# Shaped Operating Envelopes: Distribution Network Capacity Allocation for Market Services

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***Abstract*—The transition from centralised, fossil fuel-powered generating units to distributed energy resources (DER) represents a significant step forward, offering numerous benefits. However, this shift also presents operational challenges for distribution network service providers (DNSPs) and the electricity markets. In this paper, we introduce our innovative solution for allocating network capacity in the form of operating envelopes, shaped to enhance customer participation in energy and reserve markets. Our approach also provides DNSPs with network support flexibility of DER which can be leveraged to either increase network throughput for market services or postpone network augmentation. This study outlines our initial findings from Project Converge[1]—a comprehensive real-world trial involving 1000 active customers (the largest participant cohort in Australia) located in the Australian Capital Territory. Within, we discuss the challenges we have encountered, the opportunities that have arisen, potential avenues for future expansion, and the invaluable insights gained during the pre-trial phase of the project.**

***Index Terms*—Co-optimisation, Coordination, DER, DOE, FCAS Market, Network Support, Renewable Energy, SOE.**

## I. Introduction

Distributed energy resources (DER) are at the forefront of the world's swift transition towards renewable energy. The replacement of centralised fossil fuel-powered generating units with DER is largely a positive development, but it also creates new challenges for both distribution network service providers (DNSPs) and electricity markets. Upgrading networks to keep up with the rapid uptake of DER can be challenging for DNSPs, or in some cases, (especially along the transition) not economically viable. For the market operators, securing the system with enough reserves in the absence of conventional large-scale units is a difficult task.

Our approach to tackling the aforementioned challenges builds on the concept of the *dynamic operating envelope (DOE)* [1]. DOEs are convex regions that serve as protective rail guards within distribution networks, ensuring customers operate within the network's safe operational range. While ensuring network safety, DOEs are calculated purely for network safety and thus might be too limiting to enable market participation of DER. Furthermore, given the widespread adoption of rooftop PV systems, specific segments of the network might already have encountered voltage or thermal constraints (as discussed further in our results section). Nonetheless, DOEs cannot resolve these pre-existing network issues.

To solve the challenges encountered by DOEs, in this paper we extend the literature in two dimensions, 1) we shape the operating envelopes, called hereinafter *shaped operating envelopes (SOE)*, to increase DER market participation and 2) we propose a new functionality within which customers (or aggregators on behalves of the customers) submit generation and load network support offers to DNSPs. Such offers are used either to resolve networks' pre-existing issues or to increase market participation of DER. Without loss of generality, this paper studies the market participation of DER in generation and load energy markets as well raise and lower 6-second, 60-second, and 5-minute frequency control ancillary service (FCAS) markets. Therefore, together with network support offers, we co-optimise to obtain SOEs across 10 different revenue stream. We position this study within the literature and highlight our contributions in following section.

## II. Related Work

With the growing market share of coordinated DER, the role of DNSPs is expanding to allow the participation of customer/aggregators to energy and reserve markets. In this new paradigm, ensuring safe operation of the grid as well as appropriate / economic plans to upgrade network infrastructures are becoming more challenging. Optimal power flow (OPF)-based analysis, either central or distributed, have been suggested to resolve the operation challenges [2]. Yet they often struggle to scale to realistically-sized networks, require direct access to all DER assets, e.g., [3], or face computational / convergence issue, e.g., [4]–[7]. In addition, these approaches are effective when there is a well-defined operating point, whereas a known operating point may not exist when bidding into energy and reserve markets; this is because the operating point can vary depending on market output and whether reserves services are activated.

Different frameworks such as introducing new local electricity market within distribution network that coordinates DER biding with distribution networks in [8] has been suggested. A local flexibility market has been proposed in [9] and [10]. However, coordinating local markets within distribution networks prior to market clearing process of the overarching market is not only complicated but requires a structural change to the existing market.

[1]https://arena.gov.au/projects/project-converge-act-distributed-energy-resources-demonstration-pilot/

To tackle the aforementioned challenge, the concept of flexibility regions has emerged, calculated either at the substation level, as evidenced by Capitanescu et al. [11], Contreras et al. [12], and Silva et al. [13], or the distribution network node level, as demonstrated by Mahmoodi et al. [14], Petrou et al. [15], [16], and Nazir et al. [17]. Approaches aiming to determine a flexibility region at the root often do so by solving OPFs problems under distinct power factors or load/generation scenarios. However, in practical power system operation, the primary objective (second to feasibility) is to minimise the operation cost and not maximise such operating regions.

Contrary to this objective, [11], and [12] can prioritise more expensive (and less efficient) DER over cheaper (and more efficient) alternatives if such an allocation results in a larger operating region at the interconnection of DNSP-TNSP. We further discuss this in the result section. To counteract this, works such as [13] offer operating regions along with associated costs that the market could utilise to dispatch the distribution network more efficiently. However, these works still require DNSPs to centrally control all DER to deliver the operating point within the feasible region. In many jurisdictions, the DNSP does not have control over consumer-owned DER nor can operate them. Instead, consumers participate in the market through an aggregator that is not affiliated with the DNSP.

The works based on dynamic operating envelopes (DOEs) [14]–[17] can solve the above issue by providing agency for aggregator to operate their assets. However, current DOEs are calculated to solely ensure network safety, thus they might limit market participation of DER. We extend the existing DOE literature to shaped operating envelopes (SOE). SOEs allocate network capacity based on market benefit of DER to maximise social welfare. To provide a balance between increasing market share of DER and treating residential customers fairly, we provide a built in fairness index that allows DNSP to increase DER market benefit while ensuring every customer is allocated a minimum envelope.

To reduce investment costs for DNSPs, we introduce a novel functionality termed "network support" within the SOE engine. This feature has been successfully tested in the Converge project within different feeders with real-world data. Through the network support mechanism, aggregators submit their price offers along with generation (injection) or load (consumption) network support for each kWh to DNSPs. Subsequently, DNSPs can strategically dispatch this network support. The objective is two-fold: either to avert the necessity for network augmentation or to amplify network capacity, thereby facilitating increased DER integration into the market.

To incorporate uncertainty effect, such as those around market prices, background load and PV power, we employ a model predictive control framework, wherein our approach is rerun every 5 minutes. By leveraging the most current uncertainty data, we derive more representative operating envelopes for customers. This approach ensures the adaptability of our methodology to evolving circumstances, enhancing its effectiveness in practical applications.

In summary, compared to approaches centred on attaining the largest feasible operating region at the root of the distribution network, such as those presented in [11], [12], SOEs exhibit a distinct advantage by offering enhanced agency to aggregators as well as providing the market operator with the cost associated with dispatching DER. Unlike [11]–[13], our approach does not require a DNSP to directly control / operate DER to deliver DER market commitments. In contrast to DOE studies [14]–[18], SOEs demonstrate a more efficient allocation of network capacity. They strike a balance between advocating fairness and boosting the market share of coordinated DER. Furthermore, the unique network support feature inherent to SOEs empowers DNSPs to defer network augmentation, thus contributing to cost savings and optimising grid operation. The comprehensive integration of these features renders SOEs a robust and innovative framework for managing the challenges arising from DER integration into distribution networks.

### A. Contributions

In the following we present our novel contribution in terms of concept, algorithm and experiments.

*Concept:* Our principal innovation is the introduction of shaped operating envelopes – a novel extension of dynamic operating envelopes. SOEs refine DOEs by more accurately capturing consumer behaviour, thereby optimising the utilisation of existing network capacity. Additionally, we introduce the concept of a network support feature within SOEs. This feature functions as a local market within the distribution network, empowering DNSPs to postpone network augmentation / facilitate secure market participation of DER.

*Algorithm:* Our approach is methodically structured as a co-optimisation problem, wherein we tackle the joint optimisation of shaped operating envelopes and network support. This formulation incorporates all network constraints and adheres to the specific bidding prerequisites of market and aggregators.

*Real-world Experiments:* We implement our approach on real-world networks in Australia with more that 1000 customers and compare the effectiveness of our approach with the available alternatives in the literature. To the best of our knowledge, this project marks the most extensive endeavour of its kind conducted in Australia.

## III. Overview

Project Converge is exploring an approach to calculating operating envelopes that factors in and integrates aggregator / customer preferences, the value of the wholesale market services they offer, and network support. The outcome is what we refer to as shaped operating envelopes, to reflect the fact that the operating envelopes are shaped by these values that go beyond pure network constraint management.

Every 5 minutes the DNSP runs the SOE-engine prior to the wholesale market dispatch on a receding horizon manner. This allows aggregators and network to use their most recent uncertainty realisation and minimise the errors associated with uncertainty. Our receding horizon implementation also allows using the latest SOE and accounts for FCAS realisations in every 5 minute interval. In this paper, aggregators participate in both the energy and FCAS markets, which have different action times— i.e., 6 seconds, 60 seconds, and 5 minutes. Within the optimisation process, we ensure that aggregators can fulfil the FCAS requirements in the next 5 minutes[2]. SOE calculation is served as a pre-evaluation phase for aggregator bids to ensure that they comply with the distribution network constraints. The SOE framework has three key steps as presented in Figure 1. The steps are:

1) Aggregators send their network support availability, market bids, and customer contributions to the DNSP.
2) The DNSP computes shaped operating envelopes and network support requirements, and communicate them back to the aggregators.
3) Aggregators submit their final rebids to the market.

