Methodology to Accelerate the Statistical Assessment Process of Evaluating the Transmission Line Lightning Performance

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***Abstract*—The lightning performance of transmission lines is a criterion of utmost importance to be evaluated in transmission system reliability. In order to accurately estimate the expected number of shutdowns, it is imperative to account for the stochastic nature of the lightning phenomenon. In this sense, the most widely used technique to estimate these shutdowns is the Monte Carlo method (MCM). Although the MCM is a trustful method, it has a huge drawback in terms of computational burden. Therefore, this paper proposes the use of an alternative statistical technique, the Unscented Transform method (UTM). This method achieves an approximate non-linear mapping by employing a set of points (associated with input statistical data), which are deterministically selected and weighted. The set of these points and weights is then used to estimate the lightning overvoltage across insulator strings, which represents a significant reduction in the number of samples evaluated in the process in relation to the application from MCM. In this paper, the MCM and UTM methods are applied and compared in the lightning overvoltage evaluation, which is carried out with the help of the Alternative Transient Program (ATP). From the studies conducted, it was found that the application of UTM in place of MCM allowed for significant computational gains while maintaining satisfactory levels of accuracy in the estimation of indicators.**

***Index Terms*—Lightning performance; Monte Carlo; Transmission lines; Unscented transform.**

I. INTRODUCTION

The main cause of unscheduled shutdowns in the power supply of transmission systems with voltage levels up to around 230 kV is associated with the harmful interaction between lightning and transmission lines [1]. Studies that quantify the lightning-outage rates (due to direct lightning) correspond to a series of procedures that are collectively referred to as “lightning performance of transmission lines”.

Lightning is an unpredictable occurrence characterized by a group of random factors. The prevalent approaches employed for this task are grounded in the Monte Carlo method (MCM)[2], [3], [4]. Generally, MCM is employed to estimate the overvoltage distribution and lightning performance by taking into account various statistical data related to lightning current properties such as peak current, rise time, tail time. By utilizing this overvoltage distribution, it becomes feasible to project the anticipated number of shutdowns. Nonetheless, depending on the characteristics of the problem, the MCM demands substantial computational time, often necessitating several minutes or even hours to complete a single estimation.

To overcome the difficulties imposed by the application of MCM, Coelho et al. proposed the use of the Unscented Transform method (UTM) to evaluate the lightning performance of transmission lines [5]. UTM is a deterministic method used in problems dealing with random variables, and it has been recently used in transient-related studies [6], [7]. It allows obtaining statistical parameters of an output random variable resulting from transformations of input random variables. Promising results in terms of computational processing gains were verified in [5], however, in this work only the lightning overvoltage is estimated, not the lightning performance. It is worth highlighting, in terms of reducing computational time, the use of analytical methods [1]. However, in this paper, the main objective is to use methods that consider the stochasticity of the lightning phenomenon, such as MCM and UTM. Additionally, it is very common to start from deterministic simulations based on some worst-case estimation, which would be significantly faster than even reduction techniques such as UTM. On the other hand, approaches based on MCM and UTM present some advantages, such as the more adequate incorporation of uncertainty, flexibility, and adaptability. In this sense, it is possible to increase the realism and accuracy of a study, making it possible to consider more statistical data as well as power system elements.

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Given the above, as an advancement of the research conducted in [5], this work presents lightning performance studies of transmission lines, considering a probabilistic approach with application and comparison of MCM and UTM methods. There is no work in the literature that uses the UTM, to the best of the authors' knowledge, for estimating transmission line lightning performance. It is worth noting that the models used to represent the other components of the transmission system (phase conductors, shield wires, towers, insulator strings, incidence model, grounding and soil, etc.) are the most up-to-date and recommended in the technical literature.

This paper is organized into four main sections, including the first one with the introduction. Section II presents a brief description of the models used to represent the transmission system and the methodologies proposed. Section III reports the results and additional discussions. Finally, Section IV presents the main conclusions obtained from the studies carried out.

II. METHODOLOGY

The lightning performance is estimated by means of evaluating the insulator string overvoltage. Thus, knowing the statistical distribution of lightning overvoltage across insulator strings is of utmost importance. In this work, two numerical methods are used to estimate lightning overvoltage and performance, the MCM and the UTM. Moreover, the lightning transient is computed with the aid of the Alternative Transients Program (ATP), in both cases (MCM and UTM).

Two random variables are considered in this work to carry out the studies, although both methods allow considering the representation of uncertainty for more input information for the problem: one is associated with the lightning peak current (I_p), and the other is associated with the voltage phase angle at the moment of the lightning striking (θ). The dependence relation of the front time and peak current is considered in the model through its correlation [8].

Additionally, it is important to highlight that lightning performance is a composition of two phenomena [9]: i) flashover - when the lightning strikes directly the phase conductor and it causes an insulator disruption and ii) backflashover - when the lightning strikes the shield wires or tower top and it causes an insulator disruption. Considering the geometric configuration of the transmission line used in this work (see section III), only backflashover rates (BFOR) will be estimated, since the flashover rates are practically zero (effective shielding). The BFOR is the number of expected shutdowns per 100 km of line during the period of one year [1].

In this sense, the following subsections present the transmission system and lightning modeling, in addition to how the MCM and UTM methods are applied to lightning performance studies.

A. Transmission System Modeling

The correct assessment of lightning performance of transmission line is intrinsically related to its appropriate modeling. The electromagnetic transients-type software model of transmission systems, for lightning-related phenomena, has been studied and published by several authors, such as [4], [10]. To calculate backflashover rates, it is necessary to model: i) transmission line conductors (phase and shield wires), ii) the tower, iii) the grounding, and iv) the modeling of insulator strings. Here follows a brief description of each model.

1) Transmission Lines: Both phase conductors and shield wires were represented using the Lines Constants and Cable Constants (LCC) tool (already implemented in ATP) [11]. The J. Marti model [12] was considered to calculate the line parameters. In this model, the electromagnetic coupling between conductors is considered. It is considered three adjacent towers on each side of the main tower (the one that suffers the stroke) and at the end of the last tower, it is considered a 5 km-length-long transmission line.

2) Transmission Towers: For lightning-related phenomenon, transmission towers (lower than 60 meters) can be modeled considering a lossless transmission line, without losing accuracy [13]. Thus, only two parameters of the tower are necessary: i) propagation velocity; and ii) surge impedance. The first parameter, propagation velocity along the towers, depends on the tower structures (cross-arms, horizontal elements and inclined elements), here, it is considered it approximately 80% of the speed of light ($\approx 2.4 \times 10^8$ m/s) for the cases under analysis (due to the existence of trusses). This value is also indicated by [1]. