Online Optimal Scheduling for Battery Swapping Charging Systems with Partial Delivery

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***Abstract*—Battery swapping–charging system (BSCS) is a promising operating paradigm to provide centering charging and battery swapping service for electric vehicles in transportation electrification. Facing real-time information on battery demand under the limited transporting trucks, flexible online optimization of battery delivery and transportation routing is essential for meeting practical requirements. This paper investigates the real-time scheduling problem in BSCS, considering the battery partial delivery, energy demand, delivery deadline, and vehicle routing. Considering the non-deterministic polynomia hardness of battery transportation, the offline BSCS is a time-consuming task and is unsuitable for the online setting. A Lagrangian relaxation-based Benders decomposition is proposed for parallel and real-time implementation, improving the scheduling efficiency. To tackle future information such as battery demands and delivery deadlines, by introducing the dummy copy, the offline algorithm is embedded within a rolling horizon framework to solve in real-time repeatedly. Finally, case studies using real road maps in Shanghai and Belgium have verified the validity of the proposed online framework and confirmed the necessity of considering partial delivery in enhancing the operation flexibility of BSCS. The computational efficiency of the proposed algorithm is studied under different scales of the road network, and the profit from partial delivery and online implementation are highlighted.**

***Index Terms*—Battery swapping station, electric vehicle, partial delivery, online scheduling.**

I. Introduction

With the increasing shortage of fossil resources and serious environmental concerns, electric vehicles (EVs) attracted extensive attention in realizing environmental-friendly transportation systems [1], [2]. Nevertheless, the popularization of EVs is impeded by the limited power supply, which necessitates the development of extensive charging infrastructure and involves long-term charging times [3].

In state-of-the-art EV charging studies, battery charging stations, battery swapping stations (BSSs), and battery swapping-charging systems (BSCS) are included to supplement energy for EVs [4]. Compared with battery charging stations, BSSs have more advantages, e.g., shorter service time and lower cost for EVs [5]. However, battery inventory shortages of BSSs may result in severe service interruptions when facing the large battery demand from EVs [6]. BSCS centrally charges EV batteries at a centralized charging station (CCS) and subsequently distributes them to battery swapping stations (BSSs) through battery transporters (BTs), constructing a power supply network based on the traffic network. Based on the centralized charging and multi-station distribution under a logistics system, BSCS offers a pioneering option to tackle the challenges above [4]. Note that existing researches consider facilities that offer both charging and swapping services for EVs [7]. However, due to the limitations of land area and deployment costs, it is more economical to separate charging from swapping, as considered by many related works [6], [8].

In BSCS, many works have studied the logistic transportation of batteries [9]. For example, to the cleanliness of the battery power supply side and the convenience of the EV battery demand side, a cooperative operation method of electric power system and truck-based battery transportation system is proposed in [10]. Similarly, Ban et al. [11] investigate optimal schedules for battery charging, swapping, and truck routing by a Lagrangian decomposition method. In [12], a joint optimal scheduling model is developed, integrating the battery charging/discharging schedule in the BSS, the generation plan of wind power, and the vehicle routing problem (VRP) of trucks.

However, the previous studies treat solely battery transportation as the VRP, without considering the practical aspect that battery transporters (BTs) and CCS in BSCS are typically managed by the same operator [13]. Some delivery requests may be served successively by multiple coordinated BTs if the large demand is higher than the limited capacities of BTs, i.e. partial delivery [14]. However, the current study does not take into account the battery partial delivery, leading to unnecessary fuel costs and reduced flexibility in battery transportation [15]. Moreover, the BSCS problem usually becomes an non-deterministic polynomia(NP)-hard problem due to the combinatorial nature of battery logistics [16]. The aforementioned studies [11], [12], [17] have employed commercial optimization solvers to address this model. However, the calculation efficiency of solvers is hard to meet the fast solution requirements with the expansion of urban scale [18]. Apart from those, battery transport demands and future traffic information are often uncertain in practice [19]. Clearly, addressing the BSCS problem within an online setting is more valuable and promising in transportation applications [20]. To the best of our knowledge, the BSCS problem has not been fast computed within an online framework by all the

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aforementioned literature.

Motivated by the above question, we propose a battery transportation model of BSCS, which considers partial battery delivery in an online manner. Specifically, unlike conventional logistics transportation problems, the BSCS in this paper considers the battery demands of BSSs under the transportation network to be met through the collaborative transport of various vehicles. Thus, the BSCS problem fundamentally belongs to the demand management of EV batteries, which significantly influence the delivery flexibility of battery demand for power system. Meanwhile, in response to the characteristic of distributed deployment of various BSSs, we also develop an efficient algorithm to reduce the operational complexity introduced by the battery transportation for real-time implementation. The contributions of this article are concluded as follows.

1) We propose a joint optimization framework for BSCS that simultaneously considers battery partial delivery and logistic routing, with the optimization process specific to collaborative transporting under the limited capacity of BTs.
2) We propose a Lagrangian relaxation-based Benders decomposition (LRBD) algorithm to decouple the logistic routing and battery delivery to substantially improve the computational efficiency with little performance loss.
3) We propose an online implementation of the proposed method by introducing the dummy copies and rolling horizon framework to solve the BSCS problem in real-time repeatedly to tackle the future uncertainty information.

