Sizing distributed energy resources in a renewable energy community with a grid-aware internal market structure

Stegen Thomas
Glavic Mevludin
Cornélusse Bertrand
Department of Electrical
Engineering and Computer Science
University of Liège
Liège, Belgium
{tstegen, mevludin.glavic, bertrand.cornelusse}@uliege.be

Giannitrapani Antonio
Paoletti Simone
Dipartimento di Ingegneria dell'Informazione
e Scienze Matematiche
Università di Siena
Siena, Italy
{giannitrapani, paoletti}@diism.unisi.it

***Abstract*—This paper proposes a cooperative approach aimed at distributed energy resources sizing in a renewable energy community, with considerations of the community's optimal operation, impact on the electrical grid and an allocation of the benefits to its members. To this purpose, multiple investment modes are evaluated via a two-step procedure. In the first step, the size of renewable energy sources is determined by solving an optimization problem that maximizes community welfare, considering network and investments. In the second step, an optimization problem maximizing additional community member profit with price regularisation is solved. This step shares benefits among community members. The potential of the proposed procedure is illustrated using a benchmark Dickert-LV network. This is a fully cooperative framework where the community operator is ensuring adequate grid operation, operational planning and sizing of new investments.**

***Index Terms*—Distributed energy resources, renewable energy community, optimization, sizing, benefits allocation**

I. INTRODUCTION

Energy communities (ECs) in general, and renewable (RECs) in particular, appear to be a promising response to the challenges posed by the energy transition [1], [2], [3]. To be a truly viable solution, the energy communities have to be properly sized and managed while taking into account the interaction with the grid to which they are connected. Several works have dealt with the sizing of distributed energy resources (DERs) in ECs, and notable ones include [4], [5] for ECs in general and [6], [7], [8] for RECs in particular. In [4], bilevel programming is proposed as the solution approach for an EC investment and operation problem with a business model incorporation. Game theory reward distribution schemes have been used in [5] for the sizing and fair revenue sharing among community members. This work has suggested Shapley/Core, Variance/Core, and Nucleolus reward distribution schemes as the solution to the problem. In [6], the authors compare three strategies for DER sizing in REC: economic optimization, peak shaving, and self-sufficiency. These models are tested on multiple types of EC networks (urban, suburban, and rural), and their impact on the grid is quantified. The work shows that the best strategy for grid operation changes depending on the grid type. The work in [7] proposes to use a mixed integer linear program (MILP) to show that a REC can reduce the CO2 emissions related to energy usage. This reduction is amplified when the participants invest in a central DER rather than multiple personal DERs.

Two cooperative frameworks using game theory are developed in [8]. The first one uses Shapley values to determine distribution keys for profit. This method is computationally intensive, but adequately rewards the mobilized flexibility. The second one finds a natural Nash equilibrium, which tends to reward the potential flexibility of end-users. As noted in [5], ECs are designed through non-cooperative or cooperative approaches. In non-cooperative approaches, each EC member acts on its own interest in the local energy market [9], while in cooperative ones the EC operator takes the task of trading on behalf of EC members. The cooperative approaches align with the European context [1], [2], [3] and require a fair scheme for sharing the benefits to all EC members.

In this paper, we propose a procedure that considers in a single framework sizing of DERs, optimal community operation, and a fair allocation of the benefits to the community members. Moreover, building on the model presented in [9], grid operation is accounted for through the consideration of power flow equations (more precisely, a branch flow model [10]) to avoid grid limit violations. This extension thus allows us to consider the physical aspects of energy exchange through the grid, which is a key point for optimal localization of the assets. We assume that all users of a low-voltage (LV) grid are members of the REC (the selection of the optimal number of REC members is not considered), and the community operator (CO) is located at the point of common coupling (PCC) and is responsible for grid operation behind this point. The first step of the proposed procedure is formulated as a REC welfare maximization problem, with network and investment considerations. The second step optimizes profit sharing by fixing intra-community energy prices and maximizes a member's minimum additional profit from joining the community. We illustrate the potential of the proposed procedure using a benchmark Dickert-LV network. Three scenarios are compared, if optimal investment in DERs is determined:

- for each user independently without a community;
- for each community member when they invest separately;
- for the community as a whole.

The proposed two-step approach is inspired by [9], which merges the two steps in a bilevel formulation but does not include a grid model or new investments sizing. Although the problem addressed here could also be modeled as a bilevel program, it would result in a hard mixed-integer non-convex program. Our two-step approach approximates this bilevel problem by soft-linking the sizing problem, a mixed-integer second-order cone problem, and the profit-sharing problem, a quadratic program.

The paper is organized as follows: Sections II and III address the two steps of the procedure, starting by finding an optimal behavior for the entire community, before proposing a way to share the benefits. Section IV describes the case study and hypotheses. The simulation results are presented and discussed in Section V, and some conclusions are drawn in Section VI.

II. MAXIMIZING COMMUNITY WELFARE CONSIDERING NETWORK AND INVESTMENTS

The first optimization problem maximizes the community's social welfare, considering all its users' investment and operating costs. The CO also ensures reliable community grid operation. This problem has a double role, finding optimal sizes and locations for new investment while establishing the operational behavior for the EC. The overall problem can be formulated as,

$$\begin{aligned} \max \quad & \text{Total profit of the community } (2) \qquad (1)\\ \text{subject to} \quad & \textit{Price and exchange constraints } (3)-(6),\\ & \textit{Devices constraints } (7)-(14),\\ & \textit{Sizing constraints } (15)-(24),\\ & \textit{Power flow equations } (25)-(38). \end{aligned}$$

The total profit of the community is defined as,

$$\sum_{u\in\mathcal{U}} \left(J_u^{\text{com}} - \text{CAPEX}_u/I^{\text{hor}}\right) - \pi^{\text{peak}}\overline{p} - \pi_t^{\text{igr}} \sum_{t\in\mathcal{T}} w_{d_t}\Delta_T P_t^{\text{loss}}, \tag{2}$$

where $\mathcal{U}$ is the set of users of the LV grid, and $\mathcal{T}$ is the set of time periods of the simulation. This set can also be divided in several representative days d which are part of set $\mathcal{D}$. J_u^{com} is the total yearly profit of user u considering its energy exchanges over the whole time horizon, CAPEX_u is its investment in new DERs and I^{hor} is the investment horizon in years. $\overline{p}$ is the yearly peak power injection at the PCC of the community, which is charged at price π^{peak}. The last term penalizes the energy lost (P_t^{loss}) in the electrical lines. The time is discretized in time steps of duration Δ_T.

