Delay Margin Comparisons for Power Systems with Constant and Time-varying Delays

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***Abstract*—The paper shows that, for certain classes of power system models, if time-varying delays are replaced with their average value, the small-signal stability analysis returns conservative results, while showing a lower computational burden. The paper first compares, through an exact analytical approach, the delay margin of a second-order electromechanical model with inclusion of constant, square-wave, and Gamma distributed delays. Since the analytical approach is not viable for realistic power system models, the paper also develops a novel general method to calculate the eigenvalues for systems with time-varying stochastic delays that cannot be described by an analytical probability distribution function. These kind of delays are relevant for the study of wide-area measurement systems (WAMSs). The IEEE 14-bus system serves to compare the stability margin and the critical damping ratio of a constant and two WAMS delay models solved with the proposed numerical method.**

***Index Terms*—Delay Differential Algebraic Equations (DDAE), time-delay system, delay margin, small-signal stability, Wide-Area Measurement System (WAMS).**

I. Introduction

A. Motivation

The Quenching Phenomenon (QP) in time-delay systems describes the situation where a system that is stable (unstable) with a constant delay becomes unstable (stable) if the constant delay is replaced by a time-varying delay with the same average value as the constant delay [1]. The QP implies that constant and time-varying delays have different delay margins, where *delay margin* is defined as the maximum magnitude of a delay that does not make the system unstable. The existence of the QP justifies the need of precise delay models to study the stability analysis of power systems with inclusion of delays. However, the stability analysis involving time-variant delays shows a significant computational burden and hard-to-solve numerical issues [2].

This paper deals with delay systems for which the delay margin when considering constant delays is always smaller than with time-varying delays that have the same average values. For such systems, a constant delay leads to conservative results of the stability assessment. Moreover, the computational burden of the stability analysis is much lower than that required for studying system with time-varying delays. This paper aims at discussing the mathematical conditions for the existence of such delay systems and show that certain classes of power system models fall in this category.

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B. Literature Review

The delay margin of a time-delay system can be computed through time-domain or frequency-domain stability analysis methods. Time-domain methods include Lyapunov functional approaches, in particular the Lyapunov-Krasovskii Functional (LKF) [3]. Frequency-domain methods mostly consist in techniques to find a finite set of approximated roots of transcendental characteristic equations [4]–[6].

The LKF method has been utilized to study the delay margin of power systems [7]–[9]. The LKF method is a sufficient but not necessary criterion, and is generally over-conservative. The extent of its conservativeness is system and delay model dependent [10], [11]. The LKF method, therefore, is not suitable to compare the delay margin for different types of delay. On the other hand, the eigenvalue analysis is a sufficient and necessary stability criterion and ensures a fair comparison [12]. The eigenvalue analysis method has proved to be accurate and efficient to evaluate the small-signal stability of power systems [13]–[15]. The QP has been observed in power systems through a frequency-domain method [16].

References [17] and [18] show that time delays that range from a few tens to a few hundreds of milliseconds can deteriorate the damping mode and even lead the power systems to collapse. The delays introduced by Wide-area Measurement Systems (WAMSs) distribute within 50 – 700 ms and, therefore, can affect the stability of power systems [19]. WAMS delays are the results of a series of processes along with the data communication from the measurement device to the controller/monitoring center, including long-distance data delivery, data packet dropout, noise, communication network congestion [7], [20], [21]. They are necessarily time-variant.

Reference [2] proposes a detailed time-varying WAMS delay model and the eigenvalue analysis method to evaluate the impact of this time-varying delay on the stability of power systems. This method is based on an iterative Newton technique that allows determining the critical eigenvalues. This method has two limitations: (i) the analytic Probability Distribution Function (PDF) of the time-varying delay is required; and (ii) the accuracy of this method relies on the initial guess utilized to start the Newton method. This paper develops a generalized method that avoids these issues and, thus, enables the calculation of the eigenvalue for any system with time-varying delays.

C. Contribution

The contributions of the paper are as follows.

- An analytic method to evaluate the delay margin of a one-machine infinite-bus model with inclusion of both constant and time-varying delays.
- A generalized small-signal stability analysis approach for arbitrary-size power systems with generic time-varying delays.
- A WAMS delay model with more realistic assumptions than the one developed so far in the literature.
- An study on the delay margin and the optimal Power System Stabilizer (PSS) gain of a power system with a WAMS-based PSS using different WAMS delay models through the proposed small-signal stability analysis approach.

D. Organization

The paper is organized as follows. Section II reviews the existing propositions and techniques to solve the stability analysis of time-delay systems. Section III compares the delay margin of a one-machine infinite-bus model with constant, time-varying and distributed delays. Section IV proposes a generalized eigenvalue analysis method that can solve the small-signal stability of the power system with any time-varying delays, and validates the method through an illustrative example. In Section V, two WAMS delay models are discussed, and the IEEE 14-bus system is utilized to compare the effect of these two models and the constant delay on the stability of a power system. Finally, Section VI outlines the most relevant results of the paper and draws conclusions.