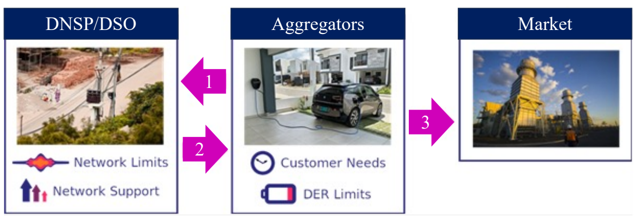


Fig. 1: Information flow for the key steps of SOE framework.

### A. Step 1: Bids and Contributions

This first step is where aggregators inform the DNSP of their intentions and capabilities. The aggregator provides their day-ahead wholesale market bids and rebids to the DNSP (before sending them to market). Alongside these market-related inputs, aggregators also provide details regarding their network support availability. Each aggregator also sends a plan for how their customers individually will contribute to delivering the offered market services. For each NMI (National Meter Identifier) this plan is made up of:

- Capacity contribution to each market and network support bid band; and
- Forecast uncontrolled consumption / production (+ optional confidence interval).

This information allows the DNSPs to effectively disaggregate the wholesale bids, from National Electricity Market (NEM) regions down to the LV distribution network level, enabling a more targeted optimisation of the envelopes to meet constraints within the distribution network.

### B. Step 2: Envelope Calculation

At this step, the DNSP solves an optimisation problem to constrain the wholesale bids of aggregators and allocate operating envelopes for customers. This is done by solving a specially formulated OPF problem. We refer to this calculation as shaping the bids and operating envelopes, with the outputs being network support requirements and shaped operating envelopes.

[2]If aggregators participate in different markets cleared at different timescales, our model would need modification to accommodate such situations.

Using wholesale market pre-dispatch prices, this calculation selects a subset of aggregator bids that remain compliant with network constraints. This DER coordination process is to maximise the following objectives:

- Expected value of the bids to the wholesale market, after accounting for any network support costs; and
- Similarity of envelopes across NMIs of similar type.

This is a multi-objective problem that in practice is solved by weighting the importance of these two objectives. At times the objectives can be in conflict, so it will be up to the DNSP to set an appropriate weighting between them, possibly under the direction of the regulator.

As part of this calculation, distribution network support instructions may be provisioned for customers where this will improve the objective. These instructions are a redirection of a part of the customer's energy market bid capacity toward network support (that will be provided irrespective of the energy market outcome). This is a form of short-term network support that is either compensated at one of several market-derived rates or based on pre-negotiated rates. The cost of this network support is factored into the SOE calculation.

The resulting shaped operating envelopes, and network support are communicated back to the aggregator.

### C. Step 3: Final Rebids

As a final step, the aggregator submits their final rebids for the upcoming dispatch interval to the wholesale market. In theory, the shaped rebids calculated by the DNSP could be forwarded to Australian Energy Market Operator (AEMO). Alternatively, an aggregator can independently calculate their final rebids. In order to avoid manipulation, the resulting rebids must be consistent with the SOEs and the original bids DNSP based its calculation on.

The rest of this paper is organised as follows. Section IV provides the detailed design and implementation of SOE and network support. Section V presents the benchmark approaches utilised to assess the effectiveness of our proposed approach. Section VI reports and discusses our results. In Section VII, we conclude this paper and discuss the challenges encountered during the real-world implementation and share insights gained, as well as outline potential future extensions.

## IV. Design and Implementation

The overarching problem can be formulated in its generic form as follows:

$$\min f(x) \tag{1a}$$
$$g_i(x_i, y_i) \leq 0 \qquad \forall i \in \mathcal{N} \tag{1b}$$
$$h_n(y_n) \leq 0 \qquad \forall n \in \Omega^{nmi} \tag{1c}$$

In the following, we will begin by introducing network constraints denoted as $g$. Subsequently, we will discuss constraints at each NMI represented by $h$, and finally, we will present the objective function $f$.

### A. Network Model

*Notations:* We introduce a tree graph denoted as $\mathcal{G} = \{\mathcal{N} \cup \{0\}, \mathcal{L}\}$ to depict a radial distribution network comprising $n+1$ nodes and a set of $\mathcal{L}$ lines interconnecting these nodes. The node 0 designates the substation node and maintains a constant voltage. Let $\mathcal{N} := \{1, ..., n\}$ represent the index set of nodes.