In this model, the CO is in charge of grid operation behind the PCC. The hypothesis of this model is that the CO is virtually connected to the slack bus of the network and is billed depending on the energy metered at this point. The cost of network losses should be covered by the community network fees (γ^{com}).

The total community profit of each user (J_u^{com}) is defined by

$$\begin{aligned} J_u^{\text{com}} = -\,\Delta_T \sum_{t\in\mathcal{T}} w_{d_t} \Bigg(& \pi_t^{\text{igr}} i_{u,t}^{\text{gri}} - \pi_t^{\text{egr}} e_{u,t}^{\text{gri}} \\ & + \gamma^{\text{com}} \left(e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}}\right) \\ & + \gamma^{\text{sto}} \left(\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}}/\eta_u^{\text{dis}}\right) \Bigg) \quad \forall u \in \mathcal{U}. \end{aligned} \tag{3}$$

Variables $e_{u,t}^{\text{gri}}$ and $e_{u,t}^{\text{com}}$ (resp. $i_{u,t}^{\text{gri}}$ and $i_{u,t}^{\text{com}}$) represent the power injected to (resp. subtracted from) the grid and the community by user u at time t. Simultaneous power import and export is prevented by a grid import price (π_t^{igr}) greater than the grid selling price (π_t^{egr}). The community operator charges a fee for the intra-community exchanges valued at γ^{com} and the storage usage is valued at γ^{sto} for battery wear. The benefits are weighted with w_{d_t} which depends on the day d of the current timestep t, this term allows the simulation horizon to represent a complete year.

Equation (4) ensures that internal community power exchanges are balanced at all time, and equation (5) determines the peak of the whole community.

$$\sum_{u\in\mathcal{U}} \left(i_{u,t}^{\text{com}} - e_{u,t}^{\text{com}}\right) = 0 \qquad \forall t \in \mathcal{T}, \tag{4}$$

$$\sum_{u\in\mathcal{U}} \left(i_{u,t}^{\text{gri}} - e_{u,t}^{\text{gri}}\right) + P_t^{\text{loss}} \leq \overline{p} \qquad \forall t \in \mathcal{T}. \tag{5}$$

The power balance for each community member defines its power exchanges depending on its internal devices and usage,

$$\begin{aligned} & e_{u,t}^{\text{gri}} - i_{u,t}^{\text{gri}} + e_{u,t}^{\text{com}} - i_{u,t}^{\text{com}} = \\ & \qquad P_{u,t}^{\text{PV}} - D_{u,t}^{\text{nfl}} + P_{u,t}^{\text{dis}} - P_{u,t}^{\text{cha}} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \end{aligned} \tag{6}$$

where $P_{u,t}^{\text{PV}}$ and $D_{u,t}^{\text{nfl}}$ are the power produced by the PV system and the fixed power consumption of user u at time t. Variables $P_{u,t}^{\text{cha}}$, and $P_{u,t}^{\text{dis}}$ are the active powers going in and out of the battery storage system (BSS).

A. Photovoltaic installations

Photovoltaic power injection ($P_{u,t}^{\text{PV}}$) is bounded for every time period t. The upper bound value is defined as the PV profile $\overline{P}_{u,t}^{\text{PV}}$ depending on both weather conditions and PV installation size,

$$P_{u,t}^{\text{PV}} \in [0, \overline{P}_{u,t}^{\text{PV}}] \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \tag{7}$$

B. Battery storage systems

The following equations model the BSS state of charge evolution. Constraint (10) enforces an identical state of charge ($s_{d,u}^{\text{init}}$) at the beginning ($t_{d,0}$) and end ($t_{d,T}$) of each representative day d. This value is defined as a variable to be optimally determined.

$$\begin{aligned} & s_{u,t_{d,1}} - \Delta_T \left(\eta_u^{\text{cha}} P_{u,t_{d,1}}^{\text{cha}} - P_{u,t_{d,1}}^{\text{dis}}/\eta_u^{\text{dis}}\right) = s_{u,d}^{\text{init}} \\ & \hfill \forall u \in \mathcal{U}, \quad \forall d \in \mathcal{D}, \end{aligned} \tag{8}$$

$$\begin{aligned} & s_{u,t} - s_{u,t-1} - \Delta_T \left(\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} - P_{u,t}^{\text{dis}}/\eta_u^{\text{dis}}\right) = 0, \\ & \hfill \forall u \in \mathcal{U}, \quad t \in \{t_{d,2}, \ldots, t_{d,T}\}, \forall d \in \mathcal{D}, \end{aligned} \tag{9}$$

$$s_{u,t_{d,T}} = s_{u,d}^{\text{init}} \qquad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \tag{10}$$

with $s_{u,t}$ the state of charge of the storage of user u at the end of period t. These are bounded by variables that depend on the sizing results introduced in the next subsection,

$$s_{u,t} \in [\underline{S}_u; \overline{S}_u] \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \tag{11}$$

$$s_{u,d}^{\text{init}} \in [\underline{S}_u; \overline{S}_u] \qquad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \tag{12}$$

$$P_{u,t}^{\text{dis}} \in [0; \overline{P}_u^{\text{dis}}] \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \tag{13}$$

$$P_{u,t}^{\text{cha}} \in [0; \overline{P}_u^{\text{cha}}] \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \tag{14}$$

C. Sizing

Some of the optimization variables of this first problem are the sizes of the new DERs. Each member can invest in new PV (C_u^{PV}) or BSS (C_u^{BSS}) capacities at its location.

