

A practical modeling of circuit breakers with unknown statuses in generalized state estimators

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Abstract—This paper proposes a new practical approach for including circuit breakers with unknown statuses in the formulation of the generalized state estimation problem. Unlike other proposals, only one single operating constraint is proposed for representing two possible and mutually exclusive breaker statuses: closed or open. This is achieved by using the concept of complementarity conditions based on the apparent power flow and complex voltages across the breaker terminals. This complementarity constraint is then transformed into an equality constraint by using the Fischer-Burmeister merit function, and it is directly included into the state estimation problem to simultaneously co-estimate the operating state of the entire network together with the breaker’s unknown operating statuses. The proposal’s effectiveness is demonstrated in the IEEE 24-bus benchmark system with different substation configurations and circuit breakers with unknown statuses.

Index Terms--Circuit breakers, state estimation, topology estimation, complementarity constraints.

I. INTRODUCTION

A. Motivation

State estimation (SE) is a fundamental tool for the static analyses of electric power systems, which are performed by assuming that the topology processor provides a correct network configuration without any uncertainty associated with circuit breaker (CB) statuses in power substations. Hence, errors in CB statuses undermine the quality of estimations because the bus-branch model representing the current connectivity of transmission elements is incorrect [1].

Different reformulations of the SE problem have been proposed to identify circuit breakers’ on/off statuses at substations based on a detailed node-breaker (NB) model to cope with topology errors. The problem of simultaneously estimating the system operating state and all circuit breakers’ statuses at explicitly modeled power substations is referred to as generalized state estimation (GSE) [1]. In these reformulations, most of these proposals assume a known status of a CB, and the possibly erroneous assumption is corrected in a post-estimation process. On the other hand, only a few proposals explicitly model the unknown status of a CB in the GSE formulation, where the breaker status is considered unknown because no status signal has been received or this

signal is suspicious. This paper addresses this problem by proposing a practical and straightforward approach for modeling and including the unknown operating status of a CB in the GSE problem such that the estimation can only converge to one of its two possible operating statuses: closed or open.

B. Literature Review

Different methodologies have been proposed to perform a GSE study considering a detailed NB model of the transmission grid. The constrained nonlinear weighted least squares-based state estimation is extended in most of the proposals, e.g., [2]-[13], to consider topological measurements associated with the operating status of CBs. The CBs are included in the SE formulation through operational constraints of zero-voltage drop across closed breakers and zero-power flow through open breakers. In addition, the active and reactive power flows through all CBs included in the formulation are added as state variables to the SE problem. If the assumed known status of a single circuit breaker is identified as being incorrectly reported to the state estimator, this status is changed at a post-processing stage, and the state estimation is newly performed. Within this context, several ways of identifying topological measurement errors have been proposed, which are broadly classified into numerical methods and rule-based methods, as detailed in [14]. Since the statuses of CBs are correctly reported to the control center most of the time [8], all the approaches mentioned above perform the co-estimation of states and topology considering only a selected number of substations. Hence, only those substations suspected of having doubtful statuses of CBs are modeled in detail. On the other hand, more recent proposals reported in [8]-[10] represent all buses through detailed NB models, which significantly increase the problem dimensions. To cope with this problem, a zone partitioning algorithm is proposed in [8] for solving the estimation problem through parallel processing.

The circuit breakers are considered as having unknown statuses if their operating statuses are not explicitly known or if the power system’s operator does not have absolute confidence in the breaker statuses provided by the network configurator. The explicit modeling of CBs with unknown statuses is only considered in [2] and [10]-[13]. The unknown

status of a breaker is modeled in [2] by considering the active and reactive powers that flow through it as state variables to be estimated. The conventional weighted least squares (WLS) method is extended in [10] to consider CBs with known and unknown statuses. The former is included in the SE formulation through operating constraints. In addition, the objective function of the SE problem is extended with a penalized function that represents the possible operating statuses of unknown CBs. The proposal reported in [11] demonstrates that considering only the power flows through CBs with unknown statuses [2] is not a sufficient condition for estimating their correct operating statuses. The possible inaccuracy in the results is prevented by adding two independent equality constraints to the state estimation problem for every CB with an unknown status. One of the constraints is given by the product of the active power flow and the voltage phase angular difference at both ends of the CB, while the other is stated by the reactive power flow multiplied by the voltage drop in the CB. The enforcement of both constraints determines the operating status of the CB: closed or open. The proposal in [12] relies on representing the unknown status of CBs through topology variables, which are added to the SE problem's state vector. For each topology variable, a quadratic constraint is also included in the problem formulation to ensure that its estimated value is 0 for an open CB and 1 when it is closed. Lastly, if a large number of measurements are available, the unknown statuses of CBs are identified in [13] using a direct current-based NB model. In this case, the possible status of a CB is modeled by its active power flow multiplied by a binary variable. This model is included through two linear inequality constraints in a mixed-integer quadratic programming problem, which also considers equality and inequality constraints associated with the substation layout.

