranteed local power balance violation on the device's bus. A local power balance violation occurs if, for example, a lone producing device on a bus is scheduled to provide more regulation down reserve than the sum of upper bounds on real power that can be transmitted from adjacent lines. For devices where this type of violation does not occur, bounds are tightened on device real power variables p to guarantee that the reserves committed by the unit commitment problem are feasible after the AC-OPF subproblems. If a large number of devices provide reserves, this method can become overly restrictive for the AC-OPF subproblems and can lead to power balance violations. For this reason, a variation of this method is considered where only some fraction γ of devices providing real power reserve have their bounds tightened. When $\gamma < 1$, devices are sorted by the value of reserve that they provide and the top γ of them are constrained in the AC-OPF subproblems. This method can be also be used in conjunction with post-AC-OPF reserve allocation.

Reserve re-dispatch via linear programs, the final method, computes reserve products post-AC-OPF by fixing commitment decisions u and dispatched real and reactive power levels p and q by solving the reserve model (zonal reserve balances, headroom constraints, costs, and penalties) as a linear program (LP). These LPs are independent at each time period and are solved in parallel.

B. Decomposition algorithms

The subroutines described in Section III-A are combined into the follow four algorithms for solving the full AC-UC problem.

Algorithm 1 is a simple decomposition designed to mimic a basic version of current industrial practice. A commitment schedule u, along with nominal real power p, reactive power q, and real reserve products p^{RES}, is computed via the copper-plate unit commitment problem described in Section III-A1. Co-optimization of commitment decisions with real power reserves is chosen to mimic a joint reserve scheduling market [20]. Real reserve products are then fixed, imposing tightened bounds on real power. AC-OPF subproblems are solved sequentially, with a bound-tightening step before each solve to ensure feasibility of ramping constraints. Reactive power reserve products are allocated after solving AC-OPF subproblems by the greedy strategy described in Section III-A3.

Algorithm 1 Reserve-preserving decomposition

Initialize: $p, q, u \leftarrow$ initial status
$p, q, u, p^{\mathrm{RES}} \leftarrow$ Copper-plate unit commitment
Tighten bounds on p by fixing p^{RES}
for $t = 1 \dots 48$ **do**
    Tighten bounds on p_t using ramping constraints
    $p_t, q_t \leftarrow$ AC-OPF at t
end for
$q^{\mathrm{RES}} \leftarrow$ Greedy reserve allocation
return $p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}}$

Algorithm 2 is designed to prioritize unit commitment-feasible, AC-feasible solutions. It solves a copper-plate unit commitment problem to obtain a commitment schedule and nominal real and reactive powers, then solves AC-OPF subproblems sequentially. After solving AC-OPF subproblems, real and reactive powers are fixed and both real and reactive reserve products are allocated using the greedy approach.

Algorithm 2 Simple greedy decomposition

Initialize: $p, q, u \leftarrow$ initial status
$p, q, u \leftarrow$ Copper-plate unit commitment
for $t = 1 \dots 48$ **do**
    Tighten bounds on p_t using ramping constraints
    $p_t, q_t \leftarrow$ AC-OPF at t
end for
$p^{\mathrm{RES}}, q^{\mathrm{RES}} \leftarrow$ Greedy reserve allocation
return $p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}}$

Algorithm 3 uses the same basic decomposition, but is refined by the observation that tightening bounds on a large number of devices can lead to AC infeasibility. It employs the same bound tightening strategy as Algorithm 1 but only for the top 5% of devices providing reserve value, as described in Section III-A3. After AC-OPF subproblems, reserves are re-computed via linear programs that balance costs with zonal reserve penalties subject to fixed real power, reactive power, and on status variables previously computed.

Algorithm 3 Reserve/AC-balancing heuristic

Parameter $\gamma = 5$
Initialize: $p, q, u \leftarrow$ initial status
$p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}} \leftarrow$ Copper-plate unit commitment
Tighten p bounds by fixing p^{RES} for top $\gamma\%$ of devices
for $t = 1 \dots 48$ **do**
    Tighten bounds on p_t using ramping constraints
    $p_t, q_t \leftarrow$ AC-OPF at t
end for
$p^{\mathrm{RES}}, q^{\mathrm{RES}} \leftarrow$ Reserve re-dispatch via linear programs
return $p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}}$

Algorithm 4 is the balancing heuristic algorithm with a parallel decomposition of the AC-OPF subproblems at individual time periods. This is followed by a sequential projection of real power into the bounds implied by ramping constraints at the previous time point. The resulting solutions satisfy the unit commitment and ramping constraints, have modest AC power balance violations due to ramping constraint projections, and have reserves that are balanced between the pre-OPF copper-plate reserve schedule and the post-OPF reserve re-dispatch. This algorithm variant is comparable to the solution method that was used for the "ARPA-E Benchmark" algorithm in Event 4 of GOC Challenge 3.