The PV and BSS sizes are optimized for each user considering its location and the investment costs, considering economies of scale,

$$\text{CAPEX}_u = \Pi^{\text{PV}}(C_u^{PV}) + \Pi^{\text{BSS}}(C_u^{BSS}) \tag{15}$$

where $\Pi^y(x)$ is the cost of installation for a system y of size x:

$$\Pi^{\text{y}}(x_u) = b^{\text{y}}_u \pi^{\text{y}}_f + x_u \pi^{\text{y}}_v - D(x_u) \quad (16)$$

$$0 \leq D(x_u) \leq d^{\text{y}}_u (x_u - 0.5\overline{x}^{\text{y}}) \pi^{\text{y}}_d \quad (17)$$

$$b^{\text{y}}_u \underline{x}^{\text{y}} \leq x_u \leq b^{\text{y}}_u \overline{x}^{\text{y}} \quad (18)$$

with π^{y}_f, π^{y}_v and π^{y}_d respectively the fixed, variable and discounted cost coefficients. Function $D(.)$ is positive and gives a discount of π^{y}_d per unit of capacity installed above a system size threshold (*i.e.*, half of the maximum). The fixed cost and discount terms allow to represent economies of scale in new investments as shown in Fig. 1. Equation (18) bounds the system sizes and fixes the value of b^{y}_u, the binary variable indicating the user's investment in technology y. The binary variable d^{y}_u indicates that the size the investment is sufficient for discounted prices.

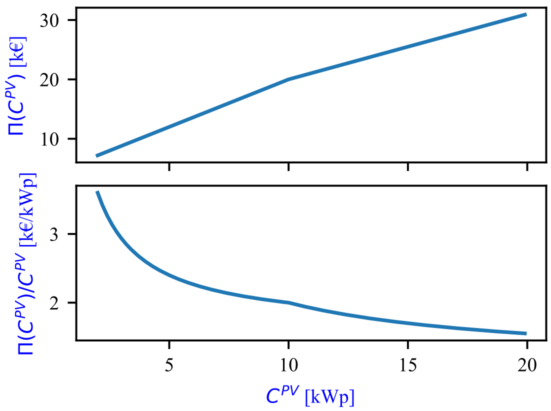


Fig. 1. Total investment costs and capacity costs for a PV installation (π^{PV}_f = 4e3€, π^{PV}_v = 1.6€/Wp and π^{PV}_d = 0.5€/Wp).

The investment capacities are also limited by the budget of the community participants ($\overline{B}_u$). The total investment costs for the whole community is limited by the budget of all participants pooled together:

$$\sum_{\forall u \in \mathcal{U}} \text{CAPEX}_u \in [0; \sum_{\forall u \in \mathcal{U}} \overline{B}_u]. \quad (19)$$

The decision variables for investment capacities C^{PV}_u and C^{BSS}_u are used to define the bounds for the power injections in the operational planning,

$$\overline{P}^{\text{PV}}_{u,t} = \overline{P}^{\text{PV,i}}_{u,t} + C^{\text{PV}}_u \overline{p}^{\text{PV}}_{u,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (20)$$

$$\underline{S}_u = \underline{S}^i_u + 0.1 C^{\text{BSS}}_u \qquad \forall u \in \mathcal{U}, \quad (21)$$

$$\overline{S}_u = \overline{S}^i_u + 0.9 C^{\text{BSS}}_u \qquad \forall u \in \mathcal{U}, \quad (22)$$

$$\overline{P}^{\text{cha}}_u = \overline{P}^{\text{cha}}{}^i_u + \overline{P}^{\text{cha}} C^{\text{BSS}}_u \qquad \forall u \in \mathcal{U}, \quad (23)$$

$$\overline{P}^{\text{dis}}_u = \overline{P}^{\text{dis}}{}^i_u + \overline{P}^{\text{dis}} C^{\text{BSS}}_u \qquad \forall u \in \mathcal{U}, \quad (24)$$

where $\overline{P}^{\text{PV,i}}_{u,t}$ is the maximum PV production of user u at time t before investment and $\overline{p}^{\text{PV}}_u$ is the scaled PV profile associated with user u. Variables $\underline{S}^i_u$ and $\overline{S}^i_u$ represent the initial minimum and maximum values for the state of charge of user u. The newly invested capacity C^{BSS}_u can only be used ranging from 10 to 90% of its maximum capacity. Variables $\overline{P}^{\text{cha}}{}^i_u$ and $\overline{P}^{\text{dis}}{}^i_u$ denote the charging and discharging rates, respectively, for u before investment and variables $\overline{P}^{\text{cha}}_u$ and $\overline{P}^{\text{dis}}_u$ represents the maximal charging and discharging power of the new BSS per unit of capacity installed.

D. Power flow model

Power flow constraints are added to this optimization problem to ensure a reliable operation of the distribution network. The radial configuration of the distribution grid allows us to use a relaxed DistFlow model [10].

First, physical network limits are represented by limits on the squared line currents (I^{sqr}) and squared node voltages (V^{sqr}),

$$\underline{V}^{\text{sqr}} \leq V^{\text{sqr}}_{u,t} \leq \overline{V}^{\text{sqr}} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (25)$$

$$I^{\text{sqr}}_{u,t} \leq \overline{I}^{\text{sqr}}_u \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (26)$$

The set of network lines is actually identical to set $\mathcal{U}$. There are $|\mathcal{U}|$ lines in the radial distribution network due to the existence of $|\mathcal{U}| + 1$ electrical nodes. The additional node is the slack bus and does not host any user.

$\mathcal{C}_u$ is the set of children of user u in the tree graph of the electrical network. Similarly, $\mathcal{A}_u$ is the set of ancestors of user u. Due to the tree structure of he graph, there is always a single ancestor to u (a_u).

We consider line u to be the line reaching user u from its ancestor with impedance $Z_u = R_u + jX_u$. The DistFlow model is implemented as:

$$P^{\text{inj}}_{u,t} + P^{\text{line}}_{u,t} - R_u I^{\text{sqr}}_{u,t} = \sum_{c \in \mathcal{C}_u} P^{\text{line}}_{c,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (27)$$

$$Q^{\text{inj}}_{u,t} + Q^{\text{line}}_{u,t} - X_u I^{\text{sqr}}_{u,t} = \sum_{c \in \mathcal{C}_u} Q^{\text{line}}_{c,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (28)$$

$$V^{\text{sqr}}_{u,t} = V^{\text{sqr}}_{a_u,t} - 2(R_u P^{\text{line}}_{u,t} + X_{u,t} Q^{\text{line}}_{u,t}) + (R^2_u + X^2_u) I^{\text{sqr}}_{u,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (29)$$

$$I^{\text{sqr}}_{u,t} V^{\text{sqr}}_{a_u,t} \geq P^{\text{line}}_{u,t}{}^2 + Q^{\text{line}}_{u,t}{}^2 \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (30)$$

where equations (27) and (28) represent power balance at each node u, with P^{line}_u and Q^{line}_u the active and reactive powers flowing in line u. Equation (29) and (30) determine respectively the voltage drop and the squared current in each line u.