For each bus $i \in \mathcal{N}$, we employ $p_i$ and $q_i$ to represent net generation (+) or load (-). The actual real and reactive power flows at node $i$ are denoted as $P_i$ and $Q_i$, respectively. Let $V_i$ denote the magnitude of the complex voltage at bus $i$, where $V_i^2$ is represented by $U_i$. In the case of each line $(i, j) \in \mathcal{L}$, the parameters $R_{i,j}$, $X_{i,j}$, and $Z_{i,j} = \sqrt{R_{i,j}^2 + X_{i,j}^2}$ are used to signify its resistance, reactance, and impedance, while $l_{i,j}$ represents the squared magnitude of the complex branch current from bus $i$ to $j$. The Distflow equations can be formulated as follows:

$$P_i = P_j + p_j - R_{i,j} l_{i,j}, \tag{2a}$$

$$Q_i = Q_j + q_j - X_{i,j} l_{i,j}, \tag{2b}$$

$$U_j = U_i + 2\Big(R_{i,j} P_j + X_{i,j} Q_j\Big) - Z_{i,j}^2 l_{i,j} \tag{2c}$$

$$P_j^2 + Q_j^2 = U_i l_{i,j} \tag{2d}$$

where (2a) and (2b) correspond to the equations for real and reactive power balance, respectively. The voltage at each node is computed using (2c). The expression for the complex power flowing through each line is lastly provided in (2d).

In order to prevent infeasible solutions within the trial, we treat the voltage ($U_i$) and thermal limits ($l_{i,j}$) as soft constraints. To do so, we introduce auxiliary variables $U_i^{aux} \in \mathbb{R}^+$ and $L_{i,j}^{aux} \in \mathbb{R}^+$ as well as the following constraints for voltage and transformer thermal limits, respectively.

$$U_i^{aux} \geq U_i - \overline{U}_i \qquad \forall i \in \mathcal{N} \tag{3a}$$

$$U_i^{aux} \geq \underline{U}_i - U_i \qquad \forall i \in \mathcal{N} \tag{3b}$$

$$L_{i,j}^{aux} \geq L_{i,j} - i_{i,j}^{max\,2} \qquad \forall i, j \in \mathcal{T} \tag{3c}$$

These auxiliary variables are incorporated into the objective function with a penalty parameter in Section IV-C. By employing a vector representation for network variables denoted as $x = [P, Q, I, U, U^{aux}, L^{aux}]$, and nodal injection variables as $y = [p, q]$, we express the OPF constraints as $g(x, y) \leq 0$. It's important to note that due to the significant alteration in the network's operating region during the provision of FCAS in both raising and lowering scenarios, we utilise two sets of OPF constraints to ensure that the network constraints remain valid even in the most extreme edge cases. Employing superscripts $+$ and $-$ to signify the most extreme scenarios for raising and lowering reserve services, respectively, the network subproblem can be succinctly summarised as follows:

$$g^+(x^+, y^+) \leq 0 \tag{4a}$$

$$g^-(x^-, y^-) \leq 0 \tag{4b}$$

It is worth mentioning that to ensure the tractability of trial, especially within the 5 minute time interval, we also employed the linear version of OPF model (2a)–(2d), known as “LinDistFlow”. In line with [19], LinDistFlow model is achieved by ignoring the loss terms in (2a)-(2c) and the nonlinear equation (2d). We discuss the accuracy of this simplification in Section VI.

### B. Constraints at each NMI

*Notations:* Each aggregator denoted as $a \in \Omega^a$ manages a group of customers $\Omega^{a_n}$. Let $e_{b,n}^g$, $e_{b,n}^l$, $f_{b,n}^l$, $f_{b,n}^r$, $s_{b,n}^g$, and $s_{b,n}^l$ denote the energy generation, energy load, FCAS lower, FCAS raise, network support generation, and network support load offers of customer $n \in \Omega^{a_n}$ within bid band $b \in \Omega^{b}$[3]. Please note that the FCAS market in Australia is rarely activated in the event of a contingency. Consequently, we have not directly modelled the operation of FCAS within our model. If FCAS is activated in a 5 minute interval, we take it into account through our receding horizon implementation for the next optimisation. We employ the placeholder variable $z_{b,n}$ to denote these nodal bid parameters. We introduce binary variables: $u_{b,n}^g$, $u_{b,n}^l$, $u_{b,n}^{f_l}$, $u_{b,n}^{f_r}$, $u_{b,n}^{s_g}$, and $u^{s_l} b$, with the placeholder variable $u_{b,n}$, which are used to calculate the bid aggregate at every NMI $n$.

Every individual costumer is associated with a unique NMI $n$ and has the following parameters / aggregate bid value :

- Reservation interval: every customer has an associated reservation interval $[R_n^l, R_n^u]$, which delineates the lower and upper bounds of the customer's baseline load. The utilisation of an interval accounts for the inherent uncertainty in customer demand. It's worth noting that this interval can comprise a single value if there is no uncertainty associated with customer's demand.
- Nodal bid variables: for every $n \in \Omega^{a_n}$, we define:
  - energy market load / generation bids: $E_n^l / E_n^g \in \mathbb{R}$.
  - FCAS lower / raise bid: $F_n^l / F_n^r \in \mathbb{R}^+$.
- Network support: customers can offer network support load / generation, shown by $S_n^l \in \mathbb{R}^+$ / $S_n^g \in \mathbb{R}^+$.