Algorithm 4 Parallel decomposition heuristic

Parameter $\gamma = 5$
Initialize: $p, q, u \leftarrow$ initial status
$p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}} \leftarrow$ Copper-plate unit commitment
Tighten p bounds by fixing p^{RES} for top $\gamma\%$ of devices
for $t = 1 \dots 48$ in parallel **do**
    $p_t, q_t \leftarrow$ AC-OPF at t
end for
for $t = 1 \dots 48$ **do**
    Project p_t to satisfy ramping constraints
end for
$p^{\mathrm{RES}}, q^{\mathrm{RES}} \leftarrow$ Reserve re-dispatch via linear programs
return $p, q, u, p^{\mathrm{RES}}, q^{\mathrm{RES}}$

These decomposition algorithms consider all hard constraints of the GOC problem formulation [5]. That is, omitted features such as contingency constraints are modeled as soft constraints. As such, solutions produced by these algorithms are guaranteed to be feasible for the GOC problem formulation. However, as all of these algorithms are heuristics, they provide no particular guarantees on solution speed or quality. To address this, future work may focus on using these decompositions in a Benders decomposition or outer approximation framework to develop approaches that, given enough time, are guaranteed to converge to a locally optimal solution. The next section conducts detailed experiments on a variety of realistic network instances to explore the strengths and weaknesses of these algorithms.

TABLE I
RESULTS OF SOLVING FULL AC-UC PROBLEM WITH KNITRO

Case ID	N. Var.	N. Con.	Objective ($)	Solve time (s)
S0N00003-003	2260	3850	9.07e+05	71
S0N00014-003	22080	26183	2.23e+06	33
S0N00037-003	43584	47494	1.07e+07	20
N00073-333	159072	195415	2.30e+08	84
N00617-002	627768	705544	N/A	>10000

TABLE II
STATISTICS FOR THE EVENT 4 NETWORKS CONSIDERED

Buses	Devices	AC lines	p res. zones	q res. zones
73 (scenario 991)	208	127	1	1
73 (other scenarios)	205	105	1	1
617	499	723	10	10
2000	1894	2345	4	10
4224	2151	2605	2	2
6049	3774	4920	6	6
6717	5826	7173	9	12
8316	5585	7723	7	7

IV. RESULTS AND DISCUSSION

A. Full AC-UC problem evaluation

To motivate the use of heuristic decomposition algorithms for large scale instances of the AC-UC problem, we first evaluate the performance of Knitro 14.0 on small instances of the full AC-UC problem, using a heuristic branch-and-bound algorithm [21] with nonlinear programming (NLP) subproblems solved by an interior point method [22]. We use small networks and construct an AC-UC problem containing only unit commitment and AC network constraints (*i.e.* omitting reserve products). The test datasets have 3, 14, 37, 73, and 617 buses, and come either from GOC Event 4 or the "sandbox" dataset, indicated by "S0" in the Case ID. Solve times and objective values for these small instances of the simplified problem are shown in Table I. The results indicate that, even for this simplified problem, Knitro is unable to solve a 617-bus instance of the full AC-UC problem as an MINLP with 627,768 variables and 705,544 constraints. The largest instance considered in this work, with 8,316 buses, has 8,260,224 variables and 8,947,330 constraints in this simplified version of the AC-UC problem. This is well beyond the scope of off-the-shelf MINLP solvers, and motivates the use of heuristic decomposition methods to produce high-quality solutions for industry-scale instances of this problem.

B. Test data

The algorithms presented in Section III-B are evaluated in terms of solution quality and solve time on 28 day-ahead AC-UC cases provided by Event 4 of the Grid Optimization Competition, Challenge 3. Networks with 73, 617, 2,000, 4,224, 6,049, 6,717, and 8,316 buses are considered. Four representative scenarios are considered for each network. Problem data for each network are given in Table II. Except for the 73-bus network, all scenarios for a given network have the same numbers of dispatchable devices, AC lines, and reserve zones. Each case is assigned a unique identifier ("Case ID") using the network size and the scenario number from GOC Event 4. The objective values of solutions are computed by an independent evaluation program provided by the GOC that considers the full problem formulation [23].

C. Computational Setting

All results in this section are computed using HPE ProLiant XL170r servers with two Intel 2.10 GHz CPUs and 128 GB of memory using Julia v1.6 [24]. These machine have 36 physical cores and up to 72 simultaneous threads. Optimization models are constructed with JuMP v1.14 [25]. The unit commitment MIP models are solved with Gurobi v10.0 and the AC-OPF subproblems are solved with Ipopt v3.14 [26] using the MA27 linear solver [27] and a symbolic automatic differentiation approach [28] that is similar to the approach of Gravity [29]. To reduce the runtime of ill-conditioned instances that require many interior-point iterations to converge, solves are terminated via callback if they have reached a 10^{-3} unscaled primal residual and a 1.0 unscaled dual residual. The reserve assignment LPs are solved using HiGHS v1.5 [30].

Implementations of the models and subroutines used in this work can be found at https://github.com/lanl-ansi/GOC3Benchmark.jl.

D. Solution quality

Solution quality results for Algorithm 1 are shown in Table III. In Tables III-VI, "Objective" is the market surplus value of the solution produced by presented algorithm, while "Best-known solution" is the value of the best solution produced by any team in the GOC, according to the leaderboards [31]. Each of these best solutions was obtained within a two hour time limit on the standardized hardware used by the competition. "Gap" is defined as the percent difference with respect to the best-known solution. "Res. penalty" is the total penalty incurred by zonal reserve shortfalls, while "p-penalty" and "q-penalty are total real and reactive power balance violation penalties. The penalty factor is 10^6 US dollars per power unit, so achieving a 10^2 power balance penalty is equivalent to a total power balance violation magnitude of 10^{-4}. "Solve time" is the time elapsed between the start of Julia code execution and the point at which the solution file is written.

While the algorithm produces acceptable solutions for many cases, 13 cases have gaps to the best-known solution of over 30%. In 12 of these 13 cases (and several others), the dominant penalties are real and reactive power balance violation penalties. In Table III, rows corresponding to solution gaps of larger than 30% are highlighted. This suggests that fixing reserve products for many devices is too restrictive to achieve AC feasibility.

Algorithm 2 prioritizes AC feasibility over reserve feasibility. It includes reserves in its copper-plate unit commitment model, but does not fix any reserve products before the AC-OPF subproblems. Table IV shows solution quality results for Algorithm 2, which produces surprisingly high-quality solutions for a simple greedy approach. Except for seven cases, these solutions are computed within the 2 hour time limit. Of the 28 cases, 12 have a gap of less than 10% to the best-known solution. In 9 of the remaining 16 cases, the dominant penalty is the reserve shortfall penalty. This suggests that post-AC-OPF greedy reserve allocation is not sufficient to minimize reserve penalty, and that solutions could be improved by including information from the reserves computed by the unit commitment subproblem.

Algorithm 3 attempts to improve reserve shortfall penalties by tightening bounds on $\gamma = 5\%$ of devices providing reserve value. In addition, reserve products are re-dispatched after the AC-OPF subproblems via linear programs with zonal balance models. The solution quality results for this algorithm are shown in Table V. The results show that this algorithm produces high-quality results in most cases, with the gap to the best-known solution less than 10% for 19 of 28 cases. Reserve penalty dominates in only one of the remaining nine cases (N02000-031). The total market surplus across all cases is 1.74×10^{10}\$ for Algorithm 3, compared to 1.37×10^{10}\$ for Algorithm 2, a 20% improvement with a value of over 3 billion US dollars. Fig. 1 compares the penalties incurred by Algorithms 1-3 on cases 4,000 buses or larger. The comparison shows large power balance violation penalties for Algorithm 1 and large reserve shortfall penalties for Algorithm 2. In the few cases where Algorithm 3 incurs large penalties, they are dominated by moderate power balance violations.

The results demonstrate that surprisingly good solutions can be obtained by a simple algorithm that does not consider contingency constraints, line switching, shunt control, or transformer control. In 16 of 28 cases considered, Algorithm 3 obtains solutions with objectives within 2% of the best-known. In case N08316-131, however, the algorithm takes longer than the 2 hour time limit to produce a solution.