C. Contributions

Based on the mentioned above, this paper puts forward an entirely different way of modeling and including CBs with unknown statuses in the generalized state estimation problem. To the best of the authors' knowledge, such a formulation has not been previously reported in the literature and substantially differs from other proposals: [2]-[14]. Within this context, the specific contributions of the proposed approach with respect to other proposals are as follows;

- A new single constraint is derived from the first principles to simultaneously consider the two possible and mutually exclusive operating conditions of a breaker's unknown status: closed or open. This constraint is formulated based on the complementarity theory to include it directly in the SE problem to co-estimate nodal voltages of the entire power system and unknown statuses of CBs in a unified frame of reference. Therefore, the proposal avoids assuming operating statuses in breakers with missing or ambiguous information that will require a post-estimation process to eliminate possible topology errors and re-running the state estimator [3]-[9].

- When a CB with an unknown status forms a loop with closed CBs, it is impossible to estimate from the CB's operational constraints if its unknown status strictly corresponds to a fully open or closed state. This ambiguity problem is solved for the first time in this paper by proposing a power flow-based comparative test that correctly identifies the CB's unknown status.
- Lastly, the proposed approach is completely general and can be used in any previously described approaches.

D. Paper Organization

This paper is organized as follows. First, the node-breaker modeling of an electric network is described in Section II, where the proposed modeling of the unknown status of a CB is detailed. Next, the proposed formulation for the GSE problem is described in Section III. Several case studies are then presented in Section VI to demonstrate the effectiveness of the proposal in correctly determining the unknown operating status of CBs. Lastly, conclusions and future work are presented in Section V.

II. NODE-BREAKER SYSTEM MODELING FOR THE GSE

A. Network and Measurement Models

The GSE formulation considers that in addition to the set of system transmission buses $\mathcal{N}_e := \{1, 2, \dots, N_e\}$ interconnected through transmission elements $\mathcal{T}_e := \{1, 2, \dots, N_{\mathcal{T}_e}\}$ there exists a set of buses $\mathcal{N}_{se} := \{1, 2, \dots, N_{se}\}$ explicitly modeled at power substations that are interconnected through a set of CBs $\mathcal{T}_{cb} := \{1, 2, \dots, N_{\mathcal{T}_{cb}}\}$. In this case, the number of CBs is given by $N_{\mathcal{T}_{cb}} = N_{\mathcal{T}_{cb}} + N_{\mathcal{T}_{ocb}} + N_{\mathcal{T}_{ucb}}$, where $N_{\mathcal{T}_{cb}}$ and $N_{\mathcal{T}_{ocb}}$ correspond to the number of CBs with known statuses: closed and open breakers, respectively. Furthermore, $N_{\mathcal{T}_{ucb}}$ denotes the number of CBs with unknown statuses. Based on that mentioned above, the total system buses are grouped in the set $\mathcal{N}_{tsb} \subseteq (\mathcal{N}_e \cup \mathcal{N}_{se})$, while the set of total transmission elements and CBs is denoted by $\mathcal{T}_{ecb} \subseteq (\mathcal{T}_e \cup \mathcal{T}_{cb})$. Lastly, the active and reactive powers flowing through all CBs, i.e., $\{P_{km}\}_{\{k,m\} \in \mathcal{N}_{se}} \in \mathbf{P}_{cb}$ and $\{Q_{km}\}_{\{k,m\} \in \mathcal{N}_{se}} \in \mathbf{Q}_{cb}$, respectively, are considered as state variables to be estimated. Therefore, the augmented state vector is represented by (1), where $\mathbf{V} \in \mathbb{R}^{N_e + N_{se}}$, $\boldsymbol{\theta} \in \mathbb{R}^{N_e + N_{se}}$, $\mathbf{P}_{cb} \in \mathbb{R}^{N_{\mathcal{T}_{cb}}}$ and $\mathbf{Q}_{cb} \in \mathbb{R}^{N_{\mathcal{T}_{cb}}}$:

$$\mathbf{x}_{e-cb} = [\mathbf{V} \ \boldsymbol{\theta} \ \mathbf{P}_{cb} \ \mathbf{Q}_{cb}]^T \in \mathbb{R}^{2(N_e + N_{se} + N_{\mathcal{T}_{cb}})}. \quad (1)$$

The set of physical measurements of electrical variables are obtained from the subsets of observable buses