We use the place holder variable $Z_n$ for $E_n^l$, $E_n^g$, $F_n^r$, $F_n^l$, $S_n^l$, $S_n^g$ an the following constraints to calculate the aggregate bid for every NMI $n$. .

$$Z_n = \sum_b z_{b,n} \times u_{b,n} \tag{5a}$$

$$g(E_z^l, E_z^g, F_z^l, F_z^r) \leq 0 \tag{5b}$$

Equation (5a) derives the admissible offers of aggregator $a$[4] for each NMI, while considering any interdependency constraints, modelled via (5b), known as trapezium in the NEM. We next limit these offers within the operating envelopes as follows:

$$S_n = S_n^g - S_n^l, \tag{6a}$$

$$O_n^l \leq R_n^l + S_n - E_n^l - F_n^l \tag{6b}$$

$$O_n^u \geq R_n^u + S_n + E_n^g + F_n^r \tag{6c}$$

[3]It's important to note that our approach is designed based on the NEM framework, wherein participants submit bids to the market across up to 10 price bands.

[4]Notice that these variables have an implicit index $a$ to account for the aggregators which for now we have omitted to increase readability.

Equation (6a) computes the overall network support for each customer, which will be utilised in determining their envelope values. The lower and upper bounds of these envelopes are established through (6b) and (6c).

In addition, shaped operating envelopes have a simple built-in fairness index. Our fairness objective strives to increase the size of the smallest envelope, ensuring that no customer is left with no network access. Notice that, if our primary concern is fairness, all customers will ultimately will get envelopes of equal sizes. To incorporate this, we introduce variables $\underline{O}^l$ and $\underline{O}^u$, along with constraint 7, applicable to all $n \in \Omega^{a_n}$:

$$\underline{O}^l \leq O_n^l \qquad \underline{O}^u \leq O_n^u \tag{7}$$

### C. Objective Function

The objective function is composed of four distinct terms, encompassing the following objectives: 1) maximising the aggregator's benefit in the market, 2) maximising fairness, 3) minimising network support costs for the DNSP, and 4) minimising the breach of soft constraints through the application of penalty measures. This can be written as follows:

$$\max\left((1-\mu)C^M + \mu C^F - C^P - C^{NS}\right) \tag{8}$$

The fairness index $\mu$, ranging between 0 and 1, serves as a means to strike a balance between enhancing the market share of DER and ensuring an equitable distribution of network capacity among all customers.

The market benefit $C^M$ represents the difference between revenue and costs. The revenue is calculated using pre-dispatch prices, which are forecasted prices provided by AEMO. These prices are denoted as $\Pi = [\Pi^g, \Pi^l, \Pi^{f_r}, \Pi^{f_l}]$. Additionally, we incorporate aggregator network secure bids, denoted as:

$$\mathcal{E} = \left[\sum_{n\in\Omega^{a}n} E_n^g, \sum_{n\in\Omega^{a}n} E_n^l, \sum_{n\in\Omega^{a}n} F_n^r, \sum_{n\in\Omega^{a}n} F_n^l\right] \tag{9}$$

The cost is determined based on the initial offers of aggregators for each bid band, denoted as capacity: $e_{b,i} = [e_{b,n}^g, e_{b,n}^l, f_{b,n}^r, f_{b,n}^l]$, and price: $\pi_b = [\pi^g, \pi_l, \pi^{f_r}, \pi^{f_l}]$. These values are further influenced by the binary decision variables $u_{b,n} = [u_{b,n}^g, u_{b,n}^l, u^{f_r}b, n, u^{f_l}b, n]$. In other words, the calculation of cost takes into account the specified offers for capacity and their corresponding prices, along with the decisions made through the binary variables. Finally, the market benefit can be obtained as follows:

$$C^M = \left(\Pi\mathcal{E} - \sum_{i\in\Omega^{a_i}} \sum_{b\in\Omega^b} e_{b,i}\pi_b u_{b,i}\right) \tag{10}$$

Considering the minimum-sized envelopes $\underline{O}^l$ and $\underline{O}^u$, the fairness component $C^F$ can be articulated as follows:

$$C^F = \underline{O}^l + \underline{O}^u \tag{11}$$

The first term within the objective function seeks to maximise overall social welfare, while the secondary term contributes to enhancing fairness in the envelope calculation process. The parameter $\mu$ represents a user-defined value that dictates the priority assigned to either objective. We posit that the initial segment of the objective function ($\mu = 0$) inherently embodies an equitable aspect, as it aligns with the overarching objective of electricity markets, which is to optimise social welfare on a global scale. Nonetheless, our approach extends the capability for DNSPs to equitably address low voltage (LV) customers if they choose to do so.