The paper is organized as follows. Section II describes the corresponding BSCS model and then formulates the battery transportation problem. Section III has further introduced the proposed offline algorithm and corresponding online form. Numerical results with discussions are reported in Section IV. Finally, case studies and conclusions are given in Section V.

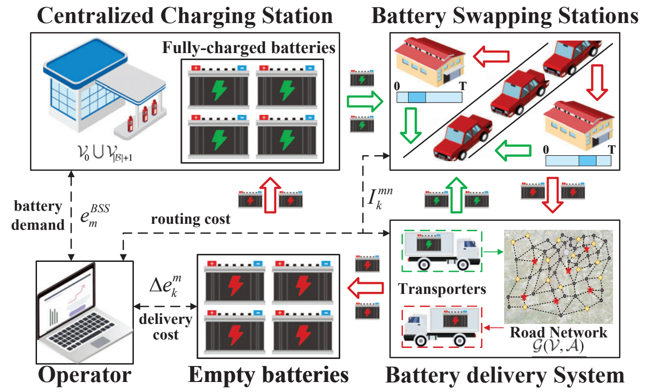

Fig. 1. Structure of BSCS

II. FRAMEWORK OF A BSCS SYSTEM

As shown in Fig.1, the CCS and BSSs are generally managed by the same operator. The BSSs will first provide battery swapping services for EVs and broadcast battery demand to CCS. Then, CCS will distribute batteries to BSSs through a fleet of BTs, and BTs will begin and finish their daily tour from the vehicle depot of CCS. In addition, BTs are required to deliver fully-charged batteries and recycle the empty ones before the demand deadline of BSSs. Note that battery transportation supports partial delivery, which allows the demand of a BSS to be fulfilled by multiple BTs [15].

A. Scheduling Description for a BSCS System

For notational convenience, the variables and parameters are described as: $\mathcal{B}$ denotes the set of BSSs, and $\mathcal{K}$ denotes the set of BTs. $\mathcal{G}(\mathcal{V},\mathcal{A})$ denotes the digraph consisted of the start CCS $\mathcal{B}_0$, the end CCS $\mathcal{B}_{|\mathcal{B}|+1}$, as well as BSSs $\mathcal{B}$, thus $\mathcal{V}=\{\mathcal{B}_0,\mathcal{B},\mathcal{B}_{|\mathcal{B}|+1}\}$. Define $\mathcal{V}_0=\mathcal{B}\cup\mathcal{B}_0$ and $\mathcal{V}_{\mathcal{B}+1}=\mathcal{B}\cup\mathcal{B}_{|\mathcal{B}|+1}$. Note that the set of arc $\mathcal{A}=\{(m,n)|m,n\in\mathcal{V},m\neq n\}$, in which each arc is associated with a specific travel cost d^{mn} and time τ^{mn}.

B. Mathematical formulation

In this section, we first elaborate on the offline battery transportation of BSCS. To optimize the battery transportation process and minimize the travel cost, flow conservation constraints should be met:

$$\sum_{n\in\mathcal{V}} I_k^{mn}-\sum_{n\in\mathcal{V}} I_k^{nm}=0,\forall m\in\mathcal{B},k\in\mathcal{K} \tag{1}$$

$$\sum_{n\in\mathcal{V}} I_k^{\mathcal{B}_0 n}-\sum_{n\in\mathcal{V}} I_k^{n\mathcal{B}_{|\mathcal{B}|+1}}=0,k\in\mathcal{K} \tag{2}$$

$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{V}} I_k^{mn}\geq 1,\forall m\in\mathcal{B} \tag{3}$$

where the binary variable I_k^{mn} is equal to 1 if BT_k travels from vertex m to n, and 0 otherwise. Constraint (1) and constraint (2) indicate that all BTs start at the CSS while returning to the CSS, and exit out the BSS after entering the same one. Constraint (3) denotes each BSS can be served at least once by the BT fleet. Note that the constraint (3) is the key to ensuring the implementation of partial delivery.

Time constraints with the arrival time τ_k^m of BT_k to BSS^m are defined in (4) and (5), which states that the arrival time of BTs should not be later than the delivery deadline of BSSs.

$$M(1-I_k^{mn})\geq\tau^{mn}+\tau_k^m-\tau_k^n,$$
$$\forall m\in\mathcal{V}_0,n\in\mathcal{V}_{\mathcal{B}+1},k\in\mathcal{K} \tag{4}$$

$$\underline{\tau}^m\leq\tau_k^m\leq\overline{\tau}^m,\forall m\in\mathcal{B},k\in\mathcal{K} \tag{5}$$

where the $\underline{\tau}^m$ and $\overline{\tau}^m$ are the broadcasting time and deadline receipt time of BSS^m for battery demand, respectively. And M is a sufficiently large positive constant.