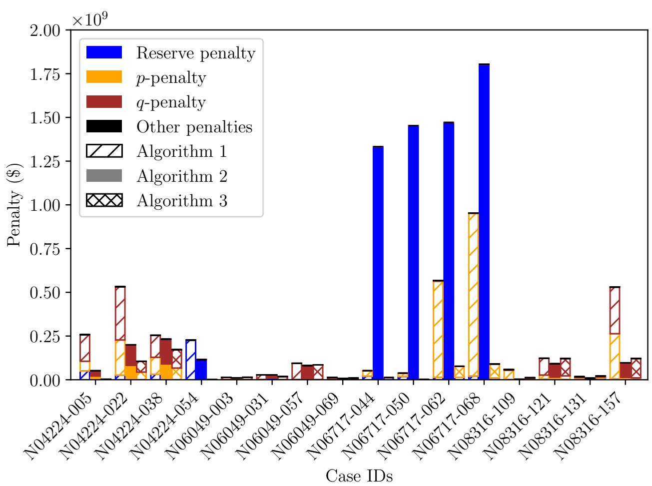

Fig. 1. Summary of objective penalties incurred for Algorithms 1-3 on the larger networks considered in this work. Algorithm 3 reliably balances AC and reserve feasibility requirements, finding solutions with low penalties.

E. Solve time

Fig. 2 shows a breakdown of the solve times of Algorithm 3 for the 28 cases presented. The results show that solving the sequential AC-OPF subproblems is the dominant computational expense, although for the 6,717-bus cases the unit commitment solve time also contributes significantly.

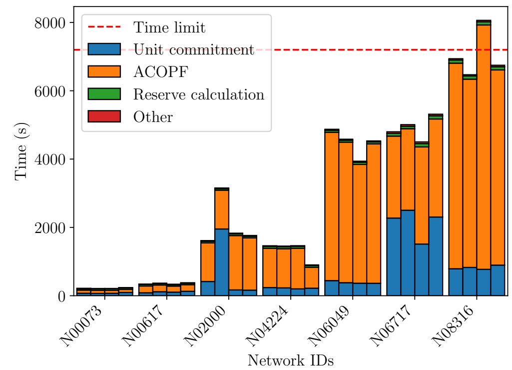

Fig. 2. A breakdown of solve times with Algorithm 3, where the scenarios for each network have the same order as in Table V. These results highlight how AC-OPF solve times are a dominant factor in the runtime of the algorithm.

Algorithm 4 solves individual AC-OPF subproblems in parallel to attempt to reduce the dominant computational burden and provide solutions within the two hour time limit. It does so at some expense to solution quality, however, as the post-processing projection step that recovers ramping feasibility can also introduce power balance violations. Solution quality results for Algorithm 4 are shown in Table VI. While the parallel solution strategy does produce high-quality solutions for most cases, it does incur a penalty compared to Algorithm 3 on cases N04224-005, N04224-022, and N04224-038 due to power balance violations.

Fig. 3 shows the solve times and speed-up factors as functions of the number of threads allocated to the Julia process running Algorithm 4 on one scenario from each of the five largest network sizes considered. While Algorithm 3 exceeds the time limit on case N08316-131, Algorithm 4 solves within the time limit when using only a single thread. With only four threads, Algorithm 4 solves within the time limit by a margin of at least 2,000 s for all cases. Additional runtime gains can be achieved using additional cores. According to the

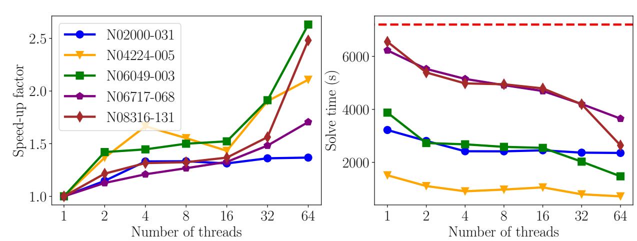

Fig. 3. Scaling of Algorithm 4's solve time with number of threads for five of the large cases considered in this work. The results show that 32 parallel processes are sufficient for satisfying a two hour runtime limit.

results presented in Fig. 2, Algorithm 3 spends between 36% and 90% of its time in the sequential AC-OPF portion of the algorithm for networks 2,000 buses and larger. As only this portion of the algorithm is parallelized, linear speedup with the number of processors is not expected. A theoretical upper bound on speedup for the most OPF-intensive instance is 10×, while an upper bound on speedup for the average instance is approximately 4×. Fig. 3 shows speedups between 1.4× and 2.6× with 64 threads for the most challenging instances, indicating the presence of some overhead. Thread scheduling is done by Julia's built-in `Threads` package. While parallelizing subproblems across time periods is sufficient to solve the presented cases within the time limit, additional gains may be obtained by decomposing in space as well. Overall, these results suggest that parallelization can be an effective approach to balancing solution quality and runtime requirements in industrial-scale AC Unit Commitment.