The penalty associated with violation of soft constraints is added with a penalty parameter $\mu$ as follows:

$$C^P = \mu\left(\sum_{i\in\mathcal{N}} U_i^{aux} + \sum_{i,j\in\mathcal{T}} L_{i,j}^{aux}\right) \tag{12}$$

Finally, given network support cost at every NMI, i.e., $\pi_{b,n}^{NS} = [\pi_{b,n}^{ns,g}, \pi_{b,n}^{ns,l}]$, the nodal network support capacities at band $b$, i.e., $s_{b,n} = [s_{b,n}^g, s_{b,n}^l]$ as well as the binary decision variables $u_{b,n}^s = [u_{b,n}^{s_g}, u_{b,n}^{s_l}]$, can be written as follows:

$$C^{NS} = \sum_{n\in\Omega^{a_n}} \sum_{b\in\Omega^b} \pi_{b,n}^{NS} s_{b,n} u_{b,n}^s \tag{13}$$

Notice that the all values above has written for a single aggregator and thus the index $a$ has been omited to increase readability.

## V. Benchmark

In this section, we introduce the benchmark approaches that we employ to assess and contrast the performance of our method. These approaches are dynamic operating envelopes that are generated using optimal power flow and each possess distinct characteristics as described in the following.

*Equal-width (EW):* This approach involves assigning envelopes of the same size to all customers, regardless of whether they own DER or the capacity of their DER. While this method is straightforward and does not require any customer information, it may result in a sub-optimal allocation of network capacity. This is because some customers may have no DER or have DER with varying capacities, and assigning them the same envelope size may lead to an inefficient distribution of network capacity.

*Proportionally Equal-Width (PEW):* This approach involves allocating envelopes that are proportional in size to customers' installed DER capacity. This method requires access to the DER capacity of every customer and enables more effective allocation of network capacity amongst customers.

*Maximising Feeder Throughput (MFT):* This approach aims to increase the throughput of the distribution network by generating envelopes that prioritise customers located closer to the root of the network. Unlike proportionally equal-width envelopes, MFT does not provide proportional equal envelopes and while some customers can have envelopes equal to their installed DER capacity, others (especially those located at end nodes) might get little or no envelope capacity.

*Fair MFT:* This approach is designed to optimise the throughput of the system while ensuring that all customers are treated fairly. Specifically, this approach generates envelopes that maximise the feeder's throughput while (similarly to our approach) ensuring that the smallest envelope in the system is wide enough. This is achieved by striking a balance between maximising throughput and increasing the size of the smallest envelope in the network.

## VI. Results

This section evaluates the effectiveness of our approach from different aspects, including accuracy in Section VI-A, data requirement in Section VI-B, customer flexibility in Section VI-C, aggregator benefit in Section Section VI-D, problem size and computation time in Section VI-E.

For our analysis, we employed real and reactive power data from customers' smart meters, specifically recorded for the month of February 2022. For the offline simulations, we generated aggregators' energy and reserve bids using the historical data from Lake Bonney Battery (LBB) and Ballarat Battery (BAL) grid-scale batteries, as reported by AEMO[5]. Capacities and trapeziums (bid feasible dispatch regions) were proportionally scaled according to the summed inverter capacity of aggregators at each node.

### *A. Power Flow Analysis*

It is important to note that we have developed our SOE engine flexibly such that DNSPs can plug in either non-linear or linear OPF model. As network size increases, adopting a linear OPF becomes more favourable due to tractability considerations. In this section, we evaluate the error associated with our linear model using exact power flow solved through Newton Raphson's method. To have a more reliable assessment, we have done this experiments across 5 different MV-LV feeders, ranging from 1458 to 2826 nodes. We utilise the actual customer load data for the month of February 2022, with a 30-minute time discretisation, i.e, 1344 scenarios (28 days $\times$ 48 time intervals).

Figure 2 plots the voltage envelope obtained by the exact and linear approaches across all 5 feeders. In our 1,344 real-world scenarios, we observed a maximum error of 0.6% with an average error of 0.002% for the linear model, compared to exact power flows. It is worth mentioning that the errors within the linear model overestimates true voltages, which implies that the actual voltages might be marginally lower than those predicted by the linear model. This overestimation is acceptable when considering the upper voltage bounds; however, it may induce infeasibility in the lower voltage bounds. To avoid infeasible results, we increase the lower voltage bounds of our linear model by 1% (from [0.94, 1.1] to [0.95, 1.1]). As a result of this modification, even for the minimum linearly obtained voltages, the exact voltages stay within the acceptable bound of [0.94, 1.1] in our experiments.