Capacity constraints denote the remaining battery capacity of BT when unloading full-charged batteries and loading empty ones in the BSSs as follows.

$$M(1-I_k^{mn})\geq e_k^m-\Delta e_k^m-e_k^n\geq -M(1-I_k^{mn}),$$
$$\forall m\in\mathcal{V}_0,n\in\mathcal{V}_{\mathcal{B}+1},k\in\mathcal{K} \tag{6}$$

$$0\leq e_k^m\leq Q_k,\forall m\in\mathcal{B},k\in\mathcal{K} \tag{7}$$

$$e_k^{\mathcal{B}_0} = Q_k, \forall k \in \mathcal{K} \tag{8}$$

where e_k^m and Δe_k^m denotes respectively the remaining battery level and delivery levels of BT_k when reaching BSS^m. Constraint (7) enforces lower and upper bounds with the battery maximum capacity Q_k. Constraint (8) states that the BTs replenish batteries to the maximum capacity when reaching CCS.

As for the battery demand of BSSs, the delivery constraints, i.e. (9)-(11), denote that BTs will only deliver batteries to the BSS passing by, and the sum of the transportation batteries to BSS^m will not exceed the demand of the BSS.

$$0 \le \Delta e_k^m \le e_m^{BSS} \sum_{n \in V} I_k^{mn}, \forall m \in \mathcal{B}, k \in \mathcal{K} \tag{9}$$

$$\sum_{k \in \mathcal{K}} \Delta e_k^m \le e_m^{BSS}, \forall m \in \mathcal{B} \tag{10}$$

$$0 \le \Delta e_k^m \le e_m^{BSS}, \forall m \in \mathcal{B}, k \in \mathcal{K} \tag{11}$$

where e_m^{BSS} denotes the battery demand of BSS^m. For the scheduling of the battery transportation process, it is difficult to establish a model to schedule each battery in BSSs. Analogously to [10], [12], we focus on the delivery power Δe_k^m instead of the quantity of EV batteries. Correspondingly, the constraints of battery quantity during the transportation process can also be transformed into the constraints of battery power level, which will not affect the operation results.

Finally, the offline battery scheduling of BSCS can be formulated into a mixed integer linear program (MILP) with decision variables $\{I_k^{mn}, \Delta e_k^m, \tau_k^m, e_k^m\}$:

Problem 1 (MILP)

$$\begin{aligned}
\min_{I_k^{mn} \in \{0,1\}, \Delta e_k^m} \quad & \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{V}} \sum_{n \in \mathcal{V}} \left\{ \gamma_k^{mn} d^{mn} I_k^{mn} \right\} \\
& + \sum_{m \in \mathcal{B}} \gamma_m^{BSS} (e_m^{BSS} - \sum_{k \in \mathcal{K}} \Delta e_k^m) \quad (12) \\
s.t. \quad & (1) - (11)
\end{aligned}$$

The objective function minimizes the total travel cost and breach penalty for not fully meeting the battery swapping demand in the BSCS, where γ_k^{mn} is the travel price and γ_m^{BSS} is the penalty price.

III. Solution Methodology

In this section, the optimization algorithm of BSCS is proposed in an online manner. For decoupling the routing and delivery in **Problem 1**, the Benders decomposition (BD) is first introduced to iteratively optimize. Due to the integer constraints, the computational efficiency of Benders decomposition is intractable to apply in the online implementation. Thus, the Lagrangian relaxation method is utilized and solved by the Lagrangian multiplier algorithm to reduce the computation time. Finally, the proposed offline algorithm is embedded into the rolling horizon framework by introducing the dummy copy for the online solution.

A. Offline Algorithm

The basic idea of BD is to decompose such a BSCS model into a master problem (MP) resolving routing and a subproblem (SP) resolving delivery. The optimal solution of the original problem is found by alternately solving the master problem and sub-problems [21].

1) Master Problem: In each iteration of BD, a Benders cut will be added to the constraint set of the MP. The MP with q infeasibility cuts and p feasibility cuts (**MP-1**) is formulated as follows.

MP-1

$$\begin{aligned}
\min_{I_k^{mn} \in \{0,1\}, Z} \quad & Z \\
s.t. \quad & (1)\text{-}(3) \\
& \mu_1(q) : D^q(I) \le 0, \forall q \in \mathcal{I} \quad (13a) \\
& \mu_2(p) : \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{V}} \sum_{n \in \mathcal{V}} \left\{ \gamma_k^{mn} d^{mn} I_k^{mn} \right\} \\
& \qquad + D^p(I) \le Z, \forall p \in \mathcal{J} \quad (13b)
\end{aligned}$$

where $D^q(I)$ and $D^p(I)$ represent the feasibility and optimal Benders cuts, respectively. And Z is a continuous variable on behalf of the objective value of the master problem.

2) Lagrangian Relaxation of MP-1: Note that a major computational bottleneck still exists, as the **MP-1** is also a MILP. When there are no cuts (13) added, the feasible region of **MP-1** satisfies the property of total unimodularity (TU) [22]. Therefore, to speed up the proposed algorithm, a Lagrangian relaxation method is applied to solve the **MP-1** by relaxing generated cuts as follows.