$\mathcal{N}_{oe} := \{1, 2, \dots, N_{oe}\} \ni \mathcal{N}_{oe} \subseteq \mathcal{N}_{tsb}$ and of transmission elements and CBs $\mathcal{T}_{oe} := \{1, 2, \dots, N_{\mathcal{T}_{oe}}\} \ni \mathcal{T}_{oe} \subseteq \mathcal{T}_{ecb}$, respectively, where \ni means such that. The measurements provided by the SCADA system correspond to nodal voltage magnitudes $\{V_k^{SCADA}\}_{k \in \mathcal{N}_{oe}}$, nodal injections of active and reactive powers, i.e., $\{P_k^{SCADA}\}_{k \in \mathcal{N}_{oe}}$ and $\{Q_k^{SCADA}\}_{k \in \mathcal{N}_{oe}}$, respectively, as well as the branch active and reactive power flows $\{P_{km}^{SCADA}\}_{\{km\} \in \mathcal{T}_{oe}}$ and $\{Q_{km}^{SCADA}\}_{\{km\} \in \mathcal{T}_{oe}}$, respectively. All these SCADA measurements are grouped in the vector $\mathbf{z}_e^{SCADA} \in \mathbb{R}^{M_e^{SCADA}}$. Similarly, the measurements provided by PMU devices correspond to nodal voltage magnitudes and phase angles, i.e., $\{V_k^{PMU}\}_{k \in \mathcal{N}_{oe}}$ and $\{\theta_k^{PMU}\}_{k \in \mathcal{N}_{oe}}$, respectively, together with the complex branch current flows that are handled in rectangular coordinates: $\Re\{\tilde{I}_{km}^{PMU}\}_{\{km\} \in \mathcal{T}_{oe}}$ and $\Im\{\tilde{I}_{km}^{PMU}\}_{\{km\} \in \mathcal{T}_{oe}}$. All these PMUs are lumped together in the vector $\mathbf{z}_e^{PMU} \in \mathbb{R}^{M_e^{PMU}}$.

The total set of physical measurements in the power grid, which commonly contains random errors, composes the vector $\mathbf{z} = [\mathbf{z}_e^{SCADA} \ \mathbf{z}_e^{PMU}]^T \in \mathbb{R}^M$. The mathematical representation of all these physical measurements is through the nonlinear model $\mathbf{z} = \mathbf{h}(\mathbf{x}_{e-cb}) + \boldsymbol{\varepsilon}$, where $\mathbf{h}(\mathbf{x}_{e-cb}): \mathbb{R}^{2(N_e + N_{oe} + N_{\mathcal{T}_{cb}})} \rightarrow \mathbb{R}^M$ is a vector of nonlinear functions relating system states \mathbf{x}_{e-cb} to the values of physical measurements \mathbf{z} . Lastly, $\boldsymbol{\varepsilon} \in \mathbb{R}^M$ is the vector of uncorrelated measurements' errors.

B. Modeling of Breakers with Known Statuses

The subsets of closed and open CBs are defined by $\mathcal{T}_{ccb} := \{1, 2, \dots, N_{\mathcal{T}_{ccb}}\} \ni \mathcal{T}_{ccb} \subset \mathcal{T}_{cb}$ and $\mathcal{T}_{ocb} := \{1, 2, \dots, N_{\mathcal{T}_{ocb}}\} \ni \mathcal{T}_{ocb} \subset \mathcal{T}_{cb}$, respectively. Furthermore, the breakers with closed and open operating statuses are connected to the node's subset $\mathcal{N}_{ccb} := \{1, 2, \dots, N_{\mathcal{N}_{ccb}}\} \ni \mathcal{N}_{ccb} \subseteq \mathcal{N}_{tsb}$ and $\mathcal{N}_{ocb} := \{1, 2, \dots, N_{\mathcal{N}_{ocb}}\} \ni \mathcal{N}_{ocb} \subseteq \mathcal{N}_{tsb}$, respectively.

If the i -th closed breaker is connected between nodes k and m , the voltage drop across its terminals must be null such that the breaker's operating conditions are mathematically defined by [2]:

$$\text{OC}_{ccb}(\mathbf{x}_{e-cb})_i = \begin{bmatrix} \theta_k - \theta_m \\ V_k - V_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \forall \{k, m\} \in \mathcal{N}_{ccb}, \quad \forall i \in \mathcal{T}_{ccb}. \quad (2)$$

On the other hand, the operating conditions for the i -th open circuit breaker connecting nodes k and m are given by the zero-power flow through it [2]:

$$\text{OC}_{ocb}(\mathbf{x}_{e-cb})_i = \begin{bmatrix} P_{km} \\ Q_{km} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \forall \{k, m\} \in \mathcal{N}_{ocb}, \quad \forall i \in \mathcal{T}_{ocb}. \quad (3)$$

C. Modeling of Circuit Breakers with Unknown Statuses

Based on the operating conditions of CBs (2) and (3), a new constraint is proposed to model the unknown statuses of CBs composing the subset $\mathcal{T}_{ucb} := \{1, 2, \dots, N_{\mathcal{T}_{ucb}}\} \ni \mathcal{T}_{ucb} \subset \mathcal{T}_{cb}$. These unknown CBs are interconnected through the subset of nodes $\mathcal{N}_{ucb} := \{1, 2, \dots, N_{\mathcal{N}_{ucb}}\} \ni \mathcal{N}_{ucb} \subseteq \mathcal{N}_{tsb}$.

One way to model the operational constraints for switching devices with unknown statuses in the GSE problem consists of using the two constraints (4) proposed in [11], which are theoretically satisfied because the open or closed statuses are mutually exclusive:

$$\begin{aligned} P_{km,i}(\theta_k - \theta_m)_i &= 0 \\ Q_{km,i}(V_k - V_m)_i &= 0 \end{aligned}, \quad \forall \{k, m\} \in \mathcal{N}_{ucb}, \quad \forall i \in \mathcal{T}_{ucb}. \quad (4)$$

When the decoupled set of constraints (4) are enforced to determine the unknown statuses of several breakers in substations with a particular type of topological configuration, e.g., open CBs having loops with closed CBs, the estimations of power flows $\{P_{km}\}_{\{k,m\} \in \mathcal{N}_{sc}}$ or $\{Q_{km}\}_{\{k,m\} \in \mathcal{N}_{sc}}$ could erroneously represent the actual operating conditions of unknown circuit breakers. By way of example, the estimated value of active or reactive power flow through open breakers can be greater than 0. In this case, one constraint would indicate that the CB is open, while the other would indicate the opposite.

A novel and different way of representing the CBs with unknown statuses is devised in this paper through the coupling of (2) and (3) by applying the concept of complementarity constraints, which results in one single complementarity constraint that simultaneously considers the two possible breakers' operating statuses for the i -th unknown CB: open or closed. In the former operating condition, both active and reactive power flows are 0, which can be mathematically represented in one single constraint expressed in terms of the apparent power:

$$S(\mathbf{x}_{e-cb})_i = P_{km,i}^2 + Q_{km,i}^2 = 0, \quad \forall \{k, m\} \in \mathcal{N}_{ucb}, \quad \forall i \in \mathcal{T}_{ucb}. \quad (5)$$

On the other hand, the voltage drop and voltage phase angle difference are both 0 when the breaker is closed. These two conditions can be merged in one single constraint expressed in terms of the polar components of both nodal voltages:

$$\Delta \vec{V}(\mathbf{x}_{e-cb})_i = \begin{pmatrix} (V_k - V_m)^2 + \\ (\theta_k - \theta_m)^2 \end{pmatrix} = 0, \quad \forall \{k, m\} \in \mathcal{N}_{ucb}, \quad \forall i \in \mathcal{T}_{ucb}. \quad (6)$$

The two operating conditions (5) and (6) can be expressed as a nonlinear complementarity problem given by (7) [15], where the complement operator \perp indicates that the product of the two functions related by the operator must be 0, i.e., $S(\mathbf{x}_{e-cb})_i \Delta \vec{V}(\mathbf{x}_{e-cb})_i = 0$, while satisfying the conditions $S(\mathbf{x}_{e-cb})_i \geq 0$ and $\Delta \vec{V}(\mathbf{x}_{e-cb})_i \geq 0$. This problem is referred to as a strict complementarity condition when only one of the two functions equals 0: i) $S(\mathbf{x}_{e-cb})_i = 0$ and $\Delta \vec{V}(\mathbf{x}_{e-cb})_i > 0$ or ii) $S(\mathbf{x}_{e-cb})_i > 0$ and $\Delta \vec{V}(\mathbf{x}_{e-cb})_i = 0$ [15]. In this case, the proposed constraint (7) strictly represents the possible open or closed operating condition associated with an unknown status of a circuit breaker, and its inclusion in the GSE problem will force the estimation to converge to either of the two excluding statuses:

$$0 \leq S(\mathbf{x}_{e-cb})_i \perp \Delta \vec{V}(\mathbf{x}_{e-cb})_i \geq 0, \quad \forall i \in \mathcal{I}_{ucb}. \quad (7)$$

In some topology configurations, both the apparent power and the nodal voltage drop can be nearly 0 in an open CB that has been considered as having an unknown status, as will be shown in the first study case in the IEEE 24-bus system. Under this circumstance, (7) corresponds to a non-strict complementarity condition, i.e., $S(\mathbf{x}_{e-cb})_i \approx 0$ and $\Delta \vec{V}(\mathbf{x}_{e-cb})_i \approx 0$, such that it would not be possible to determine the correct CB's status. This problem of determining the CB's unknown status with estimations that do not satisfy the strict complementarity condition is overcome by proposing a comparative test based on the estimated power flows, which is detailed in Section III.A.

