Multistage Day-Ahead Scheduling of Energy and Reserves

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***Abstract*—This paper presents a new economic dispatch model for the day-ahead scheduling of energy and reserves under uncertainty. The proposed model considers a multistage stochastic framework whereby scheduling decisions are dynamically updated according to observed new information about uncertain parameters such as nodal net injections. As a result, the proposed model provides a one-hour-ahead implementable energy and reserve schedule. Moreover, a particularization of this multistage model is presented, namely a two-stage model, in which a single generation and reserve schedule is provided for each hour of the day. A recently developed regularized linear decision rules framework is used to reduce the computational complexity of the multistage stochastic linear problem at hand and to prevent the in-sample overfitting issue and the threat of poor out-of-sample performance. Numerical simulations based on the IEEE 300-bus system demonstrate the effectiveness of the proposed approach, as well as its economic and operational advantages over the widely used two-stage model.**

***Index Terms*—Linear decision rules, multistage day-ahead economic dispatch, regularization, uncertainty.**

Nomenclature

Sets and Indices

$\mathcal{B}$	Set of bus indices b.
b_j^+	Origin bus index of line j.
b_j^-	Destination bus index of line j.
$\mathcal{G}$	Set of generating unit indices i.
$\mathcal{L}$	Set of indices l of lags considered for the realization of uncertainties.
L_b^+, L_b^-	Sets of transmission line indices j with origin and destination bus b.
N_Ω	Number of scenario indices ω.
Ω	Set of scenario indices ω.
$\mathcal{T}$	Set of time period indices t.
U_b	Set of indices i of generators located at bus b.

Parameters

c_b^δ	Cost coefficient for the load shedding at bus b.
c_i^{dn}, c_i^{up}	Down- and up-spinning reserve cost rates offered by generator i.
c_i^g	Generation cost coefficient of unit i.
c_b^γ	Cost coefficient for the generation curtailment at bus b.
$d_{t,\omega,b}, \hat{d}_{t,\omega,b}$	Observed and forecasted power demands at bus b in period t and scenario ω.
$\varepsilon_{t,\omega,b}$	Stochastic error term of the autoregressive model of order 1 for bus b.
F_j	Capacity of line j.
$\phi_{0,b}, \phi_{1,b}$	Parameters of the autoregressive model of order 1 for bus b.
$\bar{G}_b$	Maximum generation at bus b.
$\underline{G}_i, \bar{G}_i$	Lower and upper generation limits of unit i.
λ	Regularization coefficient.
p_ω	Probability of occurrence of scenario ω.
R_i^{dn}, R_i^{up}	Maximum down- and up-spinning reserve contributions of generator i.
RD_i, RU_i	Ramp-down and ramp-up limits of generator i.
x_j	Reactance of line j.

Decision Variables

$\beta_{t,i,0}^{(dn)}, \beta_{t,i,b,l}^{(dn)}$	Down-spinning reserve LDR coefficients of unit i in period t.
$\beta_{t,i,0}^{(g)}, \beta_{t,i,b,l}^{(g)}$	Generation LDR coefficients of unit i in period t.
$\beta_{t,i,0}^{(up)}, \beta_{t,i,b,l}^{(up)}$	Up-spinning reserve LDR coefficients of unit i in period t.

Submitted to the 23rd Power Systems Computation Conference (PSCC 2024).

The work of M. Rodrigues was funded in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

The work of J. M. Arroyo was supported in part by grants PID2021-126566OB-I00 and PID2021-122579OB-I00, funded by the Spanish Ministry of Science and Innovation MCIN/AEI/10.13039/501100011033, and by ERDF, EU "A way of making Europe", by grant SBPLY/21/180501/000154, funded by the Junta de Comunidades de Castilla-La Mancha, by the Spanish Ministry of Finance and Civil Service, by European Union Funds, and by the ERDF, and by grant 2022-GRIN-34074, funded by the Universidad de Castilla-La Mancha, under the UCLM Research Group Program, and by the European Commission, under the ERDF.