V. Conclusion

This work provides a simple decomposition framework for solving large-scale instances of a challenging AC Unit Commitment problem posed by Challenge 3 of the Grid Optimization Competition. Although the competition formulation includes several features that this work does not consider, such as contingency constraints and line switching, the proposed algorithms are successful in producing high-quality solutions to the majority of problem instances considered. A variant of the proposed method using a parallel decomposition of AC-OPF subproblems is capable of producing solutions to all instances considered within the two hour time limit, with some reduction in solution quality compared to a fully sequential computational approach. The success of this algorithm and those from other participants in the grid optimization competition suggest that industry-scale AC Unit Commitment is within reach of current numerical optimization methods.

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TABLE III
SOLUTION QUALITY RESULTS FOR ALGORITHM 1

Case ID	Best-known solution ($)	Objective ($)	Gap	Res. penalty ($)	p-penalty ($)	q-penalty ($)	Solve time (s)
N00073-303	+1.48e+08	+1.48e+08	0.03%	-4.90e+04	-2.42e+00	-3.18e+00	181
N00073-333	+2.31e+08	+2.30e+08	0.02%	-7.36e+04	-1.44e+01	-2.70e+01	180
N00073-373	+2.35e+08	+2.34e+08	0.48%	-3.43e+05	-7.28e+01	-3.91e+02	187
N00073-991	**+5.90e+07**	**+7.95e+06**	**86.51%**	**-1.16e+05**	**-6.50e+01**	**-4.92e+07**	**193**
N00617-002	+1.64e+08	+1.64e+08	0.17%	-3.58e-12	-2.68e+02	-1.45e+03	257
N00617-015	+2.65e+08	+2.65e+08	0.11%	-2.55e+04	-3.58e+02	-2.21e+03	304
N00617-039	+1.64e+08	+1.64e+08	0.18%	-2.72e+04	-2.79e+02	-1.49e+03	299
N00617-069	+2.66e+08	+2.65e+08	0.31%	-8.14e+03	-6.91e+02	-5.04e+03	295
N02000-022	+7.59e+08	+7.31e+08	3.78%	-2.45e+07	-1.27e+02	-1.08e+03	984
N02000-031	+8.26e+08	+7.84e+08	5.12%	-1.83e+07	-1.64e+07	-1.73e+03	2621
N02000-074	**+7.58e+08**	**+5.13e+08**	**32.36%**	**-2.05e+06**	**-2.44e+08**	**-1.97e+03**	**608**
N02000-080	**+8.30e+08**	**+5.51e+08**	**33.62%**	**-5.83e+05**	**-2.79e+08**	**-6.09e+03**	**641**
N04224-005	**+4.96e+08**	**+2.18e+08**	**55.99%**	**-5.01e+07**	**-5.56e+07**	**-1.52e+08**	**1813**
N04224-022	**+4.96e+08**	**-5.32e+07**	**110.71%**	**-2.59e+07**	**-2.01e+08**	**-3.04e+08**	**2013**
N04224-038	**+4.97e+08**	**+2.24e+08**	**54.82%**	**-3.11e+07**	**-9.72e+07**	**-1.26e+08**	**1729**
N04224-054	**+7.22e+08**	**+4.92e+08**	**31.90%**	**-2.26e+08**	**-3.72e+05**	**-1.22e+05**	**1074**
N06049-003	+6.09e+08	+5.71e+08	6.30%	-5.64e+05	-5.93e+05	-1.24e+07	5173
N06049-031	**+6.78e+08**	**+4.09e+08**	**39.69%**	**-1.18e+06**	**-2.54e+05**	**-2.63e+07**	**4360**
N06049-057	**+6.09e+08**	**+2.83e+08**	**53.58%**	**-3.47e+05**	**-3.19e+05**	**-9.33e+07**	**3887**
N06049-069	**+8.27e+08**	**+3.82e+08**	**53.74%**	**-3.22e+05**	**-2.65e+06**	**-9.78e+06**	**5095**
N06717-044	+9.05e+08	+8.14e+08	10.05%	-2.02e+07	-3.15e+07	-1.65e+04	3870
N06717-050	+1.32e+09	+1.23e+09	6.79%	-1.80e+07	-1.97e+07	-3.34e+04	3748
N06717-062	**+9.11e+08**	**+2.99e+08**	**67.20%**	**-1.33e+07**	**-5.50e+08**	**-4.41e+06**	**3407**
N06717-068	**+1.33e+09**	**+3.11e+08**	**76.57%**	**-2.05e+07**	**-9.31e+08**	**-1.53e+06**	**4242**
N08316-109	+1.43e+09	+1.36e+09	4.55%	-1.94e+06	-5.22e+07	-4.07e+06	4661
N08316-121	+1.16e+09	+1.03e+09	11.28%	-3.37e+06	-2.32e+07	-9.62e+07	5723
N08316-131	+1.43e+09	+1.41e+09	1.85%	-6.96e+06	-5.66e+06	-5.01e+06	4928
N08316-157	**+1.18e+09**	**+6.44e+08**	**45.54%**	**-4.01e+06**	**-2.59e+08**	**-2.66e+08**	**6957**

TABLE IV
SOLUTION QUALITY RESULTS FOR ALGORITHM 2

Case ID	Best-known solution ($)	Objective ($)	Gap	Res. penalty ($)	p-penalty ($)	q-penalty ($)	Solve time (s)
N00073-303	+1.48e+08	+1.48e+08	0.11%	-9.39e+02	-2.42e+00	-3.18e+00	163
N00073-333	+2.31e+08	+2.30e+08	0.11%	-1.94e+04	-1.44e+01	-2.70e+01	165
N00073-373	+2.35e+08	+2.34e+08	0.48%	-2.00e+05	-7.28e+01	-3.91e+02	171
N00073-991	**+5.90e+07**	**+7.96e+06**	**86.50%**	**-9.12e+04**	**-6.50e+01**	**-4.92e+07**	**188**
N00617-002	+1.64e+08	+1.64e+08	0.01%	-0.00e+00	-5.39e+01	-2.69e+02	253
N00617-015	+2.65e+08	+2.65e+08	0.12%	-2.70e+05	-2.29e+02	-1.22e+03	271
N00617-039	+1.64e+08	+1.64e+08	0.17%	-2.70e+05	-7.80e+01	-4.14e+02	262
N00617-069	+2.66e+08	+2.66e+08	0.10%	-2.70e+05	-2.46e+02	-1.30e+03	280
N02000-022	**+7.59e+08**	**-2.01e+09**	**364.29%**	**-2.77e+09**	**-6.88e+01**	**-3.14e+02**	**1345**
N02000-031	**+8.26e+08**	**-4.40e+09**	**632.79%**	**-5.23e+09**	**-8.40e+01**	**-4.31e+02**	**2886**
N02000-074	**+7.58e+08**	**+1.62e+08**	**78.64%**	**-5.96e+08**	**-5.18e+01**	**-2.89e+02**	**926**
N02000-080	**+8.30e+08**	**-1.87e+07**	**102.25%**	**-8.49e+08**	**-6.19e+01**	**-2.93e+02**	**943**
N04224-005	**+4.96e+08**	**+4.40e+08**	**11.34%**	**-2.78e+05**	**-2.08e+07**	**-3.01e+07**	**2240**
N04224-022	**+4.96e+08**	**+2.92e+08**	**41.26%**	**-0.00e+00**	**-8.21e+07**	**-1.17e+08**	**2246**
N04224-038	**+4.97e+08**	**+2.57e+08**	**48.20%**	**-0.00e+00**	**-9.04e+07**	**-1.41e+08**	**2068**
N04224-054	**+7.22e+08**	**+6.05e+08**	**16.22%**	**-1.15e+08**	**-1.82e+03**	**-9.52e+03**	**2471**
N06049-003	+6.09e+08	+5.95e+08	2.27%	-6.95e+05	-8.79e+04	-1.00e+07	5951
N06049-031	**+6.78e+08**	**+5.54e+08**	**18.32%**	**-1.13e+07**	**-1.08e+05**	**-1.61e+07**	**5283**
N06049-057	**+6.09e+08**	**+3.20e+08**	**47.57%**	**-1.00e+06**	**-2.98e+05**	**-7.96e+07**	**6050**
N06049-069	**+8.27e+08**	**+4.40e+08**	**46.72%**	**-1.26e+06**	**-1.29e+06**	**-5.07e+06**	**7597**
N06717-044	**+9.05e+08**	**-4.23e+08**	**146.70%**	**-1.33e+09**	**-1.42e+02**	**-3.82e+02**	**6385**
N06717-050	**+1.32e+09**	**-1.29e+08**	**109.81%**	**-1.45e+09**	**-2.92e+02**	**-1.65e+03**	**7302**
N06717-062	**+9.11e+08**	**-5.57e+08**	**161.19%**	**-1.47e+09**	**-1.55e+02**	**-9.73e+02**	**6585**
N06717-068	**+1.33e+09**	**-4.71e+08**	**135.40%**	**-1.80e+09**	**-4.77e+02**	**-3.40e+03**	**7614**
N08316-109	+1.43e+09	+1.42e+09	0.76%	-7.80e+05	-1.13e+06	-8.53e+05	8866
N08316-121	+1.16e+09	+1.06e+09	9.09%	-3.81e+06	-1.10e+07	-7.68e+07	9318
N08316-131	+1.43e+09	+1.42e+09	1.23%	-7.64e+06	-9.54e+05	-4.55e+05	8838
N08316-157	+1.18e+09	+1.07e+09	9.57%	-3.23e+06	-1.06e+07	-8.26e+07	10000

TABLE V
SOLUTION QUALITY RESULTS FOR ALGORITHM 3

Case ID	Best-known solution ($)	Objective ($)	Gap	Res. penalty ($)	p-penalty ($)	q-penalty ($)	Solve time (s)
N00073-303	+1.48e+08	+1.48e+08	0.09%	-2.11e+03	-6.09e+01	-3.73e+02	217
N00073-333	+2.31e+08	+2.30e+08	0.09%	-1.65e+04	-4.70e+01	-2.87e+02	212
N00073-373	+2.35e+08	+2.34e+08	0.39%	-2.69e+05	-1.32e+02	-7.99e+02	212
N00073-991	+5.90e+07	+7.36e+06	87.52%	-1.12e+05	-3.51e+02	-5.05e+07	239
N00617-002	+1.64e+08	+1.64e+08	0.09%	-3.64e-03	-6.19e+01	-2.64e+02	342
N00617-015	+2.65e+08	+2.65e+08	0.12%	-1.32e+03	-4.25e+02	-8.20e+02	368
N00617-039	+1.64e+08	+1.64e+08	0.10%	-1.26e+04	-5.03e+01	-2.24e+02	337
N00617-069	+2.66e+08	+2.66e+08	0.09%	-1.39e+03	-1.89e+02	-3.91e+02	380
N02000-022	+7.59e+08	+7.47e+08	1.61%	-1.21e+07	-1.57e+03	-5.73e+03	1614
N02000-031	+8.26e+08	+7.29e+08	11.73%	-1.00e+08	-2.13e+03	-8.05e+03	3154
N02000-074	+7.58e+08	+7.57e+08	0.08%	-1.62e+01	-6.37e+02	-2.80e+03	1831
N02000-080	+8.30e+08	+8.29e+08	0.08%	-1.50e+01	-4.47e+02	-2.45e+03	1763
N04224-005	+4.96e+08	+4.88e+08	1.57%	-1.67e+00	-9.62e+03	-2.38e+06	1462
N04224-022	+4.96e+08	+3.85e+08	22.47%	-1.19e+00	-4.38e+07	-6.12e+07	1454
N04224-038	+4.97e+08	+3.19e+08	35.77%	-6.00e-09	-6.66e+07	-1.05e+08	1464
N04224-054	+7.22e+08	+7.19e+08	0.43%	-4.63e-02	-2.60e+03	-2.59e+04	902
N06049-003	+6.09e+08	+5.75e+08	5.57%	-2.00e+05	-1.69e+05	-1.36e+07	4872
N06049-031	+6.78e+08	+5.38e+08	20.65%	-1.89e+05	-1.56e+05	-1.78e+07	4581
N06049-057	+6.09e+08	+2.92e+08	52.17%	-1.37e+03	-3.49e+05	-8.46e+07	3937
N06049-069	+8.27e+08	+3.84e+08	53.58%	-1.40e-10	-1.16e+06	-8.18e+06	4532
N06717-044	+9.05e+08	+8.94e+08	1.26%	-1.28e+07	-8.45e+02	-3.59e+03	4799
N06717-050	+1.32e+09	+1.32e+09	0.22%	-2.41e+06	-8.21e+02	-5.60e+03	5008
N06717-062	+9.11e+08	+8.31e+08	8.79%	-1.34e+07	-6.33e+07	-2.45e+05	4503
N06717-068	+1.33e+09	+1.23e+09	7.20%	-9.18e+06	-8.07e+07	-7.49e+04	5313
N08316-109	+1.43e+09	+1.41e+09	1.29%	-3.04e+05	-5.99e+06	-5.25e+06	6938
N08316-121	+1.16e+09	+1.03e+09	11.02%	-2.09e+06	-1.97e+07	-9.93e+07	6469
N08316-131	**+1.43e+09**	**+1.40e+09**	**1.96%**	**-9.57e+06**	**-5.87e+06**	**-5.61e+06**	**8061**
N08316-157	+1.18e+09	+1.06e+09	10.60%	-1.26e+06	-9.71e+06	-1.10e+08	6747

