Grid-aware Flexibility Aggregation for Zonal Balancing Markets

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***Abstract*—This paper concerns the aggregation of intra-area flexibility resources into an offer curve towards a multi-area zonal balancing market. The scope for such an aggregation is to convey both the activation cost of the available intra-area flexibility resources and the cost of intra-area transmission constraints to the multi-area zonal market. The method is motivated by the fact that the latter cost component may not be trivial, since congestion also depends on power exchanges originating from / terminating at external control areas as well as on the topology of external control areas. We leverage robust optimization to compute a worst-case upper bound to the intra-area flexibility resource aggregation cost over a plausible domain of external grid operating conditions. We translate this worst-case cost into an ordered collection of price - maximum incremental quantity pairs, at zonal resolution. Our results indicate that such an approach can be used to hedge against the fact that a TSO has no observability outside its control area of responsibility.**

***Index Terms*—Multi-area balancing, zonal markets, flexibility, congestion management, robust optimization.**

NOMENCLATURE

The main symbols used in this paper are defined as follows. Others may be defined as needed in the text.

Indices and Sets

$a \in \mathcal{A}$	Interconnected grid control areas.
$b \in \mathcal{B}$	Flexibility resources.
$n, j \in \mathcal{N}$	Network nodes.
$x \in \mathcal{X}^a \subset \mathcal{N}$	Subset of nodes outside control area a, interconnected with a node inside control area a.
$z \in \mathcal{Z}$	Balancing market zones.

N.b. Subscripts are used throughout the text to denote relevant subsets. For instance, $b \in \mathcal{B}_n$ denotes the subset of flexibility resources connected with node n and $x \in \mathcal{X}_n^a$ stands for the subset of nodes outside area a and interconnected with node n.

Parameters

c_b	Flexibility resource marginal activation cost.
e_z	Zonal incremental export (decremental import).
f_{nj}^0	Base case transmission branch flow.
$\bar{f}_{nj}$	Transmission branch thermal rating.
$p_b^{\min/\max}$	Flexibility resource lower/upper limits.
pen	A large penalty value.
X_{nj}	Branch reactance.
$\phi_{nx}^{\min/\max}$	Cross-area power flow change lower/upper limits.

Variables

p_b	Flexibility resource power output.
$s_n^{+/-}$	Nodal slack power.
θ_n	Nodal voltage angle.
ϕ_{nx}	Cross-area power flow change.

I. INTRODUCTION

Cross-area integration is a necessity towards realizing the European ambition of clean, secure and affordable electricity supply. As of October 2022, such integration has been extended to the *balancing stage* of electricity trading, with the go-live of the *Manually Activated Reserves Initiative* (MARI) platform for the exchange of *frequency restoration reserves with manual activation* (mFRR)[1] [1]. The grid model currently implemented in MARI is a simplified aggregate representation, modeling the transportation of power between market zones rather than the transmission of power between control areas.

The working approach for coping with network constraints in the MARI platform combines *ex-ante bid filtering* and *ex-post bid blocking* [2], [3]. Before communicating the available bids within their control area, TSOs can *filter* any bid that is anticipated to cause intra-area congestion. For similar reasons, once the MARI market has cleared, TSOs can also *block* the activation of a bid within their control area and replace it with another bid (from the same control area). The methodological questions related to bid filtering are not trivial, taking into account the short decision time frame, as well as the related uncertainties. Moreover, bid filtering completely masks the

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[1] The platform has been launched with the participation of 5 *Transmission System Operators* (TSOs) from the Czech Republic and Germany. Most European TSOs are expected to access the platform in the summer of 2024.

intra-area grid properties and the risk aversion of each TSO from the market. Bid blocking, on the other hand, may lead to inefficiencies. If a blocked bid can only be replaced by a bid within the same control area, opportunities to use a less expensive bid in another control area may be left unexploited.

In this paper we develop an alternative approach, based on aggregating intra-zonal flexibility resources towards a zonal balancing market. Our proposal builds on top of the *Residual Supply Function* (RSF) introduced in [4], [5] to convey the economic cost of grid congestion towards zonal balancing markets. The "original" RSF in [4], [5] was based, for any given market zone, on the *most likely* (latest available) set of power injections across all external control areas, and assumed that all activated bids would be balanced at the hub node of the interconnected grid. Here, we generalize this approach to account for these exogenous, uncertain factors. We argue that the proposed price – quantity function conveys jointly the economic cost of intra-zonal flexibility resources, intra-area grid constraints and of the limited observability of the TSO outside its individual control area. Our proposal relies on robust optimization, which typically comes with a significant computational burden for transmission grid applications. We argue that the typical topological properties of interconnected power grids and the mathematical properties of the problem at hand combine to allow for a computationally efficient solution approach. We leverage this solution approach to demonstrate the properties of our proposal both in a simple, interpretable setting and on a publicly available model of the Nordic system.

The remainder of this paper is organized as follows. Section II introduces the problem of grid-aware flexibility aggregation in the context of zonal balancing markets. Section III presents the proposal we put forward in this paper, along with its mathematical formulation and the solution approach that we have adopted. Section IV discusses the application of our proposal over a set of relevant case studies, while section V summarizes our findings and draws conclusions.

II. PROBLEM DESCRIPTION

We consider an interconnected grid that is operated according to the organizational framework of Fig. 1. In the context of physical security, it is divided into a set of *control areas* operated by respective TSOs. In the context of exchanging balancing power, each control area can be further sub-divided into several *market zones*.

A zonal balancing market matches TSO demands for increments/decrements with available flexibility resources (at the zonal resolution). In the market clearing, inter-zonal power exchanges are approximated by the *linear* transport model and subject to *Available Transfer Capacity* (ATC) constraints between any pair of directly connected zones. Since this simplified model is not suitable to represent the complex, non-linear physics of the power grid, the clearing of the zonal balancing market may not necessarily result in secure operation of the physical system. In this paper, we focus on the issue of transmission network congestion. It is already documented that a zonal market subject to ATC constraints will inevitably lead to branch flow constraint violations under the (more detailed, yet still linear approximation) DC power flow model [6].

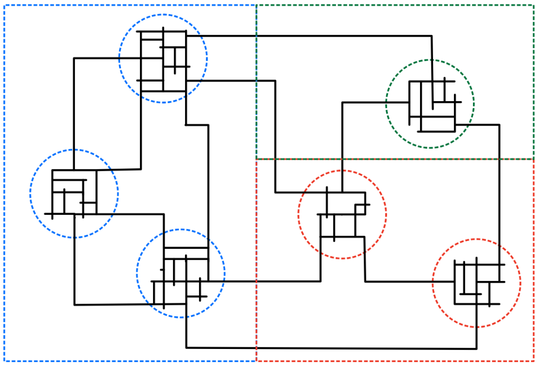

Fig. 1. Organizational framework for multi-area balancing: Control areas are separated by dashed coloured boxes, market zones are separated by dashed coloured circles, while the physical grid is sketched with solid black lines.

As originally proposed in [4], [5], we consider that TSOs could assume a hierarchical role between the balancing market and their intra-area flexibility resources. In this role, any TSO would have to anticipate the incremental export/import costs/benefits (*i.e*, energy and intra-area congestion management) associated with the potential activation of its intra-area resources, ahead of the balancing market clearing. The purpose would be to aggregate intra-area balancing flexibility into a so-called *Residual Supply Function* (RSF) and submit price – quantity offers to the balancing market that reflect the cost of the available flexibility while accounting for the intra-area transmission constraints associated with potentially sharing it. The merit of this alternative is evident. The zonal balancing market may still schedule the flexibility resources that would cause intra-area congestion, provided that the cost of resolving such congestion is economically justified from the perspective of the interconnected system.

However, anticipating intra-area congestion and its management cost so as to construct an RSF is non-trivial. Indeed, intra-area congestion depends on several factors beyond the observability of the respective TSO: the precise location of external sink/source nodes, the detailed topology of all external control areas, the balancing market activation of resources over external control areas, the congestion management actions of external TSOs, *etc.*. The original proposal in [4], [5] relied on a "*best guess*" for all such exogenous factors. The problem that we address in this paper is how to aggregate intra-zonal flexibility resources into a set of price – quantity offers for the zonal balancing market, taking into account the uncertainty of the state of the system outside the area in question, and the reliance of the process on external unobservable factors.

III. PROPOSAL

A. Overview

Let us focus on any single control area of an interconnected power grid. In the balancing market context, such an area may be sub-divided into a set of market zones. As in [4], [5], for any such market zone, we propose to develop a price – quantity resource aggregation cost curve by gradually evaluating the total cost of its enclosing control area (*i.e.*, flexibility resource activation subject to intra-area grid constraints) associated with a change in the zonal balance.

We argue here that the total cost of the control area depends on the change of the cross-area flows, between the control area of interest and the external grid. Indeed, different cross-area flow change vectors may add up to the same net change of zonal position yet result in drastically different congestion. Cross-area flow changes depend on the configuration of the complete interconnected grid, and specifically the complete power injection vector, demand vector and topology matrix. To circumvent the limited observability of any TSO outside its control area, we originally propose here to compute a *Worst-case Residual Supply Function* (WcRSF), within a so-called plausible domain of cross-area power flow changes.

The plausible domain of cross-area power flow changes expresses the range of different external operating conditions within which the TSO anticipates to share its intra-area flexibility. Within such a range, the worst-case benefit/cost of importing into/exporting out of any specific market zone is an upper bound on the flexibility resource aggregation cost associated with the net change in the position of the market zone, subject to intra-area network constraints. Gradually evaluating such cost over a range of minimum (import) and maximum (export) positions allows us to approximate a worst-case intra-zonal resource aggregation cost function. The WcRSF is the corresponding price – quantity function, defined as the slope of the worst-case intra-zonal resource aggregation cost function. It expresses the incremental cost of sharing flexibility, under the worst plausible external grid configuration. In other words, it approximates the incremental costs of exporting/importing balancing power, while internalizing the limited observability of the TSO concerning the external power system and the associated TSO risk aversion.

B. Worst-case Intra-zonal Resource Aggregation Cost

For any given market zone $\bar{z}$ and *incremental* balancing power export (resp. *decremental* import) $e_{\bar{z}}$, we evaluate the associated worst-case export (import) cost[2] over the control area that it belongs to, $a(\bar{z}) \in \mathcal{A}$, by solving the bilevel optimization problem (1 – 9).