The variables for the slack bus are defined as,

$$V^{\text{sqr}}_{\text{slack},t} = 1 \qquad \forall t \in \mathcal{T}, \quad (31)$$

$$P^{\text{inj}}_{\text{slack},t} = \sum_{c_s \in \mathcal{C}_{\text{slack}}} P^{\text{line}}_{c_s,t} \qquad \forall t \in \mathcal{T}, \quad (32)$$

$$Q^{\text{inj}}_{\text{slack},t} = \sum_{c_s \in \mathcal{C}_{\text{slack}}} Q^{\text{line}}_{c_s,t} \qquad \forall t \in \mathcal{T}. \quad (33)$$

The maximum of $P^{\text{inj}}_{\text{slack},t}$ corresponds to the peak power of the community. Indeed, equation (32) can also be expressed with the energy exchange variables as,

$$P^{\text{inj}}_{\text{slack},t} = \sum_{\forall u \in \mathcal{U}} (i^{\text{gri}}_{u,t} - e^{\text{gri}}_{u,t}) + P^{\text{loss}}_t \quad \forall t \in \mathcal{T}.$$

These power flow variables and constraints are linked to those of the community problem with:

$$P^{\text{inj}}_{u,t} = e^{\text{gri}}_{u,t} + e^{\text{com}}_{u,t} - i^{\text{gri}}_{u,t} - i^{\text{com}}_{u,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (34)$$

$$Q^{\text{inj}}_{u,t} = Q^{\text{PV}}_{u,t} + Q^{\text{BSS}}_{u,t} + Q^{\text{nfl}}_{u,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (35)$$

where $Q^{\text{nfl}}_{u,t}$ is the fixed reactive part of the load profile of u. $Q^{\text{PV}}_{u,t}$ and $Q^{\text{BSS}}_{u,t}$ are the reactive power injection or withdrawal of the PV and battery inverters. To model technical limitation, the magnitude of these variables is bounded to 30% of the current power flowing in the inverter:

$$-0.3P^{\text{PV}}_{u,t} \leq Q^{\text{PV}}_{u,t} \leq 0.3P^{\text{PV}}_{u,t} \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (36)$$

$$-0.3(P^{\text{cha}}_{u,t} + P^{\text{dis}}_{u,t}) \leq Q^{\text{BSS}}_{u,t} \leq 0.3(P^{\text{cha}}_{u,t} + P^{\text{dis}}_{u,t}) \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (37)$$

The total active power losses in the system (P^{loss}_t) for each time step are:

$$P^{\text{loss}}_t = \sum_{\forall u \in \mathcal{U}} R_u I^{\text{sqr}}_{u,t}. \quad (38)$$

E. Output of the first problem

After computing the maximum profit of the community as a whole, the solution to this problem is reused as parameters for the second step of optimization to ensure a fair sharing of the costs and benefits generated by the community. Both investments and energy usage are considered, so every community member gets billed fairly. The power flows in the network are, at this point, totally fixed, as well as the peak of the community whose bill will have to be shared among participants.

To compute the energy bills of the members, the CO has to consider all the power exchanges with the grid and the community during the simulation horizon as well as the battery charging and discharging, which is also valued.

One of the goals of the second optimization problem (39) is to account for the distribution of the co-invested benefits. The distribution key for this is defined by a simple scheme and is computed after solving (1) to be used as parameters in (39):

- If a participant has invested and there is an investment at its location, he will have priority over it.
- If a participant has more to invest than what is needed at its location, the excess goes to a community investment pool.
- If a participant has less to invest than needed at its location, the community investment pool pays the remaining.

Two parameters are computed for each participant, with a procedure illustrated in Figure 2:

- i_u the share of u in the community budget pool.
- k_u the portion of DER investment at node u that has been paid for by the community.

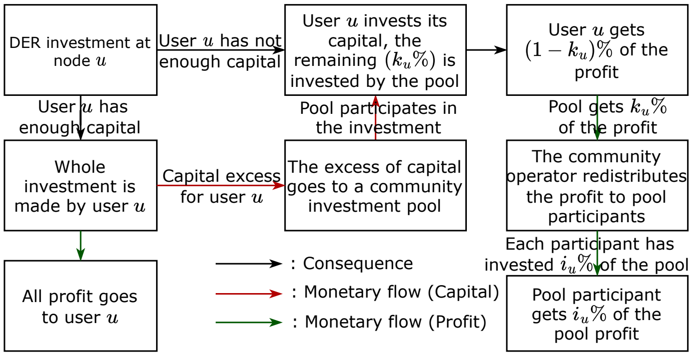


Fig. 2. Additional monetary flows in the community in case of co-investment

Using variables k_u, it is possible to compute how much the DER investment has produced. This depends on the power exchanges computed by problem (1) at every timestep. The production of the co-invested asset needs to be separated from the rest of the user's production. The value of exchanges also needs to be determined, depending on the source/destination of the energy: the community or the external grid.