II. Technical Background

The transient behavior of power systems with inclusion of time delays can be formulated as a set of nonlinear Delay Differential Algebraic Equations (DDAEs) [18]:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) \,, \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ are the state variables and algebraic variables, τ is the time delay.

Linearizing the system (1) at a given operating point, one can deduce the small-signal model of the power system in the form of linear Delay Differential Equations (DDEs):

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A_0}\boldsymbol{x}(t) + \sum_{i=1}^{i_m} \boldsymbol{A}_i \boldsymbol{x}(t-\tau_i), \tag{2}$$

where $\boldsymbol{A}_0$ is the state matrix, $\boldsymbol{A}_i$ is the delay matrix, and $i_m \in \mathbb{R}^+$ is the number of the delays. The expression of these matrices can be found in [18]. In real-world power system, τ_i is time-variant, and therefore, the DDE (2) is a Time-Varying Delay System (TVDS).

In order to simplify the eigenvalue analysis of the TVDS, we can consider the transform techniques shown in Fig. 1 – a TVDS can be transformed into a comparison Distributed Delay System (DDS), and the DDS can be further transformed into a Multiple Constant Delay System (MCDS). The two propositions shown in Fig. 1 are the following.

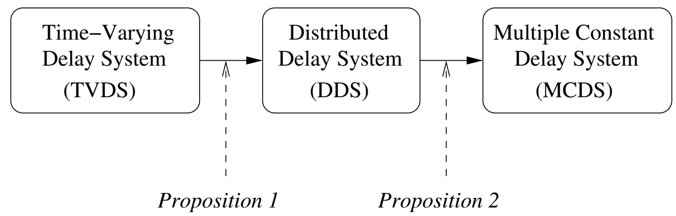

Fig. 1: Transforms of different time-delay systems.

Proposition 1: Consider that in (2), τ_i is time-varying: $\tau_i(t) : \mathbb{R}^+ \to [\tau_{\min}, \tau_{\max}], 0 \le \tau_{\min} < \tau_{\max}$. If each $\tau_i(t)$ changes *fast* enough, the stability of (2) is the same as the following *comparison system*:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\boldsymbol{x}(t) + \sum_{i=1}^{i_m} \boldsymbol{A}_i \int_{\tau_{\min}}^{\tau_{\max}} \pi_i(\xi)\boldsymbol{x}(t-\xi)d\xi \,, \tag{3}$$

where $\pi(\xi)$ is the probability distribution of ξ. The characteristic matrix of the comparison DDS is:

$$\Delta(\lambda) = \lambda\boldsymbol{I} - \boldsymbol{A}_0 - \sum_{i=1}^{i_m} \boldsymbol{A}_i h(\lambda) \,, \tag{4}$$

where

$$h(\lambda) = \int_{\tau_{\min}}^{\tau_{\max}} e^{-\lambda\xi}\pi(\xi)d\xi \,. \tag{5}$$

Remark 1: Proposition 1 is deduced in [2]. Reference [2] provides a two-step eigenvalue analysis method of (3): (i) evaluating the initial guesses for the critical eigenvalues, (ii) solving (4) through Newton iterations [14]. Note this method requires an analyzable PDF of the delay to construct the characteristic equation for Newton iterations, which limits the application of this method.

Proposition 2. The DDS:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\boldsymbol{x}(t) + \boldsymbol{A}_1 \int_{\tau_{\min}}^{\tau_{\max}} \pi(\xi)\boldsymbol{x}(t-\xi)d\xi \,, \tag{6}$$

has the same eigenvalues as the comparison system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\boldsymbol{x}(t) + \boldsymbol{A}_1 \mathfrak{K}\,\mathfrak{h} \lim_{z_m\to\infty} \sum_{z=0}^{z_m} \pi(\Xi_z)\boldsymbol{x}(t-\Xi_z) \,, \tag{7}$$

where

$$\mathfrak{h} = \frac{\tau_{\max} - \tau_{\min}}{z_m} \,,$$
$$\Xi_z = \tau_{\min} + z\,\mathfrak{h} \,,$$

$z_m \in \mathbb{R}^+$ is the number of comparison constant delays, and $\mathfrak{K}$ is a weight parameter decided by the interpretation method.

Remark 2: Proposition 2 discusses only the single delay case. The interested reader can find details on the extension to the multiple delay case in [16]. In numerical implementations,

z_m has to be truncated to a finite number without affecting the convergence of the critical eigenvalues from (7) to (6). Moreover, for a finite z_m, the eigenvalues of this MCDS can be solved through the Chebyshev discretization techniques described in [18], [14] and [15].

A numerical integration approximation approaches can be utilized to minimize the value of z_m. With an interpolation method, equation (7) can be reformulated as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\, \boldsymbol{x}(t) + \boldsymbol{A}_1 \sum_{z=1}^{z_m} \mathfrak{p}_k \, \frac{\pi(\Xi_z)}{z_m} \boldsymbol{x}(t - \Xi_z) \ , \quad (8)$$

where $\mathfrak{p}_z = \mathfrak{K}\mathfrak{h}\pi(\Xi_z)$. The optimal value of z_m depends on the selected interpolation method. The most common interpolation methods are quadratic and cubic [22]. The cubic interpolation is utilized in the remainder of the paper.

III. Stability Margins of Different Delays

This section aims at studying the delay margin of a second-order simplified power system with constant and time-varying delays through an analytical technique.

A. Simplified Power System Model

Let us consider the well-known simplified per-unit electromechanical model of a synchronous machine [23] connected to an infinite bus:

$$\begin{aligned} \Omega_b^{-1} \dot{\delta} &= \omega - 1 \ , \\ 2H\dot{\omega} &= p_m - p_e(\delta, e'_q) \ , \end{aligned} \quad (9)$$

where ω is the rotor speed; Ω_b is the reference angular speed in rad/s; H is the machine inertia constant; p_m is the mechanical power; and the electromagnetic power p_e is a function of the rotor angle position δ and the machine internal emf e'_q.

Assuming $p_e \propto e'_q \sin(\delta)$ and that the inclusion of a PSS makes $e'_q \propto \dot{\delta}$ in (9), reference [17] deduces the characteristic equation of the linearized electromechanical model as follows:

$$\lambda^2 + A\lambda e^{-\tau\lambda} + K = 0 \ , \quad (10)$$

where A is a function of the PSS control gain; $K = \partial p_e / \partial \delta$ evaluated at the equilibrium point at which (9) is linearized; and τ is the delay of the signal ω that is used as the input of the PSS. We assume that the system is stable for $\tau = 0$. In this case, the characteristic roots of (10) are:

$$\lambda = \frac{-A \pm \sqrt{A^2 - 4K}}{2} \ .$$

The system is asymptotically stable if and only if $\Re(\lambda) < 0$. Therefore, to ensure that the system is stable without delay, $A, K > 0$ must hold.

B. Constant Delay

If the delay τ_c is constant, at the delay margin $\tau_c^{\rm m}$, the characteristic equation (10) has a pair of pure imaginary roots $\lambda_{\rm m} = \pm j\beta, \ \beta \in \mathbb{R}^+$. Substituting $\lambda = j\beta$ into (10), and considering that $e^{-j\tau\beta} = \cos(-\tau\beta) + j\sin(-\tau\beta)$, one obtains:

$$0 = -\beta^2 + A\beta\sin(\tau_c^{\rm m}\beta) + K \ , \quad (11)$$

$$0 = jA\beta\cos(\tau_c^{\rm m}\beta) \ . \quad (12)$$

The general solution of (12) is $\tau_c^{\rm m} = \frac{r\pi}{2\beta}$, where $r = 1, 3, 5 \ldots$ Since the delay margin focuses on the smallest delay that fulfills the pure imaginary eigenvalues, one can limit the analysis only to the case $r = 1$.

Then, substituting $\beta = \frac{\pi}{2\tau_c^{\rm m}}$ into (11), one has:

$$-(\frac{\pi}{2\tau_c^{\rm m}})^2 + A\frac{\pi}{2\tau_c^{\rm m}} + K = 0 \ , \quad (13)$$

from where one can deduce the delay margin as:

$$\tau_c^{\rm m} = \frac{\pi}{A + \sqrt{A^2 + 4K}} \ . \quad (14)$$

C. Square-Wave Delay

Let us consider that (10) includes a fast time-varying delay $\tau_v(t)$ and that the delay margin is $\bar{\tau}_v(t) = \tau_v^{\rm m}$. Since a fast time-varying delay can be transformed into multiple constant delays (see Section II), one can deduce the following set of equations:

$$\begin{aligned} -\beta^2 + A\beta \sum_{i=1}^{i_m} \mathfrak{p}_i \sin(\tau_i\beta) + K = 0 \ , \\ jA\beta \sum_{i=1}^{i_m} \mathfrak{p}_i \cos(\tau_i\beta) = 0 \ , \end{aligned} \quad (15)$$

where $\sum_{i=1}^{i_m} \mathfrak{p}_i = 1$ holds and the delays $\tau_1 < \tau_2 < \cdots < \tau_{i_m}$ form an arithmetic series and satisfy the condition $\frac{1}{i_m}\sum_{i=1}^{i_m} \tau_i = \tau_v^{\rm m}$.

Let us consider the simplest case of (15), namely:

$$i_m = 2 \ , \ \text{and} \ \mathfrak{p}_1 = \mathfrak{p}_2 = \frac{1}{2} \ ,$$

which implies a periodic square-wave delay (see [16] for details). In this scenario, assume the two delays are $\tau_1 = \tau_v^{\rm m} - \kappa$ and $\tau_2 = \tau_v^{\rm m} + \kappa$. Since the delays are positive, it must hold $\tau_v^{\rm m} > \kappa > 0$. According to (15), one has:

$$\begin{aligned} -\beta^2 + A\beta\frac{1}{2}\big(\sin(\beta(\tau_v^{\rm m} - \kappa)) + \sin(\beta(\tau_v^{\rm m} + \kappa))\big) + K = 0 \ , \\ jA\beta\frac{1}{2}\big(\cos(\beta(\tau_v^{\rm m} - \kappa)) + \cos(\beta(\tau_v^{\rm m} + \kappa))\big) = 0 \ . \end{aligned} \quad (16)$$

Similar to the constant delay case, from (16), one can deduce that at the delay margin, $\beta = \frac{\pi}{2\tau_m}$ and, thus, equation (16) can be rewritten as:

$$-\left(\frac{\pi}{2\tau_v^{\rm m}}\right)^2 + A\frac{\pi}{2\tau_v^{\rm m}}\cos\left(\frac{\pi\,\kappa}{2\,\tau_v^{\rm m}}\right) + K = 0 \ . \quad (17)$$

Although the analytic solution of (17) cannot be obtained, by comparing (17) and (13), with $K, A, \tau_v^{\rm m}, \tau_c^{\rm m} > 0$ and $\cos(\frac{\pi\,\kappa}{2\,\tau_v^{\rm m}}) \in (0, 1]$, one can deduce that $\tau_v^{\rm m} > \tau_c^{\rm m}$ must hold.

D. Gamma Distributed Delay

Let us consider a Gamma distributed delay system:

$$\lambda^2 + A\lambda \int_0^\infty \frac{\xi^{b-1} e^{-\xi/a}}{a^b \Gamma(b)} x(t-\xi) d\xi + K = 0 \ , \tag{18}$$

where a is the scale factor and b is the shape factor.

Proposition 1 indicates that the characteristic equation of (18) is in the form of (4), where $h(\lambda) = (1 + a\lambda)^{-b}$ [2].

The mean value of the Gamma distributed delay is:

$$\bar{\tau}_g = ab \ . \tag{19}$$

Consider specific delays for $b = 2$, then one has:

$$a = \frac{\bar{\tau}_g}{2} \ . \tag{20}$$

At the delay margin $\bar{\tau}_g = \tau_g^{\mathrm{m}}$, it must have an eigenvalue $\lambda = j\beta$. With this condition, one can deduce following equations:

$$0 = -\beta^2 - \frac{2A\beta^2 a}{(1-\beta^2 a^2)^2 + 4\beta^2 a^2} + K \ , \tag{21}$$

$$0 = 1 - \beta^2 a^2 \ . \tag{22}$$

According to (20)-(22), with $A, K, \tau_g^{\mathrm{m}} > 0$, one can deduce:

$$\tau_g^{\mathrm{m}} = \frac{8}{A + \sqrt{A^2 + 16K}} \ . \tag{23}$$

Comparing (23) with (14), assume a function:

$$\phi(A, K) = \frac{A + \sqrt{A^2 + 16K}}{A + \sqrt{A^2 + 4K}} \ . \tag{24}$$

There is $\tau_g^{\mathrm{m}} \geq \tau_c^{\mathrm{m}}$, if and only if:

$$\phi(A, K) \leq \frac{8}{\pi}. \tag{25}$$

Since $A, K > 0$, one has the maximal value of $\phi(A, K)$ when $A = 0$:

$$\phi(A, K)_{\max} = 2 < \frac{8}{\pi}. \tag{26}$$

Thus, the system must hold $\tau_g^{\mathrm{m}} > \tau_c^{\mathrm{m}}$.

E. Remarks

The examples discussed above show the existence of cases for which, if the system is stable with a constant delay, it is also stable for the time-varying delays that have the same average as the constant delay. For these cases, solving the stability analysis with the constant time delay provides a conservative delay margin.

The main limitation of the analytical approach is that it is not general and the stability properties of the system can be deduced only with the full information on the parameters and delay distribution. The complexity of such an analytical approach increases rapidly with the order of the system and the complexity of the delay distribution function. Since a general analytical solution is still missing, we propose below a numerical approach to study the delay margins of large-scale power systems with realistic-modeling delays.

IV. Generalized Eigenvalue Analysis Approach for TVDSs

This section proposes a generalized eigenvalue analysis method for Time-Varying Delay Systems (TVDSs) based on an interpolation approach that computes the critical eigenvalues and improves the one currently available in the literature [2], as follows.

A. Interpolation Approaches

The first step is to obtain the PDF or, if it is not available, the probability distribution, say $\pi(\tau)$, of the time-varying delay $\tau(t)$. According to Propositions 1 and 2, a TVDS can be transformed into a MCDS, if $\pi(\Xi_z)$ is available for each interpolation point Ξ_z. If the analytic PDF of the delay is not available, one can deduce the approximated PDF through the historical data of the delay, which is feasible for real-world power systems since the time stamp of the measurements of WAMS can be recorded with a relatively high sampling rate [21].

Fig. 2 illustrates the histogram of the magnitudes of a Gamma distributed delay with $a = 0.3187$ and $b = 2$. This histogram and the other PDFs deduced on measurement data that are utilized in the remainder of the paper are obtained based on 2000 s time series with 100 Hz sampling rate. The points ι_z are the mid-points of each bin. Using the set of $\iota_z = (\Xi_z, \pi(\Xi_z))$, the original systems with Gamma distributed delay can be transformed into the comparison MCDS and the critical eigenvalues can be calculated with the Chebyshev discretization techniques discussed in [14], [18]. Moreover, if the analytic PDF is available, the obtained eigenvalues can be further corrected with the Newton iterations proposed in [2].

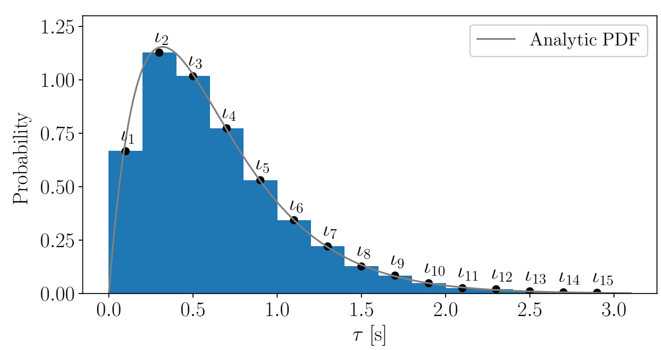

Fig. 2: Histogram of a Gamma distributed delay with $a = 0.3187$ and $b = 2$.

B. Illustrative Example

The simplified power system with Gamma DDS discussed in Section III-D serves to investigate the accuracy of the proposed interpolation method. The eigenvalue analyses discussed in the remainder of the paper are solved using the Python software tool Dome [24].

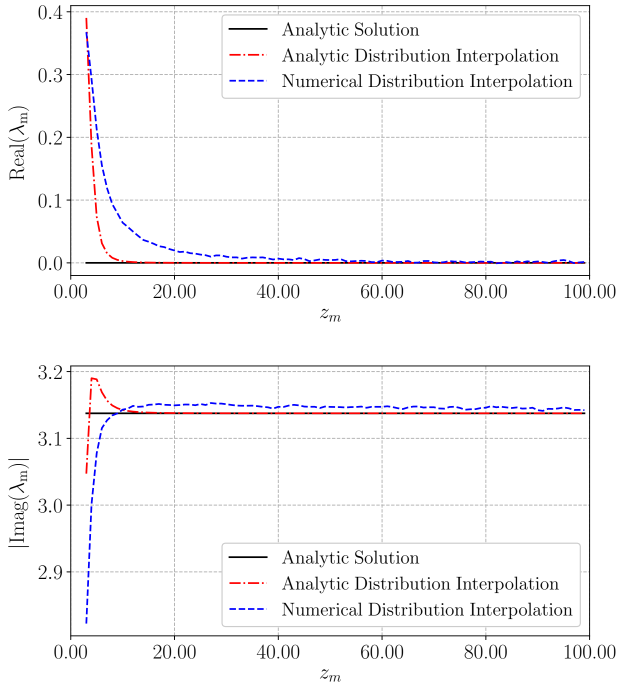

Fig. 3: Real part of the rightmost eigenvalues of (18) with $A = 5$, $K = 2$, $a = 0.3187$ and $b = 2$ as functions of $z_{\rm m}$.

1) Scenario I: Assume the simplified power system (18) with $A = 5$, $K = 2$. According to (23), the delay margin of the system is $\tau_g^{\rm m} = 0.6375$ s. For this delay margin, one has $a = 0.3187$ s, $b = 2$ and $\lambda_{\rm m} = \pm j3.1375$. Fig. 3 shows the variations of the real part and the absolute value of the imaginary part of the rightmost eigenvalues of (18) as functions of z_m. In Fig. 3, "Analytic Solution" indicates the eigenvalue is deduced by (23); "Analytic Distribution Interpolation" indicates the eigenvalue is solved by a comparison MCDS that consisted by the interpolations of the analytic PDF; and "Numerical Distribution Interpolation" considers the interpolation points ι_z.

Fig. 3 shows that, as the number of interpolation points increases, all interpolation methods converge to the exact eigenvalue. The data-driven interpolation method, however, has a slower convergence rate and introduces a small offset in the imaginary part.

2) Scenario II.: To further investigate the accuracy of the proposed method on defining the stability margin, we consider another scenario with $K = 2$, $a = 0.15$ and $z_m = 42$. The real part of the rightmost eigenvalue as a function of A is shown in Fig. 4. Note that the "Analytic Solution" is obtained from applying the Newton method to (18) using as initial guess the values of the eigenvalues obtained with the "Analytic Distribution Interpolation" method.

According to the results of Fig. 4, the two interpolation methods find that the system becomes unstable if $A > 12.5$, which is consistent with the analytic solution. These results indicate that the interpolation methods obtain an accurate estimation of the stability margin for systems with time-varying delays.

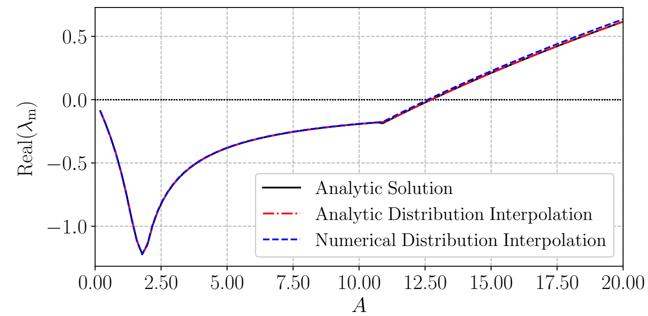

Fig. 4: Real part of the rightmost eigenvalues of (18) with $K = 2$, $a = 0.15$, $b = 2$ and $z_{\rm m} = 42$ as a function of A.

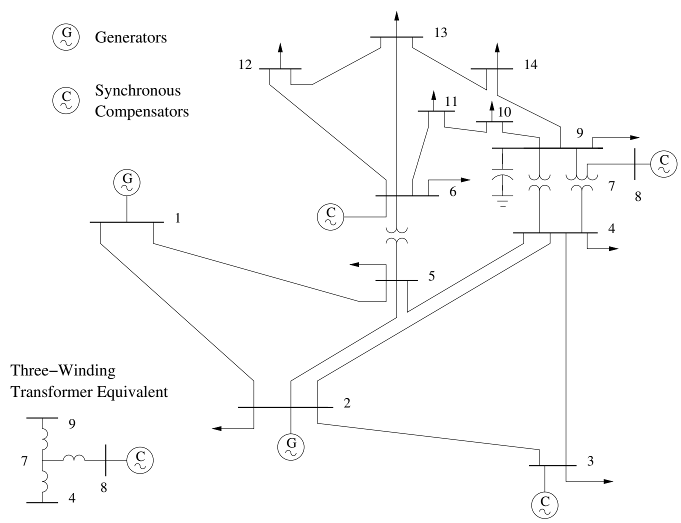

Fig. 5: IEEE 14-bus system.

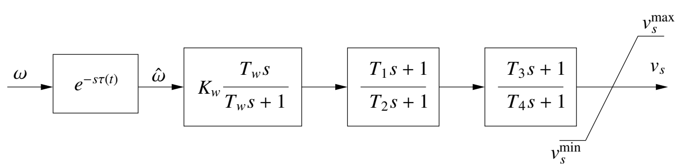

Fig. 6: Control scheme of PSS.

V. Case Study

This section investigates the delay margin for different delay models of the IEEE 14-bus system through the proposed eigenvalue analysis techniques. The topology of the IEEE 14-bus system is shown in Fig. 5. Each synchronous machine is equipped with an Automatic Voltage Regulator (AVR). A PSS implemented at Bus 1 is fed by the rotor speed of the Synchronous Generator 1. The control scheme of the PSS is shown in Fig. 6. The detail parameters of the system be found in [16]. The input signal of the PSS is obtained through a

WAMS, which introduces a delay in the system. Three WAMS delay models are considered below, namely a constant and two time-varying models.

A. Time-varying WAMS Delay Models

The two time-varying models for the WAMS delay are the following:

1) WAMS Delay Model A with Analytical PDF: This WAMS delay model is consisted by three components:

$$\tau_{\mathrm{WAMS,A}}(t) = \tau_c + \tau_p(t) + \tau_s(t) \ , \tag{27}$$

where τ_c is a constant delay; τ_p is a quasi-period delay that describes the data delivery latency T and the data packet drop at rate p, τ_s is a Gamma distributed delay. τ_s is updated every T seconds and that models the stochastic behavior of the communication system. The trajectory of this time-varying delay with $\tau_c = 50$ ms, $T = 50$ ms, $p = 10\%$, $a = 10$ ms and $b = 2$ and its PDF are shown in Fig. 7.