The upper level objective (1) is to maximize the cost of deploying flexibility resources within the market zone of interest (1st term), as well as the penalization of nodal slack variables across the control area that includes the market zone of interest (2nd term). Notice that all variables in this expression are in effect variables of the lower level of the problem, which seeks to minimize the cost incurred by the control area (4) in response to the upper-level variables which denote a change of cross-area flows.

$$\max_{\phi} \sum_{b \in \mathcal{B}_{\bar{z}}} c_b \cdot p_b + \sum_{n \in \mathcal{N}_{a(\bar{z})}} pen \cdot \left(s_n^+ + s_n^-\right), \tag{1}$$

subject to:

$$\sum_{n \in \mathcal{N}_{a(\bar{z})}} \sum_{x \in \mathcal{X}_n^{a(\bar{z})}} \phi_{nx} = e_{\bar{z}}, \tag{2}$$

$$\phi_{nx}^{\min} \leq \phi_{nx} \leq \phi_{nx}^{\max}, \ \forall n \in \mathcal{N}_{a(\bar{z})}, x \in \mathcal{X}_n^{a(\bar{z})}, \tag{3}$$

$$\min_{\mathbf{p},\theta,\mathbf{s}} \sum_{b \in \mathcal{B}_{\bar{z}}} c_b \cdot p_b + \sum_{n \in \mathcal{N}_{a(\bar{z})}} pen \cdot \left(s_n^+ + s_n^-\right), \tag{4}$$

subject to:

$$\sum_{b \in \mathcal{B}_n} p_b = \sum_{j \in \mathcal{N}_n} \frac{\theta_n - \theta_j}{X_{nj}} + \sum_{x \in \mathcal{X}_n^{a(\bar{z})}} \phi_{nx} + \left(s_n^+ - s_n^-\right), \ \forall n \in \mathcal{N}_{a(\bar{z})}, \tag{5}$$

$$p_b^{\min} \leq p_b \leq p_b^{\max}, \ \forall b \in \mathcal{B}_{\bar{z}}, \tag{6}$$

$$p_b = 0, \ \forall b \in \mathcal{B}_z, \forall z \in \mathcal{Z} \setminus \bar{z} : a(z) = a(\bar{z}), \tag{7}$$

$$-\bar{f}_{nj} \leq \frac{\theta_n - \theta_j}{X_{nj}} + f_{nj}^0 \leq \bar{f}_{nj}, \forall n, j \in \mathcal{N}_{a(\bar{z})} \tag{8}$$

$$s_n^+, s_n^- \geq 0, \forall n \in \mathcal{N}_{a(\bar{z})}. \tag{9}$$

The summation appearing on the left-hand side of equality constraint (2) adds the change of cross-area flows over all cross-area branches that originate in the control area that includes the zone of interest. This constraint enforces that the net change of cross-area flows of the control area should match the incremental export (resp. decremental import) of the zone of interest. Notice that the change of cross-area flows is a decision variable of the upper level, which seeks to identify the most challenging change with regard to intra-area congestion. Inequality constraints (3) express the aforementioned plausible domain of cross-area power flow changes[3].

As already mentioned, the lower level objective (4) is to minimize the cost incurred by the concerned control area. The nodal slack variable summation (2nd term) is included since the intra-area optimization problem may not be feasible for the worst-case cross-area flow change vector in (2 – 3)[4].

Equality constraint (5) expresses the nodal power balance, taking into account the change of the zonal position. Notice here the summation of variable ϕ_{nx} over the cross-area interconnections of each intra-area node (2nd term of the right-hand side). It applies the worst-case cross-area power flow change selected by the upper level of the problem, by way of additional demand/generation at specific *boundary* nodes of the intra-area grid. Inequality constraints (6) enforce the bounds of the balancing resources within the zone of interest,

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[2] Note that we refer to cost in the sequel, which can be negative (typically in the case of downward balancing actions), i.e. a benefit resulting from fuel savings or increased consumption of balancing resources.

[3] A naive way to set the concerned bounds per cross-area branch would be to consider the branch thermal ratings, net of the base-case flow, as per the day-ahead/intra-day market dispatch.

[4] Using these slack variables ensures that bilevel problem (1 – 9) can be solved in any case. Setting the penalty to a value greater than the maximum marginal activation cost between the intra-zonal flexibility resources is the trivial solution to ensure that this coefficient has no influence on results.

while equality constraint (7) prohibits the use of flexibility resources within other market zones of the same area. Inequality constraints (8) are the transmission capacity constraints of the intra-area grid under the DC power flow approximation and while taking into account the flows resulting from the day-ahead/intra-day market dispatch. Finally, slack variables are by definition non-negative (9).

1) Solution Approach: A very popular approach for solving linear bilevel optimization problems such as problem (1 – 9), with several power systems applications, is the so-called *KKT reformulation*. It amounts to replacing the lower level problem (4 – 9) with its optimality conditions, and applying the *big-M* technique to transform the bilinear complementary slackness conditions into mixed integer disjunctive inequality constraints. The resulting problem is a *Mixed Integer Linear Programming Problem (MILP)*. Pineda and Morales [7] extensively discuss the challenge of properly tuning the value of the big-M parameter. Moreover, the resulting MILP problem can turn out to be computationally complex, requiring extensive time to be solved by off-the-shelf branch and bound solvers. We have adopted an alternative solution approach which exploits both the mathematical properties of problem (1 – 9) and the topological properties of interconnected power grids.

It can be shown that the optimal value of the lower level problem (4 – 9) is piece-wise convex in the upper level decision variable ϕ, appearing in the right-hand side of equality constraint (5). It follows that problem (1 – 9) is the maximization of a convex function. It can further be shown that the global maximum of a convex function over a closed bounded convex set is an extreme point. In other words, the global maximizer of (1 – 9) would be a corner point of (2 – 3). It would therefore suffice to exhaustively evaluate the lower level problem (4 – 9) over all corner points of (2 – 3). Noting that these constraints express bounds on the cross-area power flow changes only, the typical interconnected power system topology is favourable regarding the computational burden of such exhaustive enumeration. Indeed, while the number of intra-area branches and transformers within a certain control area can be in the order of thousands, the number of cross-area interconnectors out of/into any single control area is typically in the order of at most tens. Hence, the number of distinct corner points of (2 – 3) does not prohibit us from solving the corresponding set of instances of (4 – 9) in an acceptable computational time. The parallelization of these linear programming problem instances is trivial.

2) Implementation environment: Our implementation of the exhaustive enumeration approach to solving (1 – 9) was developed in Julia [8] using the JuMP modeling language [9] and the PowerModels.jl framework [10] for data formatting. We further used the CDDLib.jl wrapper [11] within the Polyhedra.jl computational interface [12] to generate the set of corner points of (2 – 3). Finally, we solved all instances of the linear problem (4 – 9) with the CPLEX [13] solver.

C. Worst-case Residual Supply Function

As already mentioned, the WcRSF is a price – quantity function expressing the incremental cost of exporting/importing balancing power under worst-case assumptions regarding the corresponding change of cross-area power flows. It is defined as the slope of the worst-case intra-zonal resource aggregation cost function.

Approximating the slope of the worst-case intra-zonal resource aggregation cost function indirectly by connecting successive solution points is a byproduct of approximating the actual function by solving (1 – 9) for gradually increasing balancing volumes within a given range. However, contrary to the zero-crossing, piece-wise convex total cost function which emerges in [4], [5], the worst-case resource aggregation cost function has neither of these properties. It is intuitive to understand, from a power systems perspective, why the worst-case function should not in general cross zero. Even though the net position of a control area may not change at all, balancing power exchanges between other control areas may still change the cross-area power flows in a way that causes congestion within the area of interest. To account for the cost of such congestion, the WcRSF includes a fixed (pseudo start-up) cost. This cost is incurred, in order to access the flexibility resources within the area of interest. Non-convexity results from the definition of the WcRSF as the successive solution of bilevel optimization problem (1 – 9), rather than a single-level linear programming problem as in [4], [5]. We discuss its meaning from a power systems perspective using an example in Section IV-A.

We further express the WcRSF as an ordered collection of price ($\pi_{k,z}$) – maximum incremental quantity ($dq_{k,z}^{\max}$) pairs. The price component of any pair expresses the incremental cost (in €/MWh) of the incremental export (decremental import) of balancing power out of (into) a zone and specifically on top of the cost of the total export (import) quantity of preceding pairs in the same (import/export) direction. The maximum incremental quantity component defines the maximum amount of power that is available at the respective price and at the specific position in the sequence of pairs. In other words, it can be activated only after its predecessors have been fully activated and must be fully activated in order to activate any of its successors[5]. The constraints (which implicate binary variables) that are needed to represent the sequentiality of the ordered price – maximum incremental quantity pairs in the zonal balancing market clearing problem formulation are shown in the Appendix.

IV. RESULTS & DISCUSSION

A. Chao-Peck test case

In order to present the properties of our proposal in a transparent way, we revisit the Chao-Peck system from [4] which offers full interpretability. More specifically, we attempt

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[5] Sequentiality of activation relates to price-quantity pairs in the same (export/import) direction. Notice that the aggregated flexibility of a market zone would be activated in a single direction only.

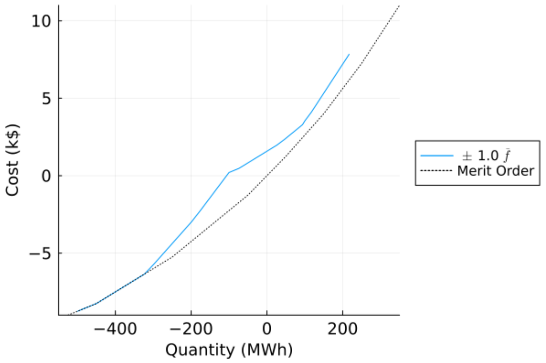

Fig. 2. Intra-zonal balancing resource aggregation cost

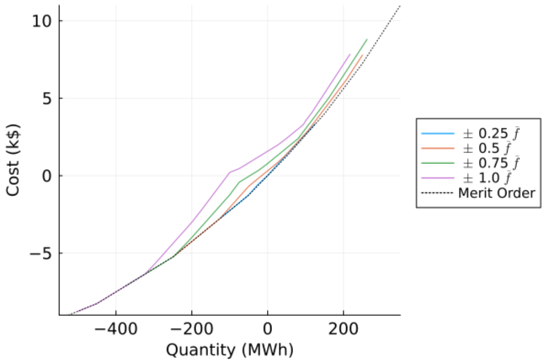

Fig. 3. Intra-zonal flexibility resource aggregation cost over alternative cross-area power flow bounds

to aggregate the intra-zonal flexibility resources of the so-called "Northern zone" (nodes 1-3) of the Chao-Peck system and following the day-ahead zonal market clearing outcome illustrated in Fig. 2 of [4]. We do so by successively solving problem (1 – 9) for incremental balancing position changes in the range [-550,350] MW (*i.e.*, the total downward/upward balancing capacity of the resources located in the Northern zone), increasing at a 1 MW resolution.

1) Intra-zonal flexibility resource aggregation cost: To start the analysis, we define a plausible domain of cross-area power flow changes by setting the upper (lower) bounds appearing in (3) equal to $\phi_{nx}^{\max / \min} = \pm \bar{f}_{nx}$. Figure 2 plots the worst-case cost of the intra-zonal flexibility resource aggregation (blue solid line) along with the cost of activating the intra-zonal balancing resources according to the economic merit order.

Let us first notice that, when importing 325 MW or more (lower left part), the two curves are identical. This implies that no intra-area congestion may occur when importing more than 325 MW, for any cross-area power flow change vector within the respective plausible domain. Conversely, when importing less than 325 MW, the curves diverge, suggesting that the worst case would cause intra-area congestion. As there is no intra-area congestion in the base-case, it may appear counter-intuitive that the worst-case outcome for a smaller incremental import volume implies congestion, whereas it does not for a larger volume. This behavior is however consistent with the considered plausible changes in the cross-area power flows (2 – 3). For the smaller incremental import volume, congestion could happen if one of the cross-area interconnectors is importing power, while the other is exporting. In order to import the larger volume while still respecting inequalities (3), both cross-area interconnectors should be importing, and this turns out not to congest the intra-area grid.

The abrupt change in the slope of the worst-case cost curve around the import quantity of 101 MW is also related to the plausible domain of cross-area power flow changes. According to our detailed results, when importing 101 MW to 324 MW, the worst-case intra-area congestion would be realized if the power flow change of the cross-area branch linking nodes (n_1, n_4) is at its lower bound from (3). Equality (2) would then determine the respective change for the 2$^{\text{nd}}$ cross-area branch of the Northern zone, linking nodes (n_3, n_5). Importing less than 101 MW with the change of (n_1, n_4) at its lower bound violates the upper bound for the (n_3, n_5) plausible power flow change. Hence, the import pattern has to switch such that the (n_3, n_5) power flow change variable is upper bounded and the (n_1, n_4) variable is determined from equality (2). Stated in mathematical terms, the optimal solution of (1 – 9) moves to an alternative corner point of (2 – 3). The next similar switch in the power flow change pattern can be observed around the point of exporting 100 MW or more.

We further highlight that the worst-case cost curve crosses the y-axis (0 MW import/export) at a positive cost. Even if the zone of interest may not produce more/less power in total, after the realization of imbalances and the activation of flexibility resources, power flows will change and may create intra-area congestion. Finally, we notice that the worst-case intra-zonal balancing resource aggregation cost curve falls shorter than the merit order curve and stops at the point of exporting 217 MW. Exporting more than this amount would cause unmanageable intra-area congestion, which is detected by non-zero values of the lower level slack variables (9).

Figure 3 plots the worst-case cost of the intra-zonal balancing resource aggregation for tighter bounds on the plausible changes of cross-area power flows in (3). Tighter power flow change bounds per cross-area branch can only make the respective worst-case intra-area congestion less problematic for the same export volume, hence less costly. For the same reason, curves computed with tighter bounds may extend further than worst-case cost curves over larger plausible domains of power flow changes. For instance, the maximum export quantity at $\phi_{nx}^{\max / \min} = \pm 0.5 \bar{f}$ is 250 MW whereas at $\phi_{nx}^{\max / \min} = \pm \bar{f}$ it is 217 MW. However, much tighter bounds restrict the total capacity of an area to share its flexibility resources. This can seen in the curve for $\phi_{nx}^{\max / \min} = \pm 0.25 \bar{f}$.

2) Zonal balancing market participation & congestion management cost: To continue the analysis, we model the submission of WcRSF bids in a zonal balancing market and the resulting congestion management outcome for the Northern control area. The purpose is to investigate the effectiveness of our proposal in reflecting both the costs of the available flexibility resources and of the intra-area congestion related to sharing such resources. With this motivation, we evaluate the following performance metrics:

TSO Quantity (Q_a)*:* The total activated balancing quantity from flexibility resources within control area a.

TSO Offer Cost (CO_a)*:* The cost of the total activated balancing quantity from flexibility resources within control area a, as per the price and quantity offers submitted by the respective TSO.

TSO Disaggregation Cost (CD_a)*:* The approximated cost of delivering the incremental balancing positions of the zones within control area a, while respecting intra-area transmission constraints.

We must briefly explain here our motivation for considering the cost of the activated intra-area flexibility resources according to the price and quantity offers submitted by the respective TSO rather than the balancing market revenue of the TSO. The latter additionally includes possible infra-marginal rents from the balancing market. We wish to factor out in our assessment the possibility that these infra-marginal rents coincidentally cover the intra-area congestion management costs.

In order to evaluate the considered performance metrics, we generate 1000 random imbalance samples by assuming that nodal imbalances are normally distributed with a zero mean and standard deviation equal to 5% of the respective nodal load. We clear a zonal balancing market subject to ATC constraints[6]. The zonal market clearing model is the model from [5], with additional binary constraints to enforce the sequential acceptance of the ordered WcRSF bids. It is included in the Appendix of this paper. The final step is the assessment of the resulting costs for the Northern area. To do so, we solve a DCOPF over the complete interconnected grid, while applying all nodal imbalances as well as the balancing market activations outside the Northern zone. Transmission constraints are only enforced in the Northern control area, the resources of which can be activated to balance the system (*i.e*, deliver the incremental balancing position of the Northern zone, while respecting intra-area transmission constraints). The resulting objective function value is our approximation of the cost of disaggregating flexibility in this area.

Table I reports the average values of the aforementioned metrics. The 1st row reports on the baseline scenario wherein intra-area flexibility resources are aggregated according to their order of economic merit, that is neglecting intra-area transmission constraints. The 2nd and 3rd rows report on scenarios wherein intra-area flexibility resources are aggregated as per our proposal, for alternative bounds on the plausible domain of cross-area power flow changes. All values have been computed over the same imbalance samples. For clarity of presentation, we split the data-set into upward (export) and downward (import) activations of balancing power. Further, the ratio $\delta C_a = 100 * {}^{(CO_a - CD_a)}/_{CD_a} (\%)$ is also displayed as a measure of the discrepancy between the TSO offer cost and its corresponding disaggregation cost.