MP-2

$$\begin{aligned}
\min_{I_k^{mn} \in [0,1], Z} \quad & \mathcal{L}_{MP} = Z + \sum_{q \in \mathcal{I}} \mu_1(q) D^q(I) + \sum_{p \in \mathcal{J}} \mu_2(p) \Big\{ \\
& D^p(I) + \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{V}} \sum_{n \in \mathcal{V}} \left\{ \gamma_k^{mn} d^{mn} I_k^{mn} \right\} - Z \Big\} \\
s.t. \quad & (1)\text{-}(3)
\end{aligned}$$

where μ_1 and μ_2 are Lagrangian multipliers corresponding to (13a) and (13b) respectively.

Note that solving **MP-2** is equivalent to solving a linear program (LP), which greatly reduces the computation burden. This claim is proved in the following lemma:

Lemma 1. *MP-2 can be exactly relaxed to an LP, in the sense that their solutions are equivalent.*

Proof. The relaxed linear program of the integer program would have only integer solutions, which happens when the constraint matrix is totally unimodular [23].

When Benders cut (13a) and (13b) are relaxed, the constraint matrix of the resultant problem **MP-2** satisfies the property of total unimodularity [24]. Then, $\{I|(1)\text{-}(3)\}$ satisfies the property of TU as well, and **MP-2** can be exactly relaxed as a linear programming problem without loss of optimality. □

Then, a Lagrangian multiplier algorithm is introduced to solve **MP-2** efficiently [7].

$$\Phi(\Lambda_k) = \sum_{m\in\mathcal{B}} \gamma_m^{BSS} e_m^{BSS} + \sum_{m\in\mathcal{B}} \sum_{k\in\mathcal{K}} [\lambda_1^{mk} \underline{\tau}^m - \lambda_2^{mk} \overline{\tau}^m - \lambda_3^{mk} Q_k - \lambda_4^{mk} e_m^{BSS} - \lambda_5^{mk} e_m^{BSS} \sum_{n\in\mathcal{V}} \bar{I}_k^{mn}] - \sum_{m\in\mathcal{B}} \lambda_6^m e_m^{BSS}$$
$$- \sum_{k\in\mathcal{K}} \lambda_7^{mk} Q_k + \sum_{m\in\mathcal{V}_0} \sum_{n\in\mathcal{V}_{\mathcal{B}+1}} \sum_{k\in\mathcal{K}} \{\lambda_8^{mnk}[\tau^{mn} - M(1-\bar{I}_k^{mn})] - \lambda_9^{mnk} M(1-\bar{I}_k^{mn}) - \lambda_{10}^{mnk} M(1-\bar{I}_k^{mn})\} \quad (14)$$

Algorithm 1: LRBD for offline problem.

Input: Related parameters of $\mathcal{G}(\mathcal{V},\mathcal{A})$, $\boldsymbol{e^{BSS}}$, $\boldsymbol{Q}$, $\underline{\boldsymbol{\tau}}$, $\overline{\boldsymbol{\tau}}$, $\boldsymbol{\tau}$, $\boldsymbol{d}$, $\boldsymbol{\gamma}$, ϵ
Output: Optimal scheduling strategy $\boldsymbol{I}^*$ and $\boldsymbol{\Delta e}^*$;

Initialize: $n=1$, $UB=\infty$, $LB=-\infty$, $\mathcal{I}=\mathcal{J}=\emptyset$; Randomly choose a set of feasible initial integer variable $\boldsymbol{I}(n)$;
while $|UB-LB|>\epsilon$ **do**
  Step1: Solve the **Dual-SP** at the n^{th} iteration to obtain $UB(n)$;
  if *dual problem (15) has a bounded solution* **then**
    $UB(n)=\Phi(\boldsymbol{\Lambda}(n))$;
    $UB=\min\{UB, UB(n)\}$;
    $\mathcal{I}=\mathcal{I}\bigcup D^p(\boldsymbol{I}(n))$;
    An optimal cut constraint (13a) is generated;
  else
    $\mathcal{J}=\mathcal{J}\bigcup D^q(\boldsymbol{I}(n))$;
    A feasibility cut constraint (13b) is generated;
  end
  Step2: Update the feasibility and infeasibility constraint sets of **MP-2**;
  Step3: Solve the **MP-2** by LMA at the n^{th} iteration to obtain $LB(n)$;
  $LB(n)=\mathcal{L}_{MP}(\boldsymbol{I}(n))$;
  $LB=\min\{LB, LB(n)\}$;
  Step4: $n=n+1$;
end
Return $\boldsymbol{I}(n), \boldsymbol{\Delta e}(n)$

3) Sub-Problem: Given the integer solution of **MP-2**, SP can be formulated as follows.
SP

$$\min_{\Delta e_k^m} \sum_{m\in\mathcal{B}} r_m^{BSS}(e_m^{BSS} - \sum_{k\in\mathcal{K}} \Delta e_k^m)$$
$$s.t. \quad (4)\text{-}(11)$$

The dual variables $\{\lambda_1^{mk},\ldots,\lambda_5^{mk},\lambda_6^m,\lambda_7^k,\lambda_8^{mnk}\ldots,\lambda_{10}^{mnk}\}$ are introduced for the **SP-1**. To produce the Benders cut, the dual problem of **SP** is derived as follows.
Dual SP

$$\max_{\Lambda_k} \quad \Phi(\Lambda_k)$$
$$s.t. \quad \text{constraints of dual SP} \quad (15)$$

where Λ_k is the set of dual variables, $\Phi(\Lambda_k)$ is shown in (14), and $\bar{I}_k^{mn}$ is the feasible solution of I_k^{mn} from the **MP-2**.