Lastly, one possible method for including (7) in the state and topology co-optimization problem consist of transforming (7) into an equality constraint by using the Fischer-Burmeister merit function [16]:

$$\text{OC}_{ucb}(\mathbf{x}_{e-cb})_i = \left(\sqrt{\left(S(\mathbf{x}_{e-cb})_i \right)^2 + \left(\Delta \vec{V}(\mathbf{x}_{e-cb})_i \right)^2 + \varepsilon} - \left(S(\mathbf{x}_{e-cb})_i + \Delta \vec{V}(\mathbf{x}_{e-cb})_i \right) \right) = 0, \quad \forall i \in \mathcal{I}_{ucb} \quad (8)$$

where a small number $\varepsilon = 10^{-12}$ is used to avoid a non-differentiable problem of (8) when the estimated state variables associated with the i -th unknown circuit breaker do not satisfy the strict complementarity condition.

III. COMPLEMENTARITY-BASED GENERALIZED STATE ESTIMATION

To estimate the unknown operating status of CBs, the GSE is performed by solving the proposed optimization problem (9). For the purpose of generality, the three possible breaker statuses are explicitly considered in the formulation:

$$\begin{aligned} \arg \min_{\mathbf{x}_{e-cb}, \mathbf{r}} \quad & J(\mathbf{r}) = \frac{1}{2} \mathbf{r}^T \mathbf{W} \mathbf{r} \\ \text{subject to} \quad & \mathbf{SC}(\mathbf{x}_{e-cb}) = \mathbf{0} \\ & \mathbf{r} - \mathbf{z} + \hat{\mathbf{z}}(\mathbf{x}_{e-cb}) = \mathbf{0} \quad . \\ & \mathbf{OC}_{ccb}(\mathbf{x}_{e-cb}) = \mathbf{0} \\ & \mathbf{OC}_{ocb}(\mathbf{x}_{e-cb}) = \mathbf{0} \\ & \mathbf{OC}_{ucb}(\mathbf{x}_{e-cb}) = \mathbf{0} \end{aligned} \quad (9)$$

In this case, $J(\mathbf{r}): \mathbb{R}^M \rightarrow \mathbb{R}$ is the objective function derived from the WLS approach, and $\mathbf{r} \in \mathbb{R}^M$ is the vector of measurements' residuals, which are considered explicit decision variables together with the components of vector \mathbf{x}_{e-cb} . The diagonal matrix $\mathbf{W} = \text{diag}\{1/\sigma_i^2\}_{i=1}^M$ weighs vector \mathbf{r} , where $\{\sigma_i^2\}_{i=1}^M$ is the i -th standard deviation that reflects the i -th meter's level of accuracy. Furthermore, $\mathbf{SC}(\mathbf{x}_{e-cb})$ is associated with the set of structural constraints associated with the zero injection measurements at the transitions' buses located in the transmission system and substations with detailed modeling. The operating constraints $\mathbf{OC}_{ccb}(\mathbf{x}_{e-cb})$, $\mathbf{OC}_{ocb}(\mathbf{x}_{e-cb})$ and $\mathbf{OC}_{ucb}(\mathbf{x}_{e-cb})$ correspond to the closed, open and unknown statuses of circuit breakers, respectively. Note that as previously indicated the first two types of statuses are known.

The optimization problem given by (9) can be solved by employing commercially available software to obtain estimations of nodal voltages of the entire system and power flows of all CBs at substations considered with an NB model. For this paper, the equality-constrained WLS problem (9) is solved by using Hachtel's augmented matrix method [17], which is implemented in a homemade program coded in MATLAB® [18].

A. Status Identification of CBs Satisfying a Non-strict Complementarity Condition

Estimating the correct operating status of a breaker flagged with an unknown status will only converge to a fully open or fully closed state if its estimated state variables satisfy the strict complementarity condition. If not, the following comparative test is proposed to ensure the correct identification of unknown statuses of breakers from the estimated power flows. After solving the optimization problem (9), the absolute values of estimated active and reactive power flows through those breakers are compared with the values of three times the active and reactive power flow measurements' standard deviations: $3\sigma_{P_{km}}^{SCADA}$ and $3\sigma_{Q_{km}}^{SCADA}$, respectively. If the absolute values of the estimated power flows are less than the value to which they are compared, the circuit breaker is identified as open; otherwise, the circuit breaker is specified as closed.