TABLE VI
SOLUTION QUALITY RESULTS FOR ALGORITHM 4

Case ID	Best-known solution ($)	Objective ($)	Gap	Res. penalty ($)	p-penalty ($)	q-penalty ($)	Solve time (s)
N00073-303	+1.48e+08	+1.48e+08	0.09%	-2.11e+03	-6.09e+01	-3.73e+02	215
N00073-333	+2.31e+08	+2.30e+08	0.09%	-1.65e+04	-4.70e+01	-2.87e+02	207
N00073-373	+2.35e+08	+2.34e+08	0.39%	-2.69e+05	-1.32e+02	-7.99e+02	209
N00073-991	+5.90e+07	+7.36e+06	87.52%	-1.12e+05	-3.51e+02	-5.05e+07	234
N00617-002	+1.64e+08	+1.64e+08	0.09%	-3.64e-03	-6.19e+01	-2.64e+02	249
N00617-015	+2.65e+08	+2.65e+08	0.12%	-1.32e+03	-4.25e+02	-8.20e+02	273
N00617-039	+1.64e+08	+1.64e+08	0.10%	-1.26e+04	-5.03e+01	-2.24e+02	265
N00617-069	+2.66e+08	+2.66e+08	0.09%	-1.39e+03	-1.89e+02	-3.91e+02	296
N02000-022	+7.59e+08	+7.47e+08	1.61%	-1.21e+07	-1.58e+03	-5.90e+03	755
N02000-031	+8.26e+08	+7.29e+08	11.73%	-1.00e+08	-1.98e+03	-7.70e+03	2390
N02000-074	+7.58e+08	+7.57e+08	0.08%	-1.59e+01	-4.11e+02	-2.30e+03	738
N02000-080	+8.30e+08	+8.29e+08	0.08%	-1.55e+01	-3.61e+02	-2.05e+03	652
N04224-005	+4.96e+08	+1.80e+08	63.61%	-6.81e-01	-2.19e+08	-9.09e+07	827
N04224-022	+4.96e+08	+2.74e+08	44.84%	-9.18e-01	-1.61e+08	-5.56e+07	775
N04224-038	+4.97e+08	+2.06e+08	58.53%	-4.14e-09	-2.01e+08	-8.42e+07	921
N04224-054	+7.22e+08	+7.19e+08	0.47%	-4.63e-02	-3.32e+05	-2.58e+04	720
N06049-003	+6.09e+08	+5.64e+08	7.39%	-2.03e+05	-1.12e+07	-1.32e+07	1934
N06049-031	+6.78e+08	+5.38e+08	20.68%	-1.88e+05	-4.29e+05	-1.76e+07	2059
N06049-057	+6.09e+08	+2.92e+08	52.06%	-1.59e+03	-3.47e+05	-8.36e+07	1537
N06049-069	+8.27e+08	+3.80e+08	53.97%	-1.57e-10	-2.16e+06	-1.01e+07	2636
N06717-044	+9.05e+08	+8.94e+08	1.26%	-1.28e+07	-8.09e+02	-3.50e+03	3868
N06717-050	+1.32e+09	+1.32e+09	0.22%	-2.41e+06	-8.65e+02	-5.65e+03	3977
N06717-062	+9.11e+08	+8.31e+08	8.79%	-1.34e+07	-6.33e+07	-2.44e+05	3217
N06717-068	+1.33e+09	+1.23e+09	7.20%	-9.18e+06	-8.07e+07	-7.49e+04	4379
N08316-109	+1.43e+09	+1.38e+09	3.14%	-3.05e+05	-3.23e+07	-5.18e+06	4114
N08316-121	+1.16e+09	+1.01e+09	13.00%	-2.10e+06	-3.78e+07	-1.05e+08	3046
N08316-131	+1.43e+09	+1.39e+09	3.24%	-9.74e+06	-2.41e+07	-5.58e+06	3995
N08316-157	+1.18e+09	+1.02e+09	13.59%	-1.20e+06	-4.37e+07	-1.12e+08	3438