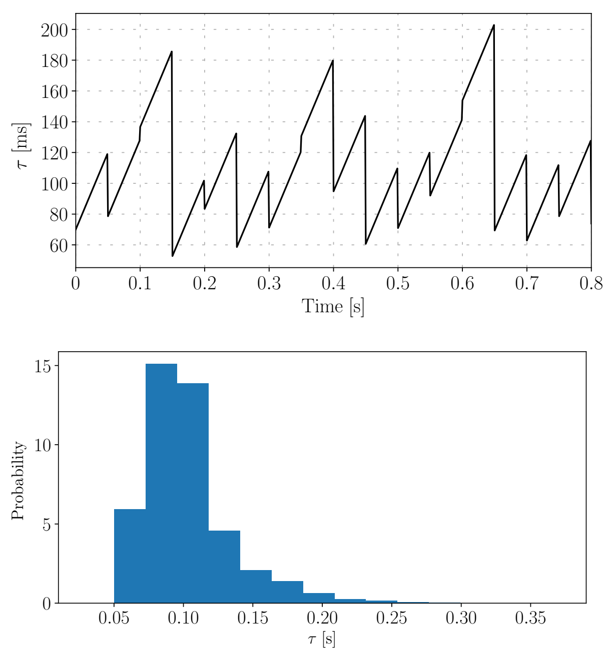

Fig. 7: WAMS Delay Model A: time-varying delay with analytic PDF.

Since the PDF of this delay model can be solved analytically, one can deduce the characteristic equation of the power system with inclusion of this WAMS delay in the form of (18), as follows [2]:

$$h(\lambda) = h_p(\lambda)\left(1 + \frac{a}{1-p}\lambda\right)^{-b} e^{-\tau_c\lambda} \ , \tag{28}$$

where

$$h_p(\lambda) = \frac{p-1}{T\lambda}\left[1 + (p-1)\frac{e^{-\lambda T}}{1 - pe^{-\lambda T}}\right] \ . \tag{29}$$

In the following, (28) is utilized in the Newton Correction included in the eigenvalue computations for the scenarios that utilizes the WAMS delay model A.

2) WAMS Delay Model B with Non-Analytical Distribution: This model has the same parameters as Model A, but different formulation:

$$\tau_{\mathrm{WAMS,B}}(t) = \tau_c + \hat{\tau}_p(t) \ , \tag{30}$$

where the period of the new quasi-periodic delay $\hat{\tau}_p$ is:

$$\hat{T} = T + \tau_s(t) \ . \tag{31}$$

This model better describes the stochastic effect for the digitalized communication processes as discussed in [25], whereas the PDF of this model is too complicated to deduced analytically. In the following eigenvalue analysis, only the data-driven interpolation method is utilized for the scenarios utilizing this delay model. The trajectory and the probability distribution of this delay model with the same parameters as the WAMS delay shown in Fig. 7 are shown in Fig. 8.

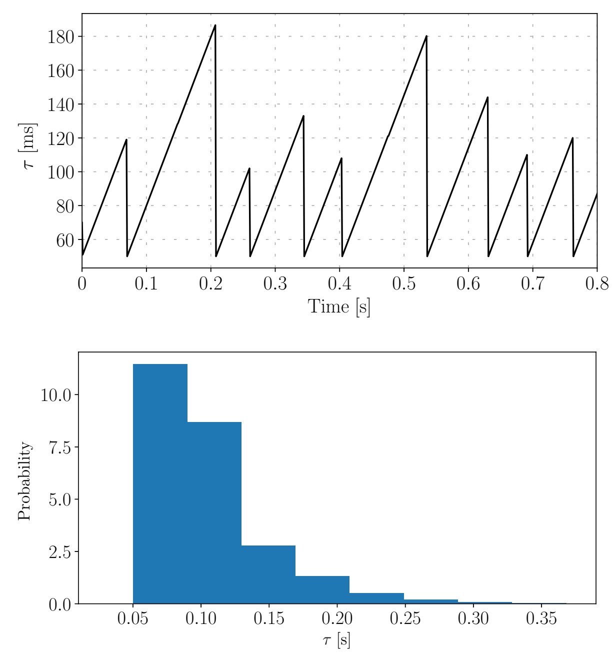

Fig. 8: WAMS Delay Model B: time-varying delay with non-analytic PDF.

B. Delay Margin

This section evaluates the delay margin of the IEEE 14-bus system for different values of the PSS gain K_w and three WAMS delay models. For all scenarios, $p = 10\%$, $b = 2$, $\tau_c = 50\mathfrak{c}$ ms, $T = 50\,\mathfrak{c}$ ms and $a = 10\,\mathfrak{c}$ ms, where $\mathfrak{c} \in \mathbb{R}^+$. The delay margins for the three WAMS delay models as functions of K_w are shown in Fig. 9.