This proposed comparison is built on the fact that errors of measurements associated with P_{km} and Q_{km} follow a

normal distribution with a mean value equal to 0 and standard deviations of $\sigma_{P_{km}}^{SCADA}$ and $\sigma_{Q_{km}}^{SCADA}$, respectively, [1]. In this case, 99.73% of random values of these errors are clustered within the range of $\pm 3\sigma_{P_{km}}^{SCADA}$ and $\pm 3\sigma_{Q_{km}}^{SCADA}$, respectively, so it is acceptable to consider $3\sigma_{P_{km}}^{SCADA}$ (resp. $3\sigma_{Q_{km}}^{SCADA}$) as the limit value that could have the estimated active (resp. reactive) power flowing through an open circuit breaker.

IV. CASE STUDIES

The proposed methodology's performance is evaluated by using the IEEE 24-bus benchmark system [19], [20] shown in Figure 1. The substations at buses 14 and 16 are modeled at the physical level. The former is composed of a ring configuration, while the substation at node 16 has a breaker-and-a-half configuration, respectively. The total number of circuit breakers for both substations equals 13, and their current operating statuses are the ones shown in Figure 1. The proposal's effectiveness in estimating the unknown statuses of CBs is validated by comparing its results with those obtained by the similar class of method described in [11], which has been coded in our homemade MATLAB program where our proposal is implemented. Lastly, the GSE study must converge to a tolerance of 1×10^{-4} by considering a flat start for all nodal voltages and null initial values for CBs' power flows to be estimated.

Even though the proposal is entirely general for considering measurements provided by a SCADA system and PMUs, from a state estimation viewpoint the power systems are predominantly metered by conventional SCADA measurements. Hence, without a loss of generality, the GSE is performed based on a set of 108 SCADA measurements consisting of 37 measurements of active and reactive power flows, and 17 measurements of active and reactive power injections. These measurements, which are located as shown in Figure 1 [21], are generated by adding Gaussian random noise with zero mean to the set of true measurements obtained from a power flow study that explicitly considers substations models. For this purpose, a power flow program with node-breaker representations was developed based on the approach reported in [22]. The power flow solutions associated with the case studies reported in the following sections are also used to validate the proposed approach. Lastly, the standard deviation values used to represent the random noise are considered as follows [23]: $\sigma_{P_{km}}^{SCADA} = \sigma_{Q_{km}}^{SCADA} = \sigma_{P_k}^{SCADA} = \sigma_{Q_k}^{SCADA} = 0.03 \text{ p.u.}$. These standard deviations are also considered for the methodology reported in [11].

A. Case 1

In this case study, all circuit breakers in the substations of Figure 1 are flagged as unknown, and measurements have no gross errors. Note that in this case, it is not possible to know in advance the existence of short-circuited open breakers, which prevents the strict complementarity condition from being met. The estimated powers flowing through the

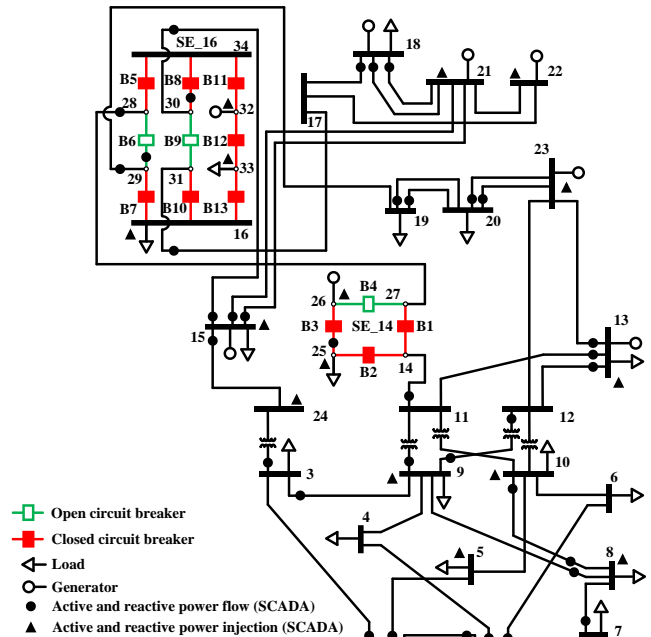


Figure 1. Measurement diagram of the IEEE 24-bus power system with substations 14 and 16 modeled at the section level.

unknown breakers, which were obtained with the proposed approach and the methodology detailed in [11], are reported in Table I. In addition, the true values of active (MWs) and reactive (MVARs) powers flowing through these breakers, obtained with a node-breaker power flow program (NB-based PFP), are also reported in this table. These results show that the operating statuses of all circuit breakers are correctly identified and correspond to the operating conditions shown in Figure 1, even when the strict complementarity condition is not satisfied in B4, B6 and B9. Within this context, from Figure 1 it is clear that the operating constraint associated with the CB B9 connecting nodes 30 and 31 does not satisfy the strict complementarity condition: the apparent power flow and the voltage drop across the CB are 0. In this case, nodes 31 and 30 have the same voltages because CB B9 forms a loop with the closed breakers B8, B11, B12, B13 and B10. A similar reasoning applies to the open CBs B6 and B4. The proposed comparison of a CB's power flows with respect to three times their corresponding standard deviations, however, permits correctly estimating the operating status for the unknown status of these circuit breakers.