$\delta_{t,\omega,b}, \delta^{RT}_{t,\omega,b}$	Load shedding at bus b in period t and scenario ω in the forecasted and real-time operation.
$\Delta_{t,\omega,i}$	Redispatch of generator i in period t and scenario ω.
$f_{t,\omega,j}, f^{RT}_{t,\omega,j}$	Power flows on line j in period t and scenario ω in the forecasted and real-time operation.
$\Phi^{(dn)}_{t,i,b,l}$	Down-spinning reserve regularization term for generator i, period t, bus b, and lag l.
$\Phi^{(g)}_{t,i,b,l}$	Generation regularization term for generator i, period t, bus b, and lag l.
$\Phi^{(up)}_{t,i,b,l}$	Up-spinning reserve regularization term for generator i, period t, bus b, and lag l.
$g_{t,\omega,i}$	Generation of unit i in period t and scenario ω.
$\gamma_{t,\omega,b}, \gamma^{RT}_{t,\omega,b}$	Generation curtailment at bus b in period t and scenario ω in the forecasted and real-time operation.
$r^{dn}_{t,\omega,i}, r^{up}_{t,\omega,i}$	Down- and up-spinning reserve contributions of generator i in period t and scenario ω.
$\theta_{t,\omega,b}, \theta^{RT}_{t,\omega,b}$	Phase angles at bus b in period t and scenario ω in the forecasted and real-time operation.

I. Introduction

The economic dispatch (ED) is one of the main tools used by system operators to ex-ante schedule available generation resources. The goal of this optimization problem is to determine the least-cost production of online generators so that the future demand is met over a specific short-term time horizon [1]. In addition to complying with generation- and network-related operational constraints, security is also ensured by co-optimizing energy and reserves [2], [3].

The ED problem constitutes a decision-making process under uncertainty in which the decision for a given time period affects the optimal decisions for the following periods. Despite its dynamic nature, the ED problem is widely addressed via a two-stage optimization framework that yields a single generation and reserve schedule for the entire day ahead [3].

In real time, however, actual generation and load typically deviate from what was scheduled in the previous day [4]. In addition, the presently large-scale integration of renewable energy resources has led to a significant increase in the intermittency of the available generation capacity, causing large fluctuations in nodal net injections [5], [6]. Thus, intraday markets become essential to complement and adjust the initial schedule and enable market parties to minimize deviations between schedules and final energy injections [3]. Moreover, since the currently used two-stage model lacks flexibility with respect to uncertainties, it may give rise to non-implementable day-ahead dispatch decisions and over-scheduling of reserves to guarantee that nodal demands are fully met.

The uncertainty in nodal net injections leads naturally to considering ED models based on stochastic programming [5]. The solution to a stochastic program yields a policy, which corresponds to a rule that specifies the decisions, based on the information available at the current stage, for any possible realization of the uncertainties present in the system [7].

This paper addresses the benefits of adopting a particular class of stochastic programming, namely multistage stochastic programming, for the day-ahead scheduling of energy and reserves under uncertainty. Unlike the widely used, albeit simpler, two-stage approaches, the decisions made at a given time stage depend on the uncertainty realizations at the previous stage while being unique for all possible future realizations so that non-anticipativity is guaranteed. Thus, generation schedules are dynamically updated according to observed new information about uncertainty, thereby outperforming existing ED approaches in terms of flexibility and adaptivity.

The predominant literature exploring multistage stochastic models for practical decision making under uncertainty, like the ED problem, customarily focuses on solution methods such as sample average approximation (SAA) techniques, the construction of scenario trees, and stochastic dual dynamic programming (SDDP). However, due to the unavailability, in general, of the probability distribution of the uncertain parameters [8] and the curse of dimensionality featured by existing methods, drastic simplifying assumptions are typically made in order to achieve tractability. An alternative to deal with the complexity of these problems is the linear decision rules (LDR) technique [9]–[11], whose first application dates back to the 50s [12].

The main idea of the LDR method is to restrict the functional form of the policy by requiring that all decisions made at each stage be a linear (or affine) function of the uncertainty realizations up to the current stage. Hence, this technique considers a sequential process in which decisions made at a particular time solely depend on the history of uncertainty realizations observed so far but not on future realizations [8]. As mentioned in [13], any convenient linear functional space of interest mapping the set of uncertainty vectors onto the set of states can be considered, and the problem linearity is preserved.

It is important to highlight, as shown in [11], that using an LDR approach allows the formulation of multistage stochastic linear problems as two-stage stochastic linear models with multiple periods. Thus, all properties, convergence results, methods, and algorithms for two-stage stochastic models are valid. Regarding its flexibility, it is possible to address multistage stochastic linear problems based on uncertainty processes with any nonlinear time-dependency structure through scenarios generated in an entirely exogenous fashion.