Fig. 9 shows that, independently from the delay model, the delay margin decreases as K_w increases. This implies that the system becomes more sensitive to delays with higher K_w. Fig. 9 also indicates that the impact of the different delay models on the stability of the power system is similar if the delays are small.

Compared with the time-varying delay models, the constant delay always leads to smaller delay margin that the time-varying models. The WAMS delay model B, which is based on the most realistic assumptions, presents the most conservative delay margins. This result implies that if the power system is stable with a constant delay, it is also stable with the time-varying WAMS delays that have the mean value equal to the constant delay.

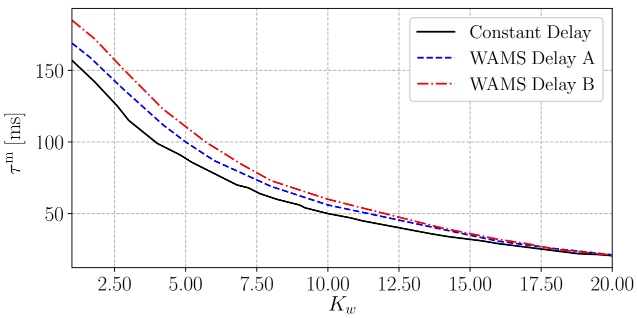

Fig. 9: Delay Margin of the IEEE 14-bus system as a function of K_w.

C. Optimal Control Gain

This section studies the optimal PSS gain of the system for different delay models. With this aim, we consider 300 tests with $K_w \in [0, 6]$. The case of 100 ms constant delay is compared with WAMS delay models A and B with $\mathfrak{c} = 0.998$ and $\mathfrak{c} = 1.02$, respectively, which make their mean values equal to 100 ms. Fig. 10 shows the smallest damping ratio ζ of the power system for different delay models as functions of K_w. Note that, in Fig. 10, if the system is unstable, the damping ratio is shown as 0.

Fig. 10 shows that the damping ratio of the 14-bus system obtained with the constant delay is always the lowest among the three delay models; while all the delay models capture the tend: with the increase of K_w from 0, the damping ratio ζ first increases until reaching the maximal value, which implies the optimal control gain, and then decreases. Each delay model shows different optimal K_w and ζ. For constant delay, the optimal K_w is 1.4, and the corresponding ζ is 4.14%; for the WAMS delay model A and B, the optimal K_w are 1.6 and 1.8, the maximum ζ are 5.2% and 5.5%, respectively. These results indicate that a less precise delay model leads to determine a suboptimal control gain. However, at least, in this case, the differences between the three optimal gains are relatively small.

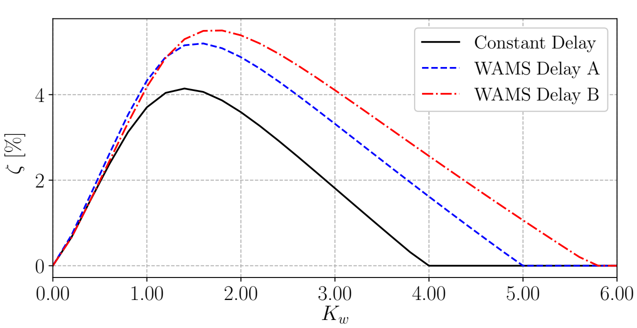

Fig. 10: Damping ratio ζ as a function of PSS gain K_w.

VI. Conclusions

This paper compares the impact of a variety of time-varying delay models on power system stability through frequency-domain analysis. A simplified second-order electromechanical model and the IEEE 14-bus system are utilized to compare such delay models. The delay margin of the electromechanical model can be determined analytically, whereas the study of larger systems require a general numerical approaches. With this aim, the paper develops a interpolation method that can accommodate both analytical and numerical probability distributions of the time-varying delays to solve the small-signal stability of large-scale power systems.

The study of a second-order electromechanical model proves the existence of cases for which a sufficient stability condition for a system with a time-varying delay is that the system is stable for the constant delay equal to the mean value of the time-varying delay. The case study shows that also for more detailed systems, such as the IEEE 14-bus system, realistic WAMS delay models lead to more conservative delay margin and higher critical damping ratio than the constant delay. This result suggests that it is possible, under certain conditions, to replace time-varying delays with their mean value and obtain a conservative estimation of the delay margin with a significant reduction of the computational burden.

Since the analytical approach is model dependent, at this time, only a numerical technique, such as the one proposed in this paper, can be use to solve complex systems. We are working on generalizing the analytical approach and this will be the focus of our future work on this topic.

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