### *B. Required Information*

Table I presents the required information for various DOEs and our SOE engine. It is worth noting that our SOE engine is capable of functioning with different sets of information, depending on the availability of data. To study the effect of

[5]https://nemweb.com.au/Reports/Archive/Yesterdays_Bids_Reports/

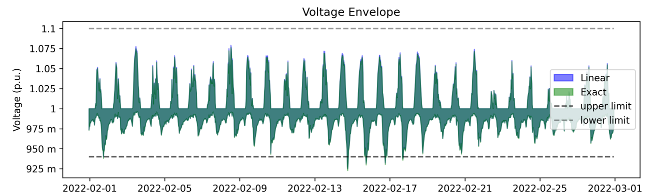


Fig. 2: Voltage envelope Feb. 2022: exact vs linear

SOE with different input data, we study the effect of 5 different cases A-E, each with a unique set of input data as reported in Table I. If an information is ticked, that model includes the associated data in the input.

TABLE I: The required information for different DOEs / SOEs

| Approach | | Information | | | | |
|---|---|---|---|---|---|---|
| | | Offers | | | DER Capacity | Reservation |
| | | NS | Energy | FCAS | | |
| DOE | EW | – | – | – | – | ✓ |
| | Fair | – | – | – | ✓ | ✓ |
| | PEW | – | – | – | ✓ | ✓ |
| | MFT | – | – | – | ✓ | ✓ |
| SOE | A | – | – | – | ✓ | ✓ |
| | B | ✓ | – | – | ✓ | ✓ |
| | C | – | – | ✓ | ✓ | ✓ |
| | D | – | ✓ | ✓ | ✓ | ✓ |
| | E | ✓ | ✓ | ✓ | ✓ | ✓ |

The "equal-width" (EW) approach requires minimal information, i.e., only requires customer reservations forecasts. Other DOEs require additional information on the behind-the-meter technologies (i.e., installed DER capacity) to calculate the envelopes. Other than the "installed DER capacity" and "reservations", SOE needs offers from aggregators, including network support, as well as energy and FCAS market offers. However, SOE engine is designed to work with or without these information. We generate 5 sets of different inputs shown by A–E, each lacking some types of information, to evaluate the performance of SOE under different set of information.

### *C. DOE and SOE for customers*

Figures 3 and 4 plots DOEs for two randomly selected customers, each served by a different aggregator. As shown in the figures, the DOEs obtained for each customer are different. Since aggregators do not provide any bidding information, the differences in DOEs are purely due to customers network connection and the type of their installed DER. Also as plotted in Figure 4, the envelope for the injection during the day where there is PV and high voltage issues in the network has shrieked compared to evening where there is need for injection.

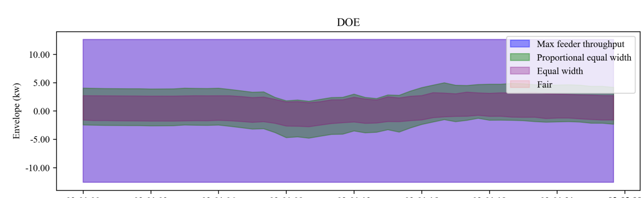


Fig. 3: Different DOEs: a customer of Aggregator 1

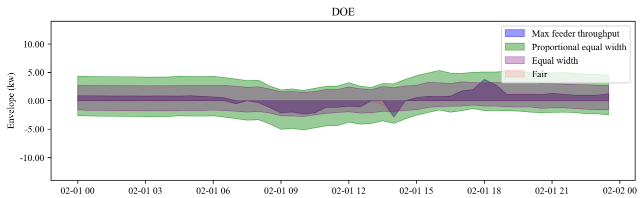


Fig. 4: Different DOEs: a customer of Aggregator 2

Contrary to DOE, SOE takes into account aggregators' bidding intentions to optimise for the most efficient operating envelopes. Figures 5 and 6 present SOEs for the same customers as in Figures 3 and 4. The inclusion of aggregators' market / network support intentions has led to more efficient envelopes that generates higher aggregate benefits. Information on the financial aspect of SOE and DOE is provided in the next section.