In the literature, different approaches have been proposed to handle the ED problem under the uncertainties related to renewable generation and demand [14]–[22]. In [14] and [15], two-stage stochastic ED models are proposed to manage the variability and uncertainty associated with wind power generation. In [16], a two-stage stochastic programming approach is adopted to solve a multiperiod network-constrained ED model with flexible demands. Recently, a two-stage stochastic ED model was addressed in [17] by a hybrid affine decision rule. Unfortunately, the two-stage stochastic optimization framework applied in [14]–[17] relies on simplifying assumptions that may not fully capture the complexity of the ED problem, thus leading to potentially suboptimal solutions.

In [18], a hybrid robust-stochastic approach is proposed whereby stochastic scenarios are replaced with probability-weighted flexibility envelopes. This approach, however, can be considered a statistical scenario-reduction process wherein each envelope encloses a corresponding percentage of scenarios. Thus, a sufficiently large number of scenarios is required to properly capture uncertainty over time. In [19], a stochastic programming model is fed with scenarios drawn directly from high-fidelity data sets, but an efficient scenario-selection method is not prescribed.

In [20], the authors propose a method to train policies for energy and reserve scheduling that can be employed in real-time operation. However, such a method relies on an offline analysis. In [21], the ED problem arising in active distribution networks is addressed by an affinely adjustable robust optimization approach, which may lead to over-conservative solutions.

In [22], a multistage stochastic programming model is proposed for the joint energy and reserve dispatch under uncertain renewable generation. The proposed framework characterizes the uncertainties over the multiple stages by a scenario tree and uses SDDP to solve the model. It is worth noting that, from a modeling perspective, the very restrictive hypotheses of SDDP constitute a significant drawback of this framework.

Motivated by the findings of [9]–[12] and the limitations of existing ED approaches [14]–[22], we propose an LDR-based multistage stochastic model for the ED problem, which provides a one-hour-ahead implementable energy and reserve schedule. More specifically, we use the two-stage LDR approach recently presented in [11]. Due to the large number of LDR coefficients that must be estimated, in-sample overfit may arise, i.e., perfectly adjusting the model to a particular set of scenarios does not guarantee an adequate performance under unseen scenarios. Hence, to address the issue of poor out-of-sample performance, we propose the application of the novel regularization method described in [13].

The main contributions of this paper are twofold:

1) A more flexible and adaptive optimization model for the ED problem under uncertainty that acknowledges the dynamic nature of the joint scheduling of energy and reserves. The proposed model is formulated in a multistage setting, thereby allowing the revision of decisions at each time stage based on the uncertainty realized so far.
2) The application of the regularized LDR method that reduces the computational burden to solve multistage models while mitigating the well-known overfitting issue and thus improving the out-of-sample performance.

To the best of our knowledge, the proposed application of the novel regularized two-stage LDR approach to the ED problem constitutes the first contribution in the literature addressing the joint scheduling of energy and reserves under a multistage stochastic programming framework without resorting to exogenous reserve requirements.

The remainder of this paper is organized as follows. In Section II, the proposed ED model is formulated. In Section III, the regularization methodology is presented. In Section IV, numerical results are provided and discussed. Finally, conclusions are drawn in Section V.

II. Problem Formulations

This section provides the formulation of the proposed model for the multistage ED problem, cast as a linear program. Subsequently, a description of how such a model allows obtaining the currently used two-stage model is presented.

A. Multistage Model

The main goal of the proposed multistage formulation for the ED problem is to estimate a dynamic policy to determine the least-cost hourly schedule of both energy and reserves for the day ahead. This approach is characterized by the following optimization model:

$$\min_{\substack{(g,r^{up},r^{dn},\Delta)_{t,\omega,i},\\ (f,f^{RT})_{t,\omega,j},\\ (\beta^{(g)},\beta^{(up)},\beta^{(dn)})_{t,i,0},\\ (\beta^{(g)},\beta^{(up)},\beta^{(dn)})_{t,i,b,l}\\ (\delta,\delta^{RT},\gamma,\gamma^{RT},\theta,\theta^{RT})_{t,\omega,b}}} \sum_{\omega\in\Omega} p_\omega \sum_{t\in\mathcal{T}} \Bigg[\sum_{i\in\mathcal{G}} \Big(c_i^g g_{t,\omega,i} + c_i^{up} r_{t,\omega,i}^{up} + c_i^{dn} r_{t,\omega,i}^{dn} \Big) + \sum_{b\in B} \Big(c_b^\delta (\delta_{t,\omega,b} + \delta_{t,\omega,b}^{RT}) + c_b^\gamma (\gamma_{t,\omega,b} + \gamma_{t,\omega,b}^{RT}) \Big) \Bigg] \quad (1)$$

subject to:

$$\sum_{i\in U_b} g_{t,\omega,i} + \sum_{j\in L_b^+} f_{t,\omega,j} - \sum_{j\in L_b^-} f_{t,\omega,j} + \delta_{t,\omega,b} - \gamma_{t,\omega,b} = \hat{d}_{t,\omega,b};\ \forall t,\omega,b \quad (2)$$

$$\sum_{i\in U_b} (g_{t,\omega,i} + \Delta_{t,\omega,i}) + \sum_{j\in L_b^+} f_{t,\omega,j}^{RT} - \sum_{j\in L_b^-} f_{t,\omega,j}^{RT} + \delta_{t,\omega,b}^{RT} - \gamma_{t,\omega,b}^{RT} = d_{t,\omega,b};\ \forall t,\omega,b \quad (3)$$

$$f_{t,\omega,j} = \frac{\theta_{t,\omega,b_j^+} - \theta_{t,\omega,b_j^-}}{x_j};\ \forall t,\omega,j \quad (4)$$

$$f_{t,\omega,j}^{RT} = \frac{\theta_{t,\omega,b_j^+}^{RT} - \theta_{t,\omega,b_j^-}^{RT}}{x_j};\ \forall t,\omega,j \quad (5)$$

$$-F_j \le f_{t,\omega,j} \le F_j,\ \forall t,\omega,j \quad (6)$$

$$-F_j \le f_{t,\omega,j}^{RT} \le F_j;\ \forall t,\omega,j \quad (7)$$

$$\underline{G}_i \le g_{t,\omega,i} \le \bar{G}_i;\ \forall t,\omega,i \quad (8)$$

$$g_{t,\omega,i} + r_{t,\omega,i}^{up} \le \bar{G}_i;\ \forall t,\omega,i \quad (9)$$

$$g_{t,\omega,i} - r_{t,\omega,i}^{dn} \ge \underline{G}_i;\ \forall t,\omega,i \quad (10)$$

$$-RD_i \le g_{t,\omega,i} + \Delta_{t,\omega,i} - g_{t-1,\omega,i} - \Delta_{t-1,\omega,i} \le RU_i;\ \forall t,\omega,i \quad (11)$$

$$-r_{t,\omega,i}^{dn} \le \Delta_{t,\omega,i} \le r_{t,\omega,i}^{up};\ \forall t,\omega,i \quad (12)$$

$$0 \le r_{t,\omega,i}^{up} \le R_i^{up};\ \forall t,\omega,i \quad (13)$$

$$0 \le r_{t,\omega,i}^{dn} \le R_i^{dn};\ \forall t,\omega,i \quad (14)$$

$$0 \le \delta_{t,\omega,b} \le \hat{d}_{t,\omega,b};\ \forall t,\omega,b \quad (15)$$

$$0 \le \delta_{t,\omega,b}^{RT} \le d_{t,\omega,b};\ \forall t,\omega,b \quad (16)$$

$$0 \leq \gamma_{t,\omega,b} \leq \bar{G}_b; \ \forall t, \omega, b \quad (17)$$

$$0 \leq \gamma^{RT}_{t,\omega,b} \leq \bar{G}_b; \ \forall t, \omega, b \quad (18)$$

$$g_{t,\omega,i} = \beta^{(g)}_{t,i,0} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \beta^{(g)}_{t,i,b,l} d_{t-l,\omega,b}; \ \forall t, \omega, i \quad (19)$$

$$r^{up}_{t,\omega,i} = \beta^{(up)}_{t,i,0} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \beta^{(up)}_{t,i,b,l} d_{t-l,\omega,b}; \ \forall t, \omega, i \quad (20)$$

$$r^{dn}_{t,\omega,i} = \beta^{(dn)}_{t,i,0} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \beta^{(dn)}_{t,i,b,l} d_{t-l,\omega,b}; \ \forall t, \omega, i \quad (21)$$

The objective function minimized in (1) comprises the expected costs of power generation and up- and down-spinning reserve allocations, as well as the costs of load shedding and generation curtailment. Using a dc power flow model, constraints (2) and (3) impose the power balance at every bus of the system for both the forecasted and the real-time observed demands, respectively, whereas line flows are characterized by the second Kirchhoff's law in (4) and (5).

Line flows are bounded in (6) and (7) according to the line rated capacities. Constraints (8)–(10) set the production limits. Constraint (11) characterizes the up and down ramp limits, whereas redispatch bounds are modeled in (12). In (13) and (14), up- and down-spinning reserve contributions are respectively limited. Constraints (15) and (16) set the bounds for load-shedding variables, where the upper bounds are equal to the forecasted and real-time observed demands, respectively. Generation curtailment is modeled by non-negative variables that are limited by the corresponding nodal maximum generation, as formulated in (17) and (18).

Finally, constraints (19)–(21) define the LDR expressions relating energy and reserves to real-time demand. Constraint (19) is associated with energy scheduling, while constraints (20) and (21) characterize up- and down-spinning reserve contributions, respectively. Note that with the resulting LDR coefficients, given by the solution to problem (1)–(21), the one-hour-ahead joint schedule of energy and reserves can be dynamically determined for any hour of the next day according to the demand information revealed by a given set of lags.

B. Two-Stage Model

The proposed multistage model can be simplified by keeping the forecasted demand as deterministic and also setting all LDR coefficients $\beta^{(g)}_{t,i,b,l}$, $\beta^{(up)}_{t,i,b,l}$, and $\beta^{(dn)}_{t,i,b,l}$ to zero. In the resulting optimization model, the schedule of energy and reserves is static, i.e., it is the same for all scenarios, and is obtained for each hour of the day ahead without updates as the demand information is revealed.

This particularization of the multistage model yields the two-stage optimization framework representing the state of the art in the technical literature. It should be noted that, as a single generation and reserve schedule is provided for each hour of the day, this formulation lacks flexibility with respect to the demand realizations.

Compared to the two-stage deterministic approach, which corresponds to the current industry practice for solving the ED problem, the standard two-stage stochastic model offers greater flexibility in endogenously defining the optimal reserve allocations and, thereby, the optimal level of reserves. This avoids relying on exogenous reserve requirements and potentially attaining suboptimal reserve allocations. Nevertheless, both two-stage approaches are static as a single generation and reserve scheduling profile is provided.

On the other hand, the proposed multistage approach combines the benefits of dynamically adjusting both reserve and generation schedules one hour ahead with endogenously defined reserve allocations. This change enables more efficient utilization of resources and better adaptation to observed system conditions. Table I summarizes the main features of these models.

TABLE I
COMPARISON OF ED MODELS

Model	Uncertainty awareness	Generation dispatch	Reserve allocation	Usage
Two-stage deterministic	No	Static	Static with exogenous requirements	Current industry practice
Two-stage stochastic	Yes	Static	Static without exogenous requirements	State of the art in the technical literature
Multistage stochastic	Yes	Dynamic	Dynamic without exogenous requirements	Proposed model

III. REGULARIZATION

As is customary in stochastic programming, solution quality for the proposed multistage model may be assessed by a two-step estimation-simulation procedure. In the first step, the LDR coefficients are estimated for a given set of in-sample scenarios. In the second step, the main goal is to find implementable decisions that follow as much as possible the estimated LDR for a large set of out-of-sample scenarios [13].

The definition of a policy for a stochastic program, however, requires the estimation of many coefficients under a limited amount of data, and may perform poorly for unseen scenarios. Moreover, in practical applications, the number of coefficients to be estimated may be greater than the number of scenarios [13]. For instance, in our proposed multistage ED model, a huge number of coefficients is needed to account for each time period, generating unit, bus with variable demand, and lags of observed data in the decision rule. As a result, the adoption of LDR may give rise to a certain degree of in-sample overfitting and, consequently, poor out-of-sample performance.

Hence, to address the overfitting issue and improve the quality of the proposed LDR-based multistage ED model, we apply the AdaLASSO technique [13], in which the objective function is penalized by the scaled l_1-norm of the coefficient vector, disregarding the intercept coefficients $\beta^{(g)}_{t,i,0}, \beta^{(up)}_{t,i,0}$, and $\beta^{(dn)}_{t,i,0}$.

Thus, the regularized multistage model is given by the following optimization problem:

$$\min_{\substack{(g,r^{up},r^{dn},\Delta)_{t,\omega,i},\\ (f,f^{RT})_{t,\omega,j},\\ (\beta^{(g)},\beta^{(up)},\beta^{(dn)})_{t,i,0},\\ (\beta^{(g)},\beta^{(up)},\beta^{(dn)})_{t,i,b,l}\\ (\delta,\delta^{RT},\gamma,\gamma^{RT},\theta,\theta^{RT})_{t,\omega,b},\\ (\Phi^{(g)},\Phi^{(up)},\Phi^{(dn)})_{t,i,b,l}}} \sum_{\omega\in\Omega} p_\omega \sum_{t\in\mathcal{T}} \Bigg[\sum_{i\in\mathcal{G}} \Big(c_i^g g_{t,\omega,i} + c_i^{up} r_{t,\omega,i}^{up} + c_i^{dn} r_{t,\omega,i}^{dn} \Big) + \sum_{b\in B} \Big(c_b^\delta (\delta_{t,\omega,b} + \delta_{t,\omega,b}^{RT}) + c_b^\gamma (\gamma_{t,\omega,b} + \gamma_{t,\omega,b}^{RT}) \Big) \Bigg] + \lambda \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{G}} \sum_{b\in\mathcal{B}} \sum_{l\in\mathcal{L}} (\Phi_{t,i,b,l}^{(g)} + \Phi_{t,i,b,l}^{(up)} + \Phi_{t,i,b,l}^{(dn)}) \quad (22)$$

subject to:

$$\text{Constraints (2)–(21)} \quad (23)$$

$$\Phi_{t,i,b,l}^{(g)} - \beta_{t,i,b,l}^{(g)} \geq 0;\ \forall t,i,b,l \quad (24)$$

$$\Phi_{t,i,b,l}^{(g)} + \beta_{t,i,b,l}^{(g)} \geq 0;\ \forall t,i,b,l \quad (25)$$

$$\Phi_{t,i,b,l}^{(up)} - \beta_{t,i,b,l}^{(up)} \geq 0;\ \forall t,i,b,l \quad (26)$$

$$\Phi_{t,i,b,l}^{(up)} + \beta_{t,i,b,l}^{(up)} \geq 0;\ \forall t,i,b,l \quad (27)$$

$$\Phi_{t,i,b,l}^{(dn)} - \beta_{t,i,b,l}^{(dn)} \geq 0;\ \forall t,i,b,l \quad (28)$$

$$\Phi_{t,i,b,l}^{(dn)} + \beta_{t,i,b,l}^{(dn)} \geq 0;\ \forall t,i,b,l \quad (29)$$

Compared to the non-regularized model (1)–(21), the objective function (22) includes a new regularization term relying on λ, which is a scalar reflecting the overall penalization level, and auxiliary variables $\Phi_{t,i,b,l}^{(g)}$, $\Phi_{t,i,b,l}^{(up)}$, and $\Phi_{t,i,b,l}^{(dn)}$. Moreover, besides the original constraints (2)–(21) included in (23), epigraph constraints (24)–(29) are added, according to [13].

Determining the optimal regularization parameter λ requires a line search procedure running the multistage optimization problem for each point inspected. However, as mentioned in [13], the best λ is, in general, stable across instances. Thus, the calibration process of a given day may use the results from previous days, and monitoring procedures can be implemented to check the validity of the current penalty parameter.

IV. Numerical Results

In order to investigate the benefits of adopting a more flexible and adaptive model for the ED problem, numerical simulations were conducted using the IEEE 300-bus system. This system comprises 300 buses, 69 generators, 411 branches, and 191 loads. The methods described in [23] and [24] were applied to obtain the generation and reserve cost data.

A day-ahead horizon of 24 hourly periods was considered. The results from the regularized multistage model described in Section III have been compared with those attained for the state-of-the-art two-stage model. For quick reference, both formulations are hereinafter denoted by MS and 2S, respectively.

For illustrative purposes, load uncertainty is assumed at the six buses with the largest original demand values, as reported in [23], which corresponds to approximately 20% of the system total load. The other loads were kept constant for the entire horizon and equal to their original demand values.

For the sake of simplicity, for both ED formulations, we consider that the real-time observed demand at the aforementioned buses follows an autoregressive model of order 1, thereby incorporating uncertainty in the load:

$$d_{t,\omega,b} = \phi_{0,b} + \phi_{1,b} d_{t-1,\omega,b} + \varepsilon_{t,\omega,b};\ \forall t,\omega,b \quad (30)$$

On the other hand, the forecasted demand is simulated differently for each dispatch model. For the multistage model, it is assumed that the forecasted demand at a given time t depends on the demand at time $t-1$, thus varying across both time and scenarios:

$$\hat{d}_{t,\omega,b} = \phi_{0,b} + \phi_{1,b} d_{t-1,\omega,b};\ \forall t,\omega,b \quad (31)$$

By contrast, for the two-stage model, the forecasted demand is assumed to be a deterministic value given by the average real-time demand over the scenarios:

$$\hat{d}_{t,\omega,b} = \frac{1}{N_\Omega} \sum_{\omega\in\Omega} d_{t,\omega,b};\ \forall t,\omega,b \quad (32)$$

Hence, the forecasted demand solely varies across time, i.e., all scenarios feature the same demand vector.