On the other hand, the results also show that the proposal detailed in [11] would lead to incorrect estimated statuses of the unknown CBs if the proposed comparative test of estimated power flows and standard deviations is not applied. For example, the results of [11] indicate that breakers B4 and B6 are closed when they are actually open. On the other hand, there is an ambiguity in the estimated statuses of CB B9.

Lastly, it could be argued that estimating the unknown status of a CB forming a loop with closed CBs is irrelevant, but note that the statuses of these last CBs can be flagged as unknown, as in this case study. Therefore, the existence of

Table I. RESULTS OF CBS' STATUS ESTIMATION, CASE 1

CBs	Decoupled Constraints [11]		Complementarity Proposed Approach		True Values (NB-PFP)	
	\hat{P}_{km}	\hat{Q}_{km}	\hat{P}_{km}	\hat{Q}_{km}	P_{km}	Q_{km}
CB1 (1)	-370	7	-370	9	-368	7
CB2 (1)	198	37	198	36	194	39
CB3 (1)	3	19	3	19	0	20
CB4 (0)	2	1	2	2	0	0
CB5 (1)	-374	-39	-375	-39	-375	-38
CB6 (0)	-1	1	-1	1	0	0
CB7 (1)	131	-45	129	-45	127	-47
CB8 (1)	119	-24	120	-23	122	-25
CB9 (0)	2	0	0	0	0	0
CB10 (1)	-325	-37	-323	-35	-325	-37
CB11 (1)	255	63	255	62	253	64
CB12 (1)	-99	-60	-98	-59	-98	-63
CB13 (1)	143	68	143	66	148	73

this type of topological configuration of the substation is also unknown and can only be determined by estimating the breakers' statuses where the strict complementarity condition is not satisfied.

B. Case 2

Measurements with gross errors will lead to incorrect estimations such that a bad data analysis must be performed to overcome this problem. Within this context, the proposed approach for modeling CBs with unknown statuses is general, and its formulation is not constrained to how measurements with gross errors are handled during the state estimation process. Even though the purpose of this paper is not to conduct a rigorous analysis of bad data, Case 1 is newly analyzed, but considering a scenario in which two active power flow measurements are corrupted. In this case, measurements P_{28-27} and P_{29-19} are modified by changing their power flow direction. The GSE problem is solved with the Hatchel-based proposed approach that includes the bad data analysis reported in [24] to detect, identify and correct the measurements with gross errors during the estimation process. The results presented in Table II demonstrate that the proposal can correctly perform the bad data analysis and identify the breakers' statuses.

C. Case 3

The system presented in Figure 1 is modified by opening the CB B12, causing a split-bus configuration because the disconnection of nodes 16 and 34 splits the substation into two electrically disconnected parts. This case study also considers that all CBs have unknown statuses. The estimations of active and reactive powers flowing through CBs are reported in Table III for the proposed approach and the GSE reported in [11]. Note that the estimations of several CBs' operating statuses obtained from the latter formulation significantly differ from their true values of active and reactive powers obtained from the NB-based PFP, which are also reported in Table III.

Lastly, the proposed approach is run 200 times using different random Gaussian measurement errors. The number of correct identifications (NCI) of unknown CB states is

Table II. RESULTS OF CBS' STATUS ESTIMATION, CASE 2

CBs	Complementarity Proposed Approach		True Values (NB-PFP)	
	\hat{P}_{km}	\hat{Q}_{km}	P_{km}	Q_{km}
CB1 (1)	-371	5	-368	7
CB2 (1)	199	37	194	39
CB3 (1)	1	23	0	20
CB4 (0)	3	-1	0	0
CB5 (1)	-375	-34	-375	-38
CB6 (0)	2	-2	0	0
CB7 (1)	-128	-44	127	-47
CB8 (1)	121	-28	122	-25
CB9 (0)	0	2	0	0
CB10 (1)	-327	-40	-325	-37
CB11 (1)	254	62	253	64
CB12 (1)	-97	-60	-98	-63
CB13 (1)	150	71	148	73

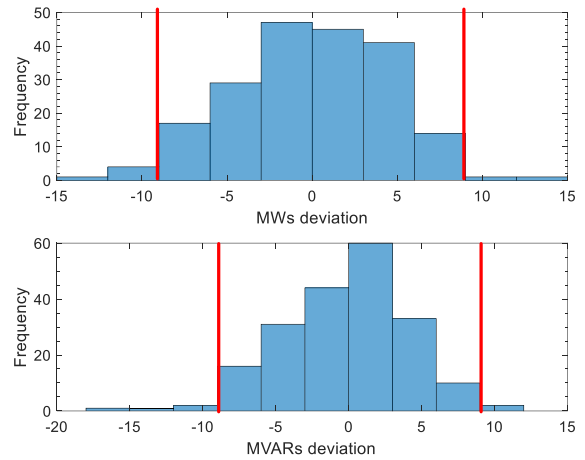


Figure 2. Histograms for the estimations of active and reactive power flows through CB B12.