The algorithm 1 shows how the energy exported by co-invested assets to the grid ($J^{\text{gr,pool}}$) and the energy imported to/exported from the community ($i^{\text{com, pool}}, e^{\text{com, pool}}$) are computed. The grid export will then be valued at a known price (π^{igr} or π^{egr}) and the power exported to the community ($e^{\text{com,pool}}_{u,t}$, which can be negative for imports) will then be valued in the profit sharing problem. The total profit will then be shared proportionally to each user's share in the community budget pool (i_u).

```
for $u$ in $\mathcal{U}$ do
    for $t$ in $\mathcal{T}$ do
        if $k_u$ != 0 then
            // Net prod. of co-invested asset:
            $e^{inv}_{u,t} = C^{\text{PV}}_u \overline{p}^{\text{PV}}_{u,t} + P^{\text{dis}}_{u,t}(C^{\text{BSS}}_u) - P^{\text{cha}}_{u,t}(C^{\text{BSS}}_u)$;
            // Net export of user $u$:
            $e_{u,t} = e^{\text{gri}}_{u,t} + e^{\text{com}}_{u,t} - i^{\text{gri}}_{u,t} - i^{\text{com}}_{u,t}$;

            // 1. User or asset self-consumes
            //    or self-produces:
            if $(e^{inv}_{u,t} > 0 \; and \; e_{u,t} < 0) \; or \; (e^{inv}_{u,t} < 0 \; and \; e_{u,t} > 0)$
              then
                $e^{\text{com,pool}}_{u,t} = k_u e^{inv}_{u,t}$;
                // Valued at community price
            end
            // 2. Net cons. and user imports:
            else if $(e^{inv}_{u,t} < 0 \; and \; e_{u,t} < 0)$ then
                $e^{\text{com,pool}}_{u,t} = k_u e^{inv}_{u,t} i^{\text{com}}_{u,t} / (i^{\text{gri}}_{u,t} + i^{\text{com}}_{u,t})$;
                $J^{\text{gr,pool}}_{u,t} = \pi^{\text{igr}}_t k_u \Delta_T e^{inv}_{u,t} i^{\text{gri}}_{u,t} / (i^{\text{gri}}_{u,t} + i^{\text{com}}_{u,t})$;
            end
            // 3. Net prod. and user exports:
            else if $(e^{inv}_{u,t} > 0 \; and \; e_{u,t} > 0)$ then
                $e^{\text{com,pool}}_{u,t} = k_u e^{inv}_{u,t} e^{\text{com}}_{u,t} / (e^{\text{gri}}_{u,t} + e^{\text{com}}_{u,t})$;
                $J^{\text{gr,pool}}_{u,t} = \pi^{\text{egr}}_t k_u \Delta_T e^{inv}_{u,t} e^{\text{gri}}_{u,t} / (e^{\text{gri}}_{u,t} + e^{\text{com}}_{u,t})$;
            end
        end
    end
end
return $J^{\text{gr,pool}}, e^{\text{com,pool}}$;
```

Algorithm 1: Computation $J^{\text{gr,pool}}$ and $e^{\text{com,pool}}_{u,t}$.

This algorithm computes for each time steps the net production of the jointly invested asset ($e^{inv}_{u,t}$) and the net export ($e_{u,t}$) at the node where the investment is located. Then, three cases can occur:

1) If the asset produces and the user consumes or the asset consumes and the user produces, the part (k_u) of energy produced by the pool investment will be valued at community price in the next problem. Indeed this energy is beneficial for the user due to the co-investment and the community exchanges.
2) If the asset and the users both have a negative net injection, both are consuming energy. This will be valued at grid tariff for a portion equal to $i^{\text{gri}}_{u,t}/(-e_{u,t})$ and the rest is considered as community exchanged and will be valued in the next problem.
3) In the last case, both have a positive net injection. Profit are computed similarly to case 2 with export prices.

III. BENEFITS DISTRIBUTION AMONG PARTICIPANTS

Using the results of (1), a second optimization problem is solved *a posteriori* to share the benefits of the community between its participants. At this point, all the exchanges and investments have been computed in the first optimization model and are fixed. This results in a given community benefit to be shared.

In practice this would be done during operation. To share all the additional benefits generated by the community, fair internal exchange prices need to be determined for the community. These prices are the main decision variables of this problem and will be used for both direct exchanges between members and return on investment from a co-invested asset to the participants of the community budget pool.

The results of this problem are highly dependent on the distribution key used by the community operator. The discussion of different sharing mechanisms and their fairness is outside the scope of this work. Here, only a simple scheme will be proposed: maximizing the minimum improvement in user's benefit compared to a case without community involvement.

$$\begin{aligned} \max \quad & \text{Minimum additional member profit (40)} && (39)\\ \text{subject to} \quad & \textit{Minimum profit } (41), \\ & \textit{Profit sharing constraints } (42) - (45), \\ & \textit{Price constraints } (46) - (47). \end{aligned}$$

The objective function of this problem is defined as,

$$F = \alpha - N(\boldsymbol{\pi}^{com}), \tag{40}$$

where $N(\boldsymbol{\pi}^{com})$ the price regularization term. The main term α represents the minimum additional benefit of a community member, compared to its standalone situation,

$$J_u \geq J_u^{\text{SU}} + \alpha, \quad \forall u \in \mathcal{U}. \tag{41}$$

The parameter J_u^{SU} is the optimal profit of user u computed with a problem similar to (1) where the community does not exist. J_u defines the optimal yearly profit of user u in the community, computed from the yearly operational cost of electricity (J_u^{energy}), negative peak profit (J_u^{peak}), negative profit to return to co-investors (J_u^{return}), and profit from co-investing (J^{coinvest}):

$$J_u = J_u^{\text{energy}} + J_u^{\text{peak}} + J_u^{\text{return}} + J_u^{\text{coinvest}}. \tag{42}$$

$$\begin{aligned} J_u^{\text{energy}} = -\Delta_T \sum_{t \in \mathcal{T}} \Big(& \pi_t^{\text{igr}} i_{u,t}^{\text{gri}} - \pi_t^{\text{egr}} e_{u,t}^{\text{gri}} \\ & + \gamma^{\text{com}} (e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}}) - \pi_t^{\text{com}} \left(e_{u,t}^{\text{com}} - i_{u,t}^{\text{com}} \right) \\ & + \gamma_u^{\text{sto}} \left(\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}} / \eta_u^{\text{dis}} \right) \Big). \end{aligned} \tag{43}$$

This equation differs from (3) by the term which represents the monetary exchanges between community members. Variable π_t^{com} represents the price at which the energy will be sold within the community at time period t of the simulation. The peak of the community ($\overline{p}$) is shared between members using equation (45), each member is then accountable to pay an amount $-J_u^{\text{peak}}$ defined by its positive share of the peak ($\overline{p}_u$) and the peak price (π^{peak}).