Given the solution Λ_k of Dual SP, Benders cuts $D^\omega(I), \forall\omega\in\{q,p\}$ in (13) can be derived, where $D^\omega(I) = f^\omega(I) + c^\omega$:

$$f^\omega(I) = \sum_{m\in\mathcal{V}_0} \sum_{n\in\mathcal{V}_{\mathcal{B}+1}} \sum_{k\in\mathcal{K}} M(\lambda_8^{mnk}+\lambda_9^{mnk}+\lambda_{10}^{mnk}) I_k^{mn} - \sum_{m\in\mathcal{B}} \sum_{n\in\mathcal{V}} \sum_{k\in\mathcal{K}} \lambda_5^{mk} e_m^{BSS} I_k^{mn}$$
$$c^\omega = \Phi(\Lambda_k) - f^\omega(\bar{I})$$

By adding the optimality or feasibility cuts to the master problem, the MP and SP are solved iteratively, until the upper and lower bound are sufficiently close [25].

4) offline algorithm for battery transportation: Based on the proposed LRBD, an iterative algorithm is developed to solve the offline battery transportation problem in Algorithm 1, where UB and LB are the upper bound and the lower bound of Benders decomposition, and the ϵ is a small number for the iteration termination.

Remark 1. *Specifically, by the Benders decomposition, the primal problem MILP is decomposed into two subproblems, corresponding to MP-1 and SP, which simplifies the difficulty of solving large-scale problems. Moreover, since SP contains independent decision variables of each BT, the battery transportation decisions at homogeneous BTs can be solved in parallel. Then, the MP-1 is relaxed to LPs from MILP by the Lagrangian relaxation method. Due to the superiority in iteratively optimizing LPs, the computation burden for real-time battery transportation is reduced. To sum up, our proposed algorithm facilitates a parallel and real-time implementation, improving the scheduling efficiency and saving computational time.*

B. Online Strategy for battery transportation

To respond the changing information arising from future battery demands or delivery deadlines in real-time, the LRBD algorithm is embedded within a rolling horizon framework to address the online battery transportation problems.

In the practical implementation, future information should be updated at every time slot. The delivery decisions of BTs should be re-optimized after the revelation of new information, so the start location of each BT will change in real-time according to the current location. Therefore, online implementation converts the battery transportation problem into a multi-CCS transportation problem, which is not in line with the LRBD algorithm for a single CCS problem.

Algorithm 2: Online implementation of LRBD.

Input: Related parameters of $\mathcal{G}(\mathcal{V},\mathcal{A})$, $\boldsymbol{Q}$, $\boldsymbol{d}$, γ
Output: Online scheduling strategy $\boldsymbol{I}$ and $\boldsymbol{\Delta e}$;
Initialize: $t=1$, overall time slot T;
while $t \le T$ **do**
  if $t = \underline{\boldsymbol{\tau}}$ **then**
    Broadcast the information of battery demand $\boldsymbol{e^{BSS}}$ and delivery deadline $\overline{\boldsymbol{\tau}}$ of the t^{th} time slot;
    if $t=1$ **then**
      Solve the offline BSCS problem by Algorithm 1;
    else
      Record the current locations $\boldsymbol{L}^t$ and the next BSS $\boldsymbol{R}^t$ of BTs;
      Add the constraint (17) to the **MP-2**;
      Solve the offline BSCS problem by Algorithm 1;
    end
  end
  $t=t+1$;
end

To this end, we introduce the concept of dummy copies to adapt the LRBD algorithm for online scenarios [26]. Initially, copies are assigned to each BSS catering to multiple battery demands across distinct time windows. Subsequently, copies are also allocated at the same location as the CCS, permitting BTs to acquire supplementary batteries for subsequent transportation missions. Additionally, we create copies corresponding to the current location of each BT, connecting them to the start CCS $\mathcal{B}_0$ and setting their travel cost and time to zero. In this way, the multi-CCS problem is equivalently transformed into a single-CCS battery transportation problem, which can be real-time addressed using the LRBD algorithm.

Formally, define a set $L^t = \{L_1^t, \dots, L_{\mathcal{K}}^t\}$ consisting of the current location of BTs and a set $R^t = \{R_1^t, \dots, R_{\mathcal{K}}^t\}$ consisting of the next request BSS of BTs at the t^{th} time slot. Additional constraints (17) are added to **MP-2** to ensure that BTs travel from the start CCS to the current location and the next BSS.

$$I_k^{1i} = 1, \forall i \in L^t \tag{17a}$$
$$I_k^{L_i^t R_i^t} = 1, \forall i \in \mathcal{K} \tag{17b}$$

Remark 2. *Apparently, constraint (17) meets the property of total unimodularity, thus the introduction of the constraint (17) does not affect the optimal equivalence when embedding to the BSCS problem.*

Now we present the details of the online LRBD method with dummy copies under the rolling horizon framework in Algorithm 2.

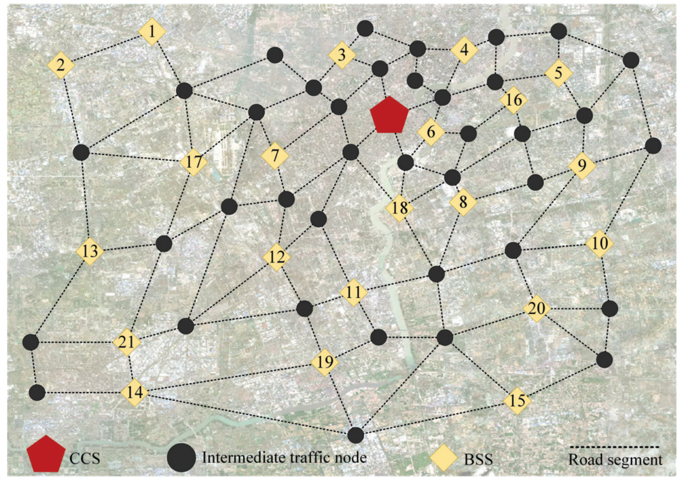

Fig. 2. Road network topological graph of Shanghai

IV. Case Studies

A. Simulation setup

In this section, simulation results for the battery transportation problem of BSCS are provided to evaluate the effectiveness of the proposed model and method. A real road network in Shanghai, China, is selected for model simulation. As shown in Fig.2, the road network is composed of 21 BSSs, 38 intermediate traffic nodes, 1 CCS, and 107 highway roads, where the intermediate traffic nodes are road nodes that bears path transfer without battery requirements. Then, the map data of Belgium[1] is adopted for algorithm simulation, which records the geographic coordinates of 100 nodes with each demand and time window. Note that the transportation network and service data of BSSs, i.e., $e_m^{BSS}, \underline{\tau}^m, \overline{\tau}^m$, are randomly selected from the Belgian dataset and implemented 10 times for some simulations. In this case, the performance of the proposed algorithm on different battery demands and transportation networks is evaluated. Each BT has a 3,250 kWh maximum battery carrying level and can take about 50 standard batteries with a capacity of 65 kWh battery. In addition, the average speed of BTs is set at 60 km/h. The driving price is assumed as 1.25 \$/km, involving the fuel fee and toll charges. And the breach penalty price for not fully meeting battery demands is 6.175 \$/kWh. All numerical studies are implemented on a personal computer with AMD Ryzen 9 5900HX CPU using Python with Gurobi 9.1.

B. Simulation of Offline Battery transportation

For a convenient illustration of partial delivery, the battery transportation results as an example of two BTs on the Shanghai road network is shown in Fig.3, where Fig.3(a) and Fig.3(b) denote transportation results without partial delivery while Fig.3(c) and Fig.3(d) depict ones involving partial delivery. Due to the limitation of BSS demand and BT capacity, each BT will return to CCS multiple times to supplement the battery. In this paper, a trip refers to a circuit in which

[1] http://www.vrp-rep.org/datasets/item/2017-0001.html

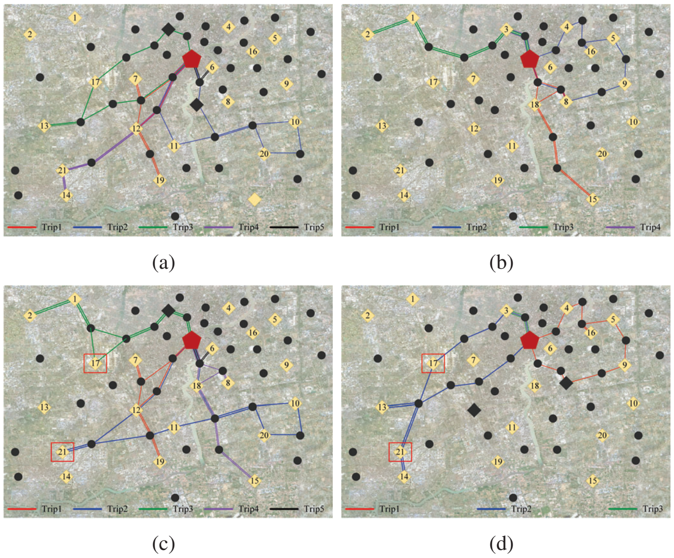

Fig. 3. Routing comparison for partial delivery: (a) BT_1 without partial delivery; (b) BT_2 without partial delivery; (c) BT_1 with partial delivery; (d) BT_2 with partial delivery.

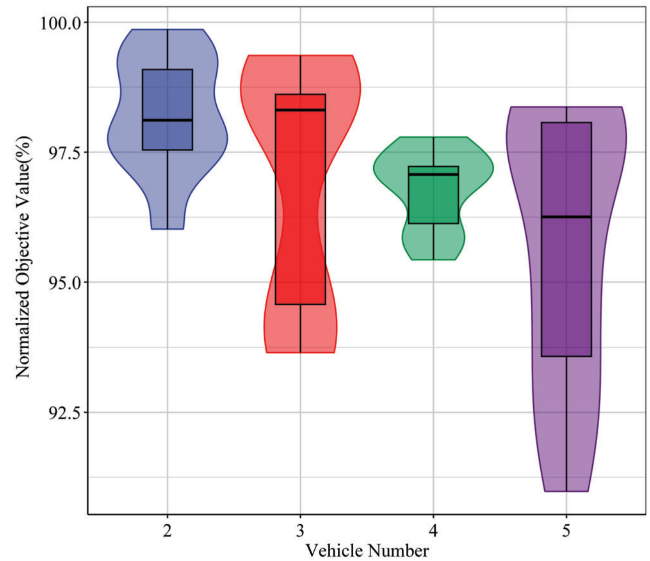

Fig. 4. Performance of partial delivery under different vehicle numbers

TABLE I
DELIVERY RESULT OF BATTERY NUMBER WITH PARTIAL DELIVERY

	BT_1	BT_2
Trip1	0-7-12-19-0 0-19(19)-14(14)-17(17)-0	0-9-5-16-4-0 0-2(2)-17(17)-28(28)-3(3)-0
Trip2	0-21-11-10-20-0 0-**16(27)**-10(10)-9(9)-15(15)-0	0-17-21-14-13-0 0-**11(23)**-**11(27)**-9(9)-19(19)-0
Trip3	0-17-1-2-0 0-**12(23)**-28(28)-10(10)-0	0-3-0 0-26(26)-0
Trip4	0-8-18-15-0 0-1(1)-28(28)-16(16)-0	
Trip5	0-6-0 0-23(23)-0	

a BT departs from the CCS, delivers batteries, and returns to the CCS. Take the BT_2 with partial delivery in Fig.3(d) as an example, the Trip2 is a journey that starts from CCS, sequentially reaches BSS^{17}, BSS^{21}, BSS^{14}, BSS^{13}, and finally returns to CCS to recharge the full battery for the next trip. In addition, Table I shows the delivery result of battery number for corresponding partial-delivery routings in Fig.3(c) and Fig.3(d). Note that each result in Table I consists of two rows, where the first row indicates the sequence of BSSs that BT_k passes through on the trip, with CCS being labeled as 0; the second row shows the number of batteries transported to the corresponding BSS in sequence, with the total battery demand of the BSS in parentheses.

Compared with two transportation routings in Fig.3, the total number of trips under partial delivery is lower than the one without partial delivery, i.e. partial delivery can reduce the travel distance in battery transportation. In addition, BSS^{17} and BSS^{21} have been transported multiple times by different BTs, as shown in the red box of Fig.3. Specifically, the BSS^{17} and BSS^{21} have large battery demand (23 and 27, as highlighted in bold in Table I), accounting for about 50% of the total BT capacity. Due to the mismatch of the BSS demand and the BT remaining capacity, traditional battery transportation models must consume more trip costs to meet high demand of BSSs. Conversely, partial delivery endeavors to coordinate transportation among multiple BTs to address the proposed challenge, thereby mitigating the contradiction between the sudden increase of BSS demand and the transportation scheduling costs of CCS.

Furthermore, the violin plot of Fig.4 depicts the ratio between the objective value (i.e. normalized objective value) with and without partial delivery under different networks, battery demands and vehicle numbers. In Fig.4, the shaded area surrounding boxes reflects the density of the normalized objective value; the wider the contour, the higher the aggregation degree in that area. The violin plot quantifies the effect of partial delivery for transportation costs under different vehicles, standing for the probability density distribution in distinct operation conditions. Fig.4 shows that the objective value under partial delivery is lower than the baseline, which increases with the number of vehicles. Thus, it demonstrates the effectiveness of the proposed model in complex road topology and variable BSS demand. In other words, the proposed BSCS can improve the flexibility in battery delivery of EV demands as the increase of controlled BTs.

To show the scalability of our proposed algorithm, Fig.5 and Table II show the comparison of runtime and operation cost among Gurobi, Benders decomposition, and our proposed algorithm under different-scale BSCS problems, where the superscript of LRBD in Table II is the cost improvement ratio between LRBD and Gurobi result.

Empirically, the decomposition of large-scale battery transportation facilitates efficient computing. Nevertheless, the runtimes of Gurobi and Benders decomposition are almost the same level when $|\mathcal{B}| < 23$. Due to the time-consuming

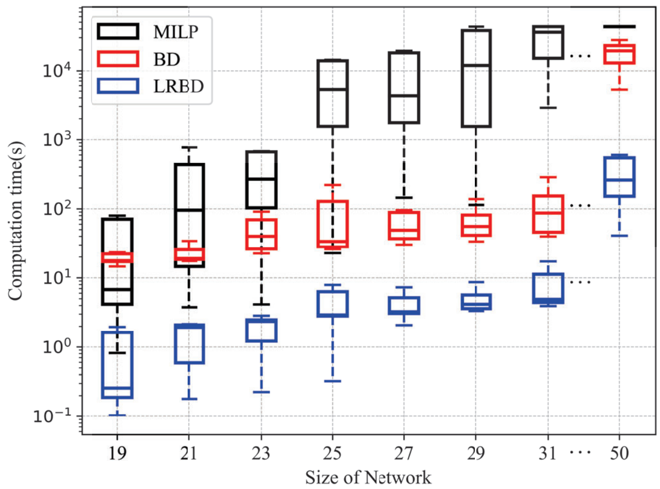

Fig. 5. Comparison on computation time

limitations of **MP-1**, the BD algorithm struggles for significant computational advantages in small-scale road networks, and thereby, disables to effectively battery transport on different network topologies. Note that the LRBD algorithm can solve the BSCS problem with 50 nodes in the minute order, which is essential in the online framework. As shown in Fig.5, although our proposed algorithm cannot maintain stable performance across all cases under a large transportation network, LRBD has a faster computation speed and outperforms Gurobi and BD by introducing the Lagrangian relaxation. This performance gain becomes more prominent with the increase in network size. This massive difference in performance demonstrates our proposed algorithm is suitable for the practical implementation of large-scale problems. In addition, as shown in Table II, compared with MILP which can obtain a global optimal solution for offline BSCS, LRBD achieves a near-optimal solution. The reason is that the Lagrangian relaxation introduces approximations, leading to the loss of accuracy. In general, there is a dilemma between solution speed and accuracy. Note that the average optimality gap is less than 10% over all instances. Hence, LRBD sacrifices a certain object accuracy and improves the calculation efficiency, which is acceptable for online optimization.

C. Simulation of Online Battery transportation

In the online simulation part, the entire rolling horizon consists of 12 time slots with each slot set as 1 hour, i.e. the operating hours of BSCS are set from 7:00 to 19:00. Given an assumption on the BSS demand at the first time slot is equal to the network size, and 2 new demands randomly emerge at every time slot in two BSSs. In the attempt to come to a contrastive conclusion, a time-prior algorithm is introduced to fully validate the superiority of the proposed online framework. In practical circumstances, the order in which BSS needs to be earlier meet is often determined by the demand deadline, which means that the earlier the deadline, the earlier the battery will be transported. In addition, if the

TABLE II
COMPARISON ON AVERAGE OPERATING COST ($)

Network Scale	19	21	23	25
MILP	82.06	95.72	97.53	98.12
BD	82.06	95.72	97.54	98.12
LRBD	$85.77^{4.51}$	$100.15^{4.62}$	$102.28^{4.87}$	$103.16^{5.13}$
Network Scale	**27**	**29**	**31**	**50**
MILP	119.62	127.55	133.48	-
BD	119.62	127.56	133.48	253.21
LRBD	$125.67^{5.50}$	$135.27^{6.04}$	$142.71^{6.92}$	$280.15^{10.63}$

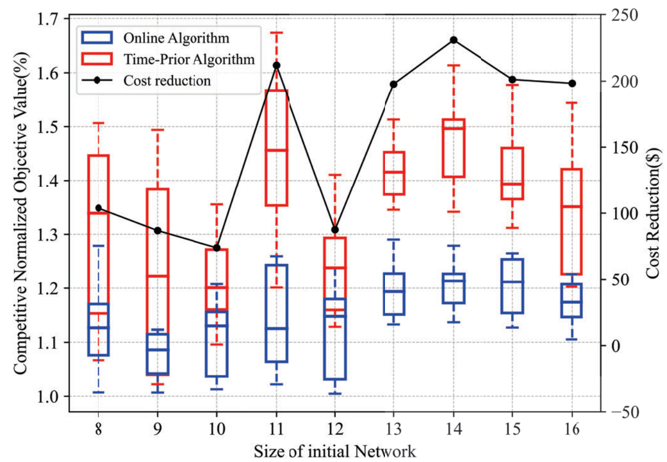

Fig. 6. Online algorithm comparison over different networks

deadline of multiple BSSs is equal, then the priority of BSS is determined according to the distance between the BTs and BSSs. Due to the difficulty of applying the offline MILP method over the whole time horizon, the network size used in this subsection is limited to a small size.

As shown in Fig.6, the proposed online algorithm outperforms the time-prior algorithm as the increasing of initial BSS size. Here, the normalized objective value is defined as the ratio between the objective value of each algorithm and the ideal offline MILP. It is noteworthy that the normalized objective value of our algorithm is less than 1.3 over different road topologies. In addition, we present the cost reductions between two online algorithms. Evidently, the transportation revenue has an overall rise as the augmentation of the road network, with each travel yielding a maximum profit of $230.9. Consequently, the proposed online algorithm outperforms the commonly used algorithm in practice (i.e. time-prior algorithm) in terms of the battery transportation problems in real-time, while the optimality loss compared to ideal circumstances is minor.

V. CONCLUSION

In this paper, a novel online scheduling framework incorporating partial delivery for BSCS is proposed to bridge the gap in the operation management of BSCS with a battery logistics model. To tackle the computational difficulty of solving the offline BSCS, a Benders decomposition method integrating Lagrangian relaxation is employed for the battery delivery, and improved to a corresponding online algorithm. Finally, our study demonstrates the effectiveness of partial delivery under the limited capacity of BTs, thereby enabling transporting collaboration and cost reduction. And the superior performance of the proposed framework on computation time is validated on different scales of road networks, both in offline and online settings. In future research, we will consider both the charging scheduling and logistic transportation of the batteries in BSCS to meet the requirements of grid regulation.

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