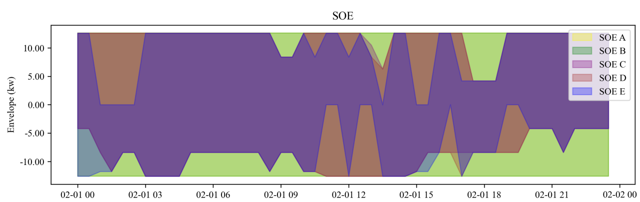


Fig. 5: Different SOEs: a customer of Aggregator 1

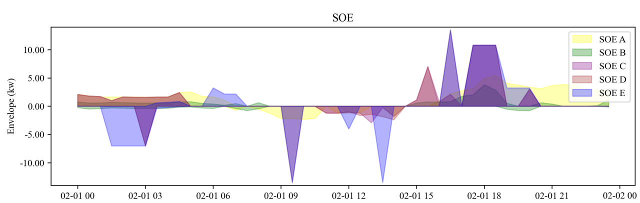


Fig. 6: Different SOEs: a customer of Aggregator 2

### D. Cost benefit: SOE vs. DOE

Table II provides information on envelope capacity, market accepted capacity and total market benefit obtained by every envelope. Envelope capacity is sum of the raise and lower provided by all envelopes. This might not be a reliable indicator of the effectiveness of each approach, since large envelopes can be allocated to customers who have no need for. In contrast, the raise and lower market accepted power is a better indicator of efficiency since it shows how effectively the envelopes were allocated to customers who wish to participate in (and be dispatched by) the electricity market.

Table II provides information on envelope capacity, market-accepted capacity, and the overall market benefits achieved by each envelope. Envelope capacity represents the summed raise and lower capacity provided by all envelopes. However, it may not be a good measure of envelope efficacy, as large envelopes can be assigned to customers who do not wish to participate in the market. In contrast, the market-accepted capacities is a better indicator of efficiency, as it shows how effectively the envelopes allowed market access.

The total benefit is the benefit obtained by all aggregators through participation in the electricity market, including energy, 3 FCAS raise, and 3 FCAS lower markets. The results, presented in Table II, indicates that SOE E is capable of achieving outcomes comparable to those when network is ignored (i.e., infinite network assumption). Note that Table II only report the benefit for a single day, yet we have seen the same performance, i.e., SOE outperforming DOE over longer time period such as a month. Further details are provided in Converge Project report[6]

It is worth mentioning that SOE that includes all the information, i.e., SOE E, obtains approximately 20% higher that the best DOE under study, i.e., DOE Fair.

TABLE II: Envelop Capacity and Total Benefits on 01/02/2022

| Approach | | Envelope (MW) | | Market (MW) | | Total |
|---|---|---|---|---|---|---|
| | | Raise | Lower | Raise | Lower | Benefit ($) |
| DOE | EW | 101.18 | 59.21 | 42.99 | 25.31 | 1116.18 |
| | Fair | 249.87 | 203.33 | 153.51 | 104.70 | 3681.26 |
| | PEW | 92.76 | 62.22 | 63.15 | 43.18 | 1597.43 |
| | MFT | 249.87 | 203.33 | 153.51 | 104.70 | 3680.18 |
| SOE | A | 249.87 | 203.33 | 153.44 | 104.68 | 3679.44 |
| | B | 228.01 | 164.89 | 152.19 | 116.93 | 3739.87 |
| | C | 227.48 | 165.79 | 152.15 | 116.61 | 3895.70 |
| | D | 177.85 | 184.51 | 142.63 | 130.85 | 4105.78 |
| | E | 180.11 | 190.36 | 152.36 | 133.43 | 4421.13 |
| No network | | $+\infty$ | $+\infty$ | 190.70 | 153.67 | 4660.73 |

### E. Problem size: SOE vs DOE

Table III illustrates the problem size and computation times for DOE and SOE calculation. All experiments were conducted using Pyomo in Python on an 8-core 64-bit, 8GB PC. Given the 5-min market clock, these computational time provide aggregators enough time to update their decisions. Since our model is linear, it can scale to realistically-sized networks.

TABLE III: Problem size and solve time (a single time step)

| Approach | Problem Size | | Time (s) |
|---|---|---|---|
| | # Variables | # Constraints | |
| DOE | 16,664 | 16,614 | 0.97 |
| SOE | 110,619 | 54,549 | 2.42 |

## VII. Conclusion, Challenges, Future Directions

This paper illustrated the importance of optimising various aspects to enhance the integration of DER, get the most out of the electricity market while supporting the grid. We reported our findings during the pre-trial phase of the Converge project in which we utilise real-world MV-LV feeders and consumer data. Our real-world experiments revealed that, on average, SOE can yield 20% higher benefits for aggregators compared to DOE. Despite the fact that SOE requires a longer solving time, 2.4 seconds as opposed to 0.97 seconds for DOE (due to solving a more sophisticated optimisation problem), it remains well below the 5-minute market rebid limit in the NEM.

[6]Project converge available at https://arena.gov.au/projects/project-converge-act-distributed-energy-resources-demonstration-pilot/

The project has also encountered its fair share of challenges, shedding light on areas that demand attention in the future. These include the need for enhanced coordination between stakeholders, especially between DNSPs and aggregators. Addressing issues related to obtaining granular NMI-level data, will be pivotal for future projects seeking to optimise DER participation. The evolution of commercial and contractual requirements in network support procurement and settlement remains an area for exploration.

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