$$J_u^{\text{peak}} = -\pi^{\text{peak}} \overline{p}_u, \tag{44}$$

$$\overline{p} = \sum_{u \in \mathcal{U}} \overline{p}_u. \tag{45}$$

This problem aims to optimize the personal share of each user by adapting the community energy prices π_t^{com}. To have sensible results, additional constraints are added to the prices to make them coherent,

$$\pi_t^{\text{com}} \in [\pi_t^{\text{egr}} + \gamma^{\text{com}}; \pi_t^{\text{igr}} - \gamma^{\text{com}}] \qquad \forall t \in \mathcal{T}, \tag{46}$$

$$N(\boldsymbol{\pi}^{com}) = \omega_N \sum_{t \in \mathcal{T}} (\pi_t^{\text{com}} - (\pi_t^{\text{igr}} + \pi_t^{\text{egr}})/2)^2, \tag{47}$$

where (46) ensures that there is no time period where the community prices are less interesting than the grid fees, be it for the buyer or seller. Equation (47) defines the regularization term, where ω_N is a weight factor such that $\alpha >> N(.)$. The role of this term is to reduce the variability in the price by driving it closer to a constant value.

The additional profit of u that needs to be returned to the pool (J_u^{return}) and the additional profit of a pool participant (J_u^{coinvest}) can be computed using the profit made by the asset from grid exchanges with fixed price ($J_{u,t}^{\text{gr,pool}}$) and the quantities exchanged inside the community ($e_{u,t}^{\text{com,pool}}$) with a price to be defined (π_t^{com}).

$$J_u^{\text{return}} = - \sum_{\forall t \in \mathcal{T}} (\Delta_T \pi_t^{\text{com}} e_{u,t}^{\text{com,pool}} + J_{u,t}^{\text{gr,pool}}) \qquad \forall u \in \mathcal{U}, \tag{48}$$

$$J_u^{\text{coinvest}} = -i_u \sum_{\forall u \in \mathcal{U}} J_u^{\text{return}} \qquad \forall u \in \mathcal{U}. \tag{49}$$

IV. CASE STUDY AND SIMULATION

A. *Case study*

The problem is run on a benchmark Dickert-LV network [11] consisting of three feeders for a total of 45 buses. The community spans the whole network and involves 39 residential users, five industrial consumers, and 1 PCC, which is the slack bus in the OPF model. The role of this slack bus is to provide for the net imbalance in the community and to compensate for the power losses in the network.

TABLE I
PARAMETERS OF THE DICKERT LV-NETWORK.

Range	Linetype	Customer	Case	Voltage level	Line length
Middle	Cable	Multiple	Good	230 V	40 m

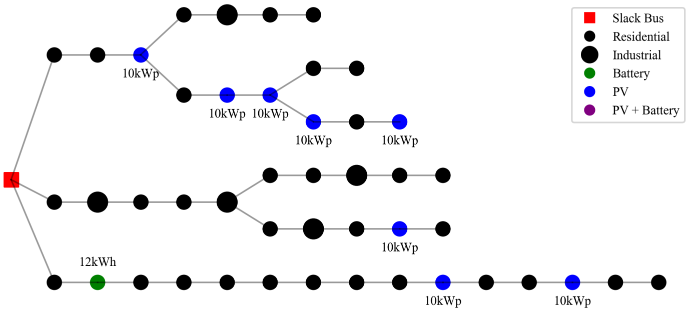


Fig. 3. Initial network of the energy community

Before new investments, the network already includes eight PV installations and one BSS, in locations shown in Fig. 3. In this case study, all users have an equal price for energy sold and bought to/from the retailer, which is constant for the whole simulation, $\pi^{\text{egr}} = 0.05$€/kWh and $\pi^{\text{igr}} = 0.40$€/kWh. The fee imposed by the CO is $\gamma^{\text{com}} = 0.01$€/kWh.

B. Simulation

The simulation is run with a one-hour resolution using 12 representative days each divided in 24 periods of one hour. This models a full a one-year period with one day per month weighted with values of w_{d_t}. Data for hourly consumption and PV production profiles are extracted from [12]. The operational planning costs of each day is weighted to have a total amount corresponding to the annual OPEX of the system. These days are all modeled and solved by a single optimization model to have common variable DER sizes determining the yearly CAPEX of each user. The problem is implemented using the Julia JuMP package [13] and solved with the Gurobi solver [14]. The results of the model are then run using the Pandapower python library [15] to verify the grid state at every timestep and to produce figures for the results of the system with new investments.

C. Scenarios

We compare three investment scenarios to assess the impact of the energy community on the grid and its members.

1) Individual model: In this model, each user can only invest at his electrical node with an amount limited by his budget,

$$\mathrm{CAPEX}_u \in [0; \overline{B}_u] \qquad \forall u \in \mathcal{U}, \tag{50}$$

and community exchanges are not allowed,

$$i^{\mathrm{com}}_{u,t} = e^{\mathrm{com}}_{u,t} = 0 \qquad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \tag{51}$$

The second optimization problem is thus irrelevant since each user can only exchange energy with the grid (*i.e.* its retailer). The value of J^{energy}_u obtained in this case defines, for each user u, the parameter J^{SU}_u used for the benefit distribution in the next scenarios.

2) REC without co-investment: In this scenario, each member's budget can only be used for personal DERs, ensured by constraint (50). A community is created, and constraint (51) is thus relaxed. This represents a community where each participant can invest in devices located behind its own energy meter. The benefits of the community mostly come from the gap between the import (π^{igr}) and export (π^{egr}) prices. When the community price is set inside this range, the community creates value for both the buyer and the seller. Equation (42) is reduced to

$$J_u = J^{\mathrm{energy}}_u + J^{\mathrm{peak}}_u \qquad \forall u \in \mathcal{U}, \tag{52}$$

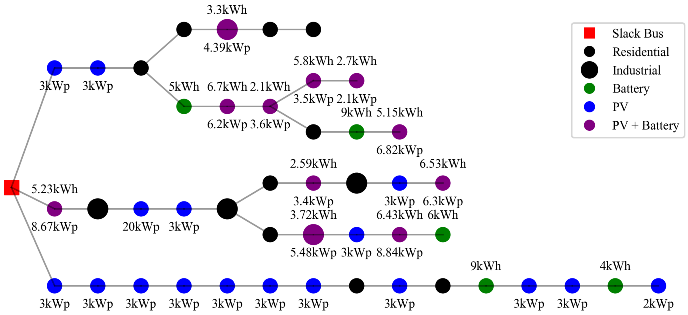
which does not account for DER co-investment.