For the in-sample and out-of-sample studies, 40 and $2,000$ demand scenarios were considered, respectively. For reproducibility purposes, system data are available at [25]. All simulations were run to optimality utilizing Julia and Gurobi 10.0.1 on an Intel Xeon E5-2680 processor at 2.50 GHz with 125 GB of RAM.

In order to evaluate the effect of the regularization method, we compare the performances of the regularized policies for different values of the regularization parameter λ, including $\lambda = 0$, which yields the non-regularized LDR policy. Fig. 1 shows the resulting in-sample and out-of-sample expected total costs. It is worth noting that if we "super regularize" the model by choosing a large λ, the dynamics of generation and reserves will be disregarded. Hence, since the load scenarios vary over time, it will be necessary to shed load and/or curtail generation, which leads to a great increase in the total costs. As can be also observed in Fig. 1, the lowest out-of-sample cost is found for $\lambda = 1,700$. For such a value of λ, the best regularization improves by 12.97% upon the non-regularized policy identified for $\lambda = 0$.

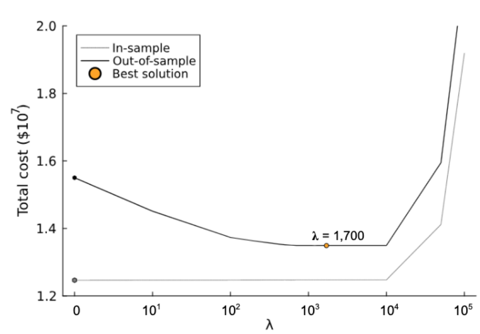

Fig. 1. Expected total costs for the regularized multistage model.

Table II summarizes both the in-sample and out-of-sample cost results for the solutions to the two-stage model and the

TABLE II
COST RESULTS ($)

	In-sample results		Out-of-sample results	
	2S	MS	2S	MS
Generation	12,456,286.17	12,458,924.59	12,456,286.13	12,458,465.37
Up-spinning reserve	27,231.40	14,696.56	27,231.39	14,596.61
Down-spinning reserve	6,449.71	2,857.96	6,449.72	2,857.99
Load shedding	0.00	3.47	993,360.53	267,732.56
Real-time load shedding	0.00	3.30	425,180.26	394,027.00
Generation curtailment	0.00	0.78	1,160,996.19	242,866.62
Real-time generation curtailment	0.00	0.80	257,505.51	110,622.69
Total	12,489,967.28	12,476,487.45	15,327,009.74	13,491,168.84

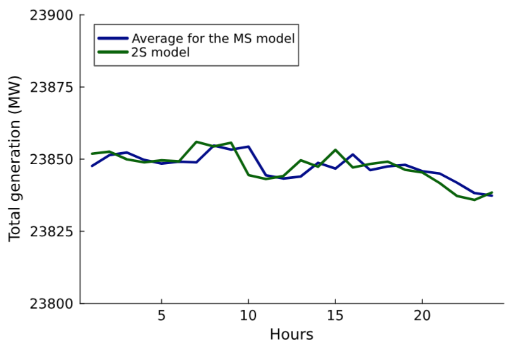

Fig. 2. In-sample total power generation.

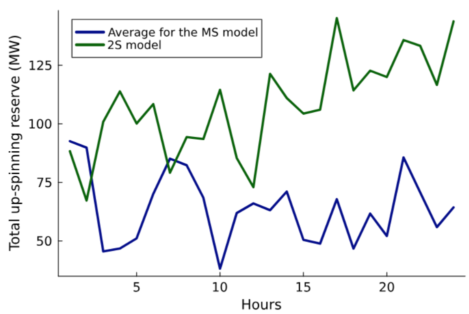

Fig. 3. In-sample total up-spinning reserve allocation.

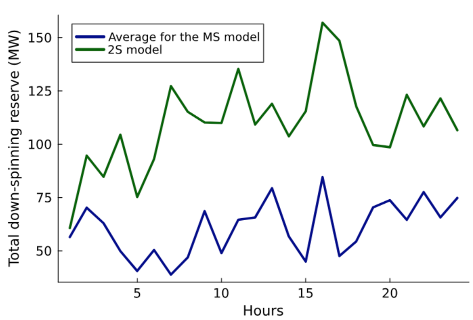

Fig. 4. In-sample total down-spinning reserve allocation.

best instance of the proposed regularized model, i.e., for $\lambda = 1,700$. It can be seen that the solution to the multistage model significantly departs from the two-stage solution due to its adaptive generation profile. As a consequence, the regularized multistage model attains an overall 11.98% improvement upon the two-stage model in the out-of-sample evaluation.

Furthermore, in Figs. 2–4, we compare the in-sample scheduling results featured by the aforementioned solutions, namely the hourly values of total power generation, total up-spinning reserve, and total down-spinning reserve. Note that, for the proposed multistage model, we depict the average results over all scenarios.

Fig. 2 shows that the multistage model yields a generation profile that is, on average, similar to that obtained for the two-stage model. As for the total reserves shown in Figs. 3 and 4, since the multistage approach allows revising the scheduling decisions at each stage, this method allocates only 59% and 55% of the two-stage total up- and down-spinning reserve amounts, respectively. Not only does this result reduce the associated costs, but it also facilitates the system operation since coordinating large reserves across complex grids requires sophisticated control and communication systems.

Regarding the computational effort, solving the two-stage model took 156.54 s, whereas the running time required to solve the best instance of the regularized multistage formulation amounted to $3,284.18$ s, which is compliant with practical time requirements. Moreover, we highlight that the computational burden featured by the multistage model can be significantly reduced by using efficient decomposition-based approaches.

Finally, we also examine the impact of the choice for the set of in-sample demand scenarios on the regularization of the multistage model. To that end, ten different in-sample scenario sets are used. As a result of this sensitivity analysis, the values for the best λ range in the interval between $1,000$ and $1,800$, whereas the average and the relative standard deviation of the corresponding out-of-sample total costs amount to $\$13,207,012.47$ and 1%, respectively.

V. CONCLUSION

This work has addressed the adoption of a more flexible and adaptive model based on multistage stochastic programming for the joint scheduling of energy and reserves within an economic dispatch setting. The related literature has mainly focused on approaches wherein this problem is formulated as a two-stage model relying on both forecasts of the uncertain parameters and simplifying assumptions. Existing approaches are thus prone to suboptimal solutions, such as non-implementable dispatch decisions and over-scheduling of operating reserves. Unlike the conventional two-stage framework, the proposed multistage formulation allows the revision of scheduling decisions at each time stage based on the uncertainty realized so far.

Although the proposed multistage stochastic model offers more flexibility for scheduling decisions, this benefit comes at the expense of solving a much larger and complex optimization problem. Due to the limitations featured by classical solution methods for multistage stochastic programming, a recently developed two-stage linear decision rules framework is adopted.

Furthermore, a novel regularization method, based on the AdaLASSO technique, is applied to prevent the threats of in-sample overfitting and poor out-of-sample performance, both due to the large number of coefficients that must be estimated when using the linear decision rules framework.

Numerical simulations allow drawing four main conclusions:

- The proposed multistage model provides more flexibility in the generation schedule. Hence, when compared to the two-stage model used for assessment purposes, smaller amounts of reserves need to be allocated. Consequently, the multistage framework achieved cost savings of up to 11.98% compared to the two-stage model.
- Since the two-stage model can be derived as a particular case of the proposed multistage formulation, the latter is the best model in terms of the in-sample total cost.
- The two-stage model, which is the state-of-the-art approach in the literature for solving the economic dispatch problem, does not allow any flexibility with respect to the demand realizations. With the increasing integration of renewable energy resources, this might become a challenge for system operators.
- In the out-of-sample evaluations, the regularized policy achieved substantial cost savings, up to 12.97%, in comparison to the non-regularized multistage model.

Admittedly, the computational effort required to solve the 300-bus case study might indicate potential scalability difficulties for larger systems. As an alternative to the monolithic solution applied here, it should be emphasized that the proposed model is suitable for state-of-the-art decomposition techniques such as Benders decomposition and progressive hedging, which may significantly improve the computational performance. The application of these decomposition methods to the proposed model will be addressed in future research.

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