TABLE I

Average performance metrics - Chao Peck test case

	Q_a (MWh)	CO_a (k\$)	CD_a (k\$)	δC_a (%)	
Merit Order	18.04	0.45	0.47	-4.1	*export*
$\pm 0.25\bar{f}$	18.02	0.5	0.47	6.4	
$\pm 0.5\bar{f}$	16.15	0.54	0.42	28.6	
Merit Order	-8	-0.2	-0.2	0	*import*
$\pm 0.25\bar{f}$	-8	-0.17	-0.2	15	
$\pm 0.5\bar{f}$	-8	0.15	-0.2	175	

It can be seen that the merit order approach underestimates the TSO disaggregation (*i.e.*, energy and transmission congestion) cost of exporting out of the Northern zone of the Chao Peck system. Our detailed results indicate a shortfall for 45.7% of the random samples, while the Northern zone would be exporting for 77% of the random samples. The alternatives based on the WcRSF approach over-anticipate the cost associated to managing intra-area congestion. The variant presented in the 2nd row would be guaranteed to recover enough payments to recover the disaggregation cost of the intra-area flexibility resources in 45.3% of the instances, irrespective of potential infra-marginal rents. The variant presented in the 3rd row, wherein cross-area power flows are assumed to change in the $\phi_{nx}^{\max/\min} = \pm 0.5\bar{f}$ interval, would recover the disaggregation cost over 100% of the random samples. It is however an ultra-conservative approximation of such cost, as evidenced by the values in the 5th column of Table I. This extreme conservativeness has a minimal effect on the setting of the illustrative Chao-Peck example. In general systems, it may undermine the economic competitiveness of the intra-area flexibility resources more severely.

B. Nordic 46-node test case

To further assess the effectiveness of our proposal in a more representative, rather than simple and interpretable, setting we also consider the 46-node model of the Nordic system [5][7]. We adopt the perspective of the TSO that is responsible for controlling a single control area, which includes zones NO1, NO2 and NO5 of the system. We focus on the use case for the WcRSF, which anticipates both the intra-area flexibility resource activation costs and intra-area congestion management costs. To do so, we repeat the procedure introduced in section IV-A2 and simulate both the zonal balancing market clearing problem and the subsequent DCOPF problem for the area of interest, with the final aim of evaluating the same metrics[8]

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[6] The Northern zone is export-constrained at the balancing stage, as per the test case parameters in [4]. We modify the test case parameters to assume additional 5% ATC at the balancing stage.

[7] The full dataset is available at http://users.ntua.gr/papavasiliou/DatasetEEM2022.zip

[8] The only difference in our implementation, with respect to the Chao-Peck test case, is the addition of the so-called *Nordic security constraints*, which restrict the total power flow between market zones and are shown as (12) in Appendix B of [5].

TABLE II
AVERAGE PERFORMANCE METRICS - NORDIC TEST CASE

	Q_a (MWh)	CO_a (k€)	CD_a (k€)	δC_a (%)	
Merit Order	227.5	7.6	8.21	-7.4	*export*
$\pm 0.25\bar{f}$	150	5.27	5.46	-3.5	
$\pm 0.5\bar{f}$	145.6	5.14	5.3	-3	
$\pm \bar{f}$	144	5.08	5.23	-2.8	
$\pm 1.5\bar{f}$	144	5.08	5.23	-2.8	
Merit Order	-238.8	-7	-7	0	*import*
$\pm 0.25\bar{f}$	-188.2	-5.42	-5.24	3.4	
$\pm 0.5\bar{f}$	-192	-5.51	-5.36	2.8	
$\pm \bar{f}$	-194.8	-5.58	-5.545	2.4	
$\pm 1.5\bar{f}$	-194.8	-5.58	-5.545	2.4	

TABLE III
AVERAGE PERFORMANCE METRICS - MODIFIED NORDIC TEST CASE

	Q_a (MWh)	CO_a (k€)	CD_a (k€)	δC_a (%)	
Merit Order	194.8	6.44	6.8	-5.3	*export*
$\pm 0.25\bar{f}$	121.5	4.2	4.17	0.7	
Merit Order	-229.7	-6.76	-6.76	0	*import*
$\pm 0.25\bar{f}$	-170.2	-4.9	-5	2	

Table II presents the average values of the considered performance metrics over 1000 samples. We first identify the difference in terms of total activated balancing quantity between the merit order offer scenario and all WcRSF variants. The merit order baseline turns out to be balanced in terms of upward and downward activations from the control area of interest. Intra-area congestion has an asymmetric effect, bearing more heavily on the ability of area a to export its flexibility resources. For instance, using the $\phi_{nx}^{\max/\min} = \pm 0.25\bar{f}$ alternative, the average export quantity would reduce by approximately 34% while the average import quantity would only reduce by 21.2%. In other words, the control area of interest is more congested towards the export direction. The 5th column of Table II can also be used to confirm this. Indeed, it shows that the merit order offer would only under-approximate the so-called TSO disaggregation cost for exporting instances.

More importantly, the 5th column of Table II shows that even though the WcRSF variants perform much better than the baseline, they still under-approximate the disaggregation cost for exporting instances only. Further, the values reported in the exporting rows for variants $\phi_{nx}^{\max/\min} = \pm\ \bar{f}$ and $\phi_{nx}^{\max/\min}$ =$1.5\bar{f}$ suggest that the shortfall may not be related to cross-area exchanges. Indeed, even though the plausible domain of cross-area power flow changes increases between these two variants, the performance is identical. Besides cross-area exchanges, the alternative suspected cause for under-approximating the TSO disaggregation cost is the uncertainty regarding the intra-area power injections. To explore this, we have performed an additional simulation where we assume that the nodal imbalance may not be observed within market zones NO1, NO2 and NO5. In other words, we model an ideal situation wherein the TSO could compute the WcRSF based on a perfect forecast of the intra-area power injections and would only be liable to face congestion due to imbalances over external control areas as well as balancing resource activations anywhere in the interconnected grid. We report these results in Table III.