3) REC with co-investment: All the community members can pool their capital together, the CO can optimally size and place the new DERs. This scenario uses the full model developed in Sections II and III.

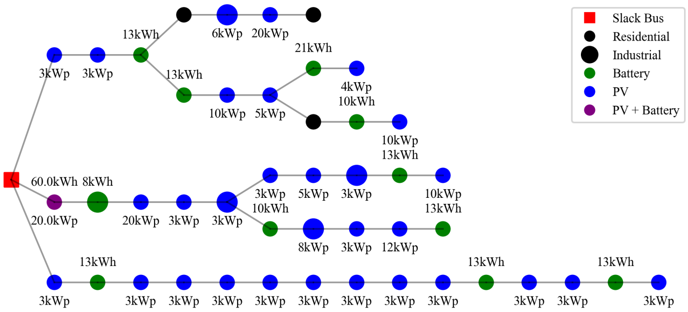
V. RESULTS

The results show that the community can create benefits for all users. The optimal investments considering the three scenarios are compared in Fig. 4, which shows that the no-community and community with self-investment have similar results.

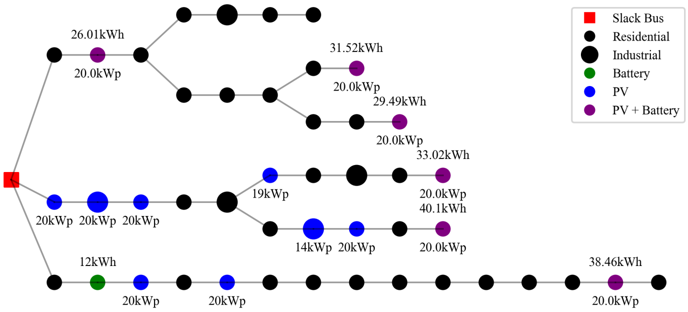
This is because the personal budget is highly limiting in these two first cases. The optimum of each member primarily depends on its own energy usage. In scenario 2, we can see an increase in BSS investment. This can be explained by the added profitability for the community to be able to maximize its self-consumption. These results are defined by the first step, which considers the community as a whole. The results for the second scenario show a totally different approach. Co-investment allows users to invest in fewer assets with a bigger capacity to take advantage of the economies of scale.



(a) No community



(b) Community with self-investment



(c) Community with co-investment

Fig. 4. Optimal new investment locations and sizes for the three scenarios.

One hypothesis made in problem (1) was that the community would be accountable for loss compensation. For this assumption to be realistic, the CO would need enough income to pay for this energy. In the basic simulation design with $\gamma^{\mathrm{com}} = 0.01$ €/kWh, this hypothesis is already validated with 3246 € of CO revenue per year for 1115 € of costs for losses in scenario 2 and 3160 € of revenue for 918 € of costs for scenario 3.

Figure 5 shows the intra-community energy prices for the 12 representative days. With the proposed profit-sharing strategy, the prices tend to be equal to their upper or lower bound, depending on the excess or deficit of power. Some prices are equal to the mean value due to the regularization term and the absence of energy to be sold in the community. In the scenario with co-investment, the prices are higher due to the necessity for co-investors to have a good return on investment.

The proposed model ensures profitability for all community members compared to their optimal behavior in the same conditions without the community's existence. Figure 6 shows how much the profit can be increased with the two community scenarios.

TABLE II
SUMMARY OF NETWORK USAGE FOR THE THREE SCENARIOS

Case	Ind. model	EC w/o co-inv.	EC w. co-inv.
Total losses [MWh]	3.04	2.79	2.29
Max voltage [pu]	1.03	1.04	1.05
Min voltage [pu]	0.95	0.95	0.95
Max line loading [%]	83.54	84.83	71.21
Self-consumption [%]	37.49	88.23	81.38
Self-sufficiency [%]	70.95	73.87	79.48
PCC peak [kW]	160.56	56.58	69.50

Table II summarizes the impact of the EC on the distribution grid. For the self-investment models, the results are comparable as the topology of new investments is very similar.

The third scenario's improvement in most aspects is achieved through increased flexibility in DER placement and sizes. This is because the CO can place the assets at the optimal place, with diminished limitation on system size. However, this larger overall DER capacity slightly increases maximum voltage and decreases self-consumption.

TABLE III
SUMMARY OF FINANCIAL RESULTS FOR THE THREE SCENARIOS

Case	Ind. model	EC w/o co-inv.	EC w. co-inv.
Total cost [k€/y]	147	108	97
Total OPEX [k€/y]	123	77	66
Total CAPEX [k€]	486	607	636
PV inv. [kWp]	157	217	260
BSS inv. [kWh]	119	172	253
Min additional profit [€/y]	0	483	938

From these results, we can see that the community can greatly improve the users' benefits in both cases. The co-investment scenario allows the minimal additional benefit (α) to be nearly doubled compared to the self-investment.

The total investment of the community is also increased in the co-investment community. This is mainly due to specific users for which the co-investment can bring new possibilities. Indeed, one of the members' budget was greater than the maximum investment at one location, which is limited by maximum size for BSS and PV installation. Bigger and more profitable investment can then be done from this budget. Some other members which did not have enough budget to realize new investments on their own, due to fixed costs and minimal capacities, now can use co-investment to generate profit.

VI. CONCLUSION

When sizing DERs in RECs, it is important to consider the REC operation (including network constraints and market awareness) and a fair distribution of the benefits to all REC members. We proposed in this paper a procedure involving two steps. The first step solves the problem of REC welfare maximization while the second one maximizes the minimum additional profit that a member can get. Based on the results obtained using a benchmark Dickert-LV network, the following conclusions can be drawn.