reported in the fourth column of Table III. These results clearly show that the proposal correctly estimated the operating states of all the breakers, except for CBs B4 and B12, in all the study cases. In this context, the proposal's worst performance corresponds to the estimations of CB B12, which failed in 13 case studies. As indicated by the red lines in the histograms shown in Figure 2, which are obtained from the entire set of estimations for the CB B12, the estimated active power is outside the $\pm 3\sigma_{P_{km}}^{SCADA}$ range 7 times. In contrast, the estimated reactive power is outside the $\pm 3\sigma_{Q_{km}}^{SCADA}$ range 6 times.

V. CONCLUSIONS AND FUTURE WORK

A static estimator that simultaneously co-estimates the system's operating state and breakers' unknown statuses is proposed in this paper. The novelty of this proposal is as follows. Firstly, the two possible operating statuses of an unknown CB, i.e., closed or open, are naturally modeled by one single operating constraint derived from the complementarity constraint concept. Secondly, this operating constraint is directly included in the static state estimation

Table III. RESULTS OF CBS' STATUS ESTIMATION, CASE 3

CBs	Decoupled Constraints [11]		Complementarity Proposed Approach			True Values (NB-PFP)	
	\hat{P}_{km}	\hat{Q}_{km}	NCI	\hat{P}_{km}	\hat{Q}_{km}	P_{km}	Q_{km}
CB1 (1)	-427	-2	200	-340	13	-340	13
CB2 (1)	196	40	200	196	40	194	39
CB3 (1)	-1	21	200	1	21	0	20
CB4 (0)	-2	2	189	1	3	0	0
CB5 (1)	-437	-54	200	-347	-28	-346	-25
CB6 (0)	1	3	200	1	3	0	0
CB7 (1)	222	-20	200	160	-17	160	-16
CB8 (1)	197	30	200	195	27	191	24
CB9 (0)	-9	-4	200	-6	0	0	0
CB10 (1)	-398	-22	200	-250	-4	-260	-4
CB11 (1)	240	24	200	153	1	155	0
CB12 (0)	-81	-27	187	5	-5	0	0
CB13 (1)	129	33	200	44	11	50	10

problem as an equality constraint, obtained using the Fischer-Burmeister merit function. Finally, the unknown statuses of breakers are directly estimated together with the nodal voltages of the entire network in a unified framework of analysis. The results numerically demonstrate that the proposed approach can satisfactorily estimate the breakers' statuses considering a given measurement redundancy, even though all circuit breakers at power substations are initially regarded as unknown and loops with closed CBs exist.

Computational time and convergence characteristics are paramount for the state estimation process. The proposed methodology was programmed in MATLAB and executed on an intel® Xeon® E3-1505M, 3 GHz computer with 40GB of RAM. The number of iterations and CPU time required during the GSE process for case 1 (resp. case 2) are 7 and 0.54 sec (resp. 11 and 0.56 sec), respectively, considering a convergence tolerance of 1×10^{-4} . As expected, the case study with gross errors took the longest time during the estimation process. Lastly, the GSE approach reported in this paper is general, so assuming known statuses in doubtful circuit breakers is not required. Furthermore, the proposed model for representing unknown CBs can be directly implemented in constraint-based generalized state estimators.

Since the known statuses of circuit breakers are assumed to be correct, the topological error analysis is not required. The system operator, however, could incorrectly assume that an open breaker is closed or vice versa, yielding a topology error. Under this circumstance, a post-estimation process is necessary to identify and correct the erroneous assumed CB's operating status. Considering that the contribution of this proposal focuses on determining the operating state of CBs with an unknown status, the topic of topology error processing is out of the scope of the paper and will be analyzed in a forthcoming paper.

Lastly, the level of redundancy required in a substation to ensure the proposed method's success must render the substation's state variables observable such that the state estimation is solvable. Based on the authors' own experience with the IEEE-24 bus system reported in Section IV, a minimum redundancy level of 0.529 is required for

estimating the state vector of nodal voltages and circuit breaker power flows within the breaker-and-a-half substation array. On the other hand, a minimum redundancy level of 0.375 is necessary for the ring substation array. In this vein, identifying the measurement redundancy level to make the system observable and the state estimation robust against gross errors in the measurements could be formulated based on numerical approaches by extending the network observability analysis reported in [25] to [27], which will be addressed in our future work.

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