Table III first indicates that, even in the absence of random imbalances within the control area of interest, the merit order aggregation approach would still under-approximate the cost of exporting the intra-zonal flexibility resources. In other words, intra-area congestion is attributable to the cross-area exchanges. The results in Table III further indicate that the WcRSF can effectively anticipate the cost of managing such congestion, which is induced by cross-area trading. Indeed, both over the exporting samples and the importing samples, the TSO offer cost, as computed by the WcRSF, over-approximates the TSO disaggregation cost. In both cases, the difference between these costs is relatively insignificant. The message here is that the WcRSF, which is only hedging against uncertainties of the external unobservable power grid, should be computed on the basis of an accurate forecast of the intra-area power injections.

V. CONCLUSIONS

This paper concerns the problem of intra-area flexibility resource aggregation in the context of zonal balancing markets. The problem is rather timely, in light of the growing progression of cross-area balancing in Europe. We adhere to the concept originally introduced in [4], [5] and calling upon TSOs to aggregate the flexibility resources within their control area in a grid-aware manner. The purpose of this aggregation is to communicate both the costs of intra-zonal flexibility resources and the constraints of intra-area congestion towards the zonal balancing market.

We revisited the methodological tools allowing the intra-zonal flexibility resource aggregation, and more specifically the statement of the optimization problem serving to evaluate the incremental change in the net balance of the enclosed market zones of an area, while accounting for internal network constraints. We proposed to evaluate said costs in a robust manner, so as to account for the risk associated with the fact that intra-area congestion is not only the product of intra-area resource activations but also of unobservable external factors.

Mathematically, proposal relies on collapsing the power injection vector, demand vector and topology matrix of the external interconnected grid into a vector of port variables between the control area of interest and the external system. The plausible domain of such port variables can be seen as a *proxy* for the external system. On this premise, the worst-case cost of evacuating power from a market zone, within the plausible domain of the port variables, is a pessimistic approximation of the actual cost under a wide range of operating conditions.

A small-scale test case, offering complete interpretability, has been used to investigate the properties of the proposed approach. The analysis showcases the rationale of our proposal, and most importantly, the challenge of defining a plausible domain for the cross-area power flow change variables. A representative test case, based on the Nordic power system, further reveals that, while our approach can effectively hedge against the uncertainties of cross-area trading, its performance is contingent on the quality of the available forecasts of intra-area power injections.

The fine balance between anticipating congestion management costs within a zone and reducing the economic competitiveness of intra-zonal flexibility resources is not easy to strike. It becomes obvious that determining efficient plausible domains for the cross-area power flow change variables is the predicament for advancing our proposal. We intend to devote particular research effort in this question, starting from an investigation of the usefulness of historical power flow records, in order to be able to extend the merits of grid-aware intra-zonal flexibility aggregation to realistic power grid instances.

We must finally acknowledge that the intra-zonal resource aggregation question has a complementary settlement question of distributing any market profits back to the stakeholders associated with the intra-zonal resources. The approach presented in this paper is in principle compatible with any intra-area settlement scheme, including the nodal intra-area settlement discussed in [4]. In other words, any surplus/deficit arising from the intra-area resource aggregation can be directly distributed to the intra-zonal stakeholders and should not be considered as a profit/loss for the TSO. However, the potential effect of combining this aggregation proposal with a specific ex-post settlement scheme is also a topic for future research.

APPENDIX

The zonal balancing market clearing formulation, integrating the WcRSF submitted by a given control area $\bar{a}$, is as stated in (10 – 21).

The 2nd row of objective function (10) and constraints (11 – 17) are expressing the WcRSF. For any balancing market zone within the control area of interest ($z \in \mathcal{Z}_{\bar{a}}$), the WcRSF is an ordered collection of $k \in \mathcal{K}_z$ price – maximum incremental quantity pairs. Continuous variable $q_{z,k}$ denotes the accepted quantity of incremental export (import) at the respective price ($\pi_{k,z}$). We adopt the notational convention that positive indices $k \in \mathcal{K}_z^+ = [1; \ldots; |\mathcal{K}_z^+|]$ correspond to incremental exports out of the respective zone (and similarly negative indices $k \in \mathcal{K}_z^- = [-1; \ldots; -|\mathcal{K}_z^-|]$ to incremental imports). Auxiliary binary variable $u_{k,z}$ indicates whether the maximum incremental quantity of the k-th pair is fully accepted in the zonal balancing market clearing, while auxiliary binary variable $v_{k,z}$ indicates whether it is partially accepted. We finally integrate the fixed (pseudo start-up) cost component by setting $dq_{0,z}^{\max} = 0$. Binary constraints (13 – 17) combine to enforce that any pair may be accepted fully or partially only after its predecessor has been fully accepted. This also implies that accepting the $k = 0$ pair and paying the corresponding pseudo-start-up cost is a prerequisite for accepting any no-zero quantity.

$$\min_{dq,p,q,u,v} \Bigg\{ \sum_{z\in\mathcal{Z}\setminus\mathcal{Z}_{\bar{a}}} \sum_{b\in\mathcal{B}_z} c_b \cdot p_b + \sum_{z\in\mathcal{Z}_{\bar{a}}} \Bigg(\pi_{0,z} u_{0,z} + \sum_{k\in\mathcal{K}_z^+} \pi_{k,z} q_{k,z} - \sum_{k\in\mathcal{K}_z^-} \pi_{k,z} q_{k,z} \Bigg) \Bigg\} \tag{10}$$

subject to:

$$q_{k,z} = u_{k,z} \cdot dq_{k,z}^{\max} + dq_{k,z}, \;\; \forall k \in \mathcal{K}_z, \forall z \in \mathcal{Z}_{\bar{a}}, \tag{11}$$

$$0 \le dq_{k,z} \le v_{k,z} \cdot dq_{k,z}^{\max}, \forall k \in \mathcal{K}_z, \forall z \in \mathcal{Z}_{\bar{a}}, \tag{12}$$

$$v_{k,z} + u_{k,z} \le u_{k-1,z}, \;\; \forall k \in \mathcal{K}_z^+, \forall z \in \mathcal{Z}_{\bar{a}}, \tag{13}$$

$$v_{k,z} + u_{k,z} \le u_{k+1,z}, \;\; \forall k \in \mathcal{K}_z^-, \forall z \in \mathcal{Z}_{\bar{a}}, \tag{14}$$

$$\sum_{k\in\mathcal{K}_z} v_{k,z} \le 1, \forall z \in \mathcal{Z}_{\bar{a}}, \tag{15}$$

$$u_{-1,z} + u_{1,z} \le 1, \;\; \forall z \in \mathcal{Z}_{\bar{a}}, \tag{16}$$

$$v_{k,z}, u_{k,z} \in \{0; 1\}, \;\; \forall k \in \mathcal{K}_z, z \in \mathcal{Z}_{\bar{a}}, \tag{17}$$

$$p_b^{\min} \le p_b \le p_b^{\max}, \;\; \forall b \in \mathcal{B}_z, \forall z \in \mathcal{Z} \setminus \mathcal{Z}_{\bar{a}}, \tag{18}$$

$$\sum_{b\in\mathcal{B}_z} p_b = -r_z + \sum_{l\in\mathcal{L}_z^{out}} t_l - \sum_{l\in\mathcal{L}_z^{in}} t_l, \;\; \forall z \in \mathcal{Z} \setminus \mathcal{Z}_{\bar{a}}, \tag{19}$$

$$\sum_{k\in\mathcal{K}_z^+} q_{k,z} = \Bigg\{ -r_z + \sum_{k\in\mathcal{K}_z^-} q_{k,z} + \sum_{l\in\mathcal{L}_z^{out}} t_l - \sum_{l\in\mathcal{L}_z^{in}} t_l \Bigg\}, \;\; \forall z \in \mathcal{Z}_{\bar{a}}, \tag{20}$$

$$t_l^{\min} \le t^l \le t_l^{\max}. \tag{21}$$

Inequality constraints (18) express the operating bounds for the flexibility resources within zones not aggregated into a WcRSF. Equality (19) is the power balance constraint for such zones, with parameter r_z denoting the random zonal imbalance and variable t_l corresponding to the transport outflows and inflows through the links of this zone. Similarly, equality (20) states the zonal power balance constraint for zones within control area $\bar{a}$ and aggregated into a WcRSF. Finally, inequalities (21) enforce the ATC constraints of the zonal market.

REFERENCES

[1] "mFRR-platform accession roadmap V7," ENTSO-e, Apr. 2023. [Online]. Available: https://www.entsoe.eu/network{_}codes/eb/mari/

[2] M. Håberg, H. Bood, and G. Doorman, "Preventing internal congestion in an integrated European balancing activation optimization," *Energies*, vol. 12, no. 3, p. 490, 2019.

[3] C. Guntermann, N. Wahl Gunderson, E. Lindeberg, and M. Håberg, "Detecting unavailable balancing energy bids due to risk of internal congestions," in *2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe)*, 2018, pp. 1–4.

[4] A. Papavasiliou, M. Bjørndal, G. Doorman, and N. Stevens, "Hierarchical balancing in zonal markets," in *2020 17th International Conference on the European Energy Market (EEM)*. IEEE, 2020, pp. 1–6.

[5] A. Papavasiliou, G. Doorman, M. Bjørndal, Y. Langer, G. Leclercq, and P. Crucifix, “Interconnection of Norway to European balancing platforms using hierarchical balancing,” in *2022 18th International Conference on the European Energy Market (EEM)*, 2022, pp. 1–7.

[6] A. Ehrenmann and Y. Smeers, “Inefficiencies in European congestion management proposals,” *Utilities Policy*, vol. 13, no. 2, pp. 135–152, 2005, electricity Transmission.

[7] S. Pineda and J. M. Morales, “Solving linear bilevel problems using big-Ms: Not all that glitters is gold,” *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 2469–2471, 2019.

[8] J. Bezanson, A. Edelman, S. Karpinski, and V. Shah, “Julia: A fresh approach to numerical computing,” *SIAM Review*, vol. 59, no. 1, pp. 65–98, 2017. [Online]. Available: https://doi.org/10.1137/141000671

[9] I. Dunning, J. Huchette, and M. Lubin, “JuMP: A modeling language for mathematical optimization,” *SIAM Review*, vol. 59, no. 2, pp. 295–320, 2017. [Online]. Available: https://doi.org/10.1137/15M1020575

[10] C. Coffrin, R. Bent, K. Sundar, Y. Ng, and M. Lubin, “PowerModels.jl: An open-source framework for exploring power formulations,” in *2018 Power Systems Computation Conference (PSCC)*, June 2018.

[11] B. Legat, R. Deits, M. Forets, D. Oyama, F. Pacaud, and E. Saba, “Juliapolyhedra/cddlib.jl,” May 2019. [Online]. Available: https://doi.org/10.5281/zenodo.1214581

[12] B. Legat, “Polyhedral computation,” in *JuliaCon*, Jul. 2023. [Online]. Available: https://pretalx.com/juliacon2023/talk/JP3SPX/

[13] Cplex, IBM ILOG, “V12. 1: User’s manual for CPLEX,” *International Business Machines Corporation*, vol. 46, no. 53, p. 157, 2009.