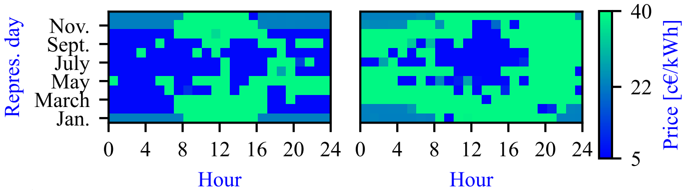


Fig. 5. Price in the community for energy exchange during 12 representative days, without co-investment on the left, with co-investment on the right.

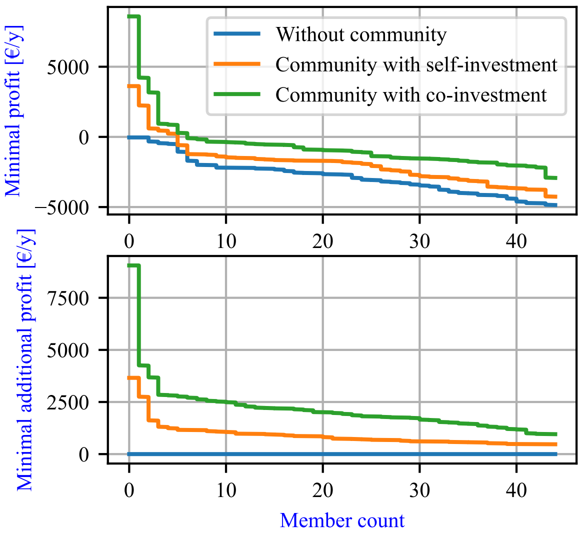


Fig. 6. Community members' increase in benefits.

- The energy community can be beneficial in terms of grid operation. With a grid-aware model of the energy community, the impact of a higher DER penetration in the distribution grid can be diminished. Regarding energy losses, line loading and self-consumption, both community models allowed for improvement.
- Energy communities greatly incentivize new DER investments. They can boost energy transition in two ways: by increasing the maximum profitable capacity and by improving profitability for DER usage. This is particularly true for a co-investment scenario.
- If the community operator is fully cooperative, it can be a useful intermediate between the distribution system operator and the community members. Long-term planning, as well as short-term operational planning, can both be beneficial with a CO. There is profitability for each participant, including community and grid operators.

Future research efforts will include a problem extension considering the demand flexibility of the members (an important aspect because of increasing electrification of heating, transportation, etc.), comparison of different profit sharing or price-fixing schemes, consideration of different notions of fairness [16], and improvement of the model to handle non-cooperative grid users that are not willing to take part in a community.

REFERENCES

[1] F. Cappellaro, G. D'Agosta, P. De Sabbata, F. Barroco, C. Carani, A. Borghetti, L. Lambertini, and C. A. Nucci, "Implementing energy transition and SDGs targets throughout energy community schemes," *Journal of Urban Ecology*, vol. 8, no. 1, pp. 1–9, 2022.

[2] European Commission, "Clean planet for all: a european strategic long-term vision for a prosperous, modern, competitive and climate neutral economy," *A European long-term strategic vision for a prosperous, modern, competitive and climate neutral economy*, 2018.

[3] A. Berka and E. Creamer, "Taking stock of the local impacts of community renewable energy: a review and research agenda," *Renewable and Sustainable Energy Reviews*, vol. 82, 11 2016.

[4] C. Aviles, S. Oliva, and D. Watts, “Single-dwelling and community renewable microgrids: Optimal sizing and energy management for new business models,” *Applied Energy*, vol. 254, p. 113665, 2019. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261919313522

[5] D. Fioriti, A. Frangione, and D. Poli, “Optimal sizing of energy communities with fair revenue sharing and exit clauses: Value, role and business model of aggregators and users,” *Applied Energy*, vol. 299, p. 117328, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261921007376

[6] T. Weckesser, D. F. Dominković, E. M. Blomgren, A. Schledorn, and H. Madsen, “Renewable Energy Communities: Optimal sizing and distribution grid impact of photo-voltaics and battery storage,” *Applied Energy*, vol. 301, no. April, p. 117408, 2021. [Online]. Available: https://doi.org/10.1016/j.apenergy.2021.117408

[7] A. Cosic, M. Stadler, M. Mansoor, and M. Zellinger, “Mixed-integer linear programming based optimization strategies for renewable energy communities,” *Energy*, vol. 237, p. 121559, 2021. [Online]. Available: https://doi.org/10.1016/j.energy.2021.121559

[8] M. Hupez, J. F. Toubeau, Z. De Greve, and F. Vallee, “A New Cooperative Framework for a Fair and Cost-Optimal Allocation of Resources within a Low Voltage Electricity Community,” *IEEE Transactions on Smart Grid*, vol. 12, no. 3, pp. 2201–2211, 2021.

[9] B. Cornélusse, I. Savelli, S. Paoletti, A. Giannitrapani, and A. Vicino, “A community microgrid architecture with an internal local market,” *Applied Energy*, vol. 242, pp. 547–560, 2019. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261919305173

[10] M. Farivar and S. H. Low, “Branch flow model: Relaxations and convexification—Part I,” *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.

[11] J. Dickert, M. Domagk, and P. Schegner, “Benchmark low voltage distribution networks based on cluster analysis of actual grid properties,” in *2013 IEEE Grenoble Powertech Conference*. IEEE, 2013, pp. 1–6.

[12] Open power system data. 2020. data package household data. version 2020-04-15. [Online]. Available: https://data.open-power-system-data.org/household_data/2020-04-15/

[13] M. Lubin, O. Dowson, J. D. Garcia, J. Huchette, B. Legat, and J. P. Vielma, “Jump 1.0: Recent improvements to a modeling language for mathematical optimization,” *Mathematical Programming Computation*, 2023.

[14] Gurobi Optimization, LLC, “Gurobi Optimizer Reference Manual,” 2023. [Online]. Available: https://www.gurobi.com

[15] L. Thurner, A. Scheidler, F. Schäfer, J.-H. Menke, J. Dollichon, F. Meier, S. Meinecke, and M. Braun, “Pandapower—an open-source python tool for convenient modeling, analysis, and optimization of electric power systems,” *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 6510–6521, 2018.

[16] F. Moret and P. Pinson, “Energy collectives: A community and fairness based approach to future electricity markets,” *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3994–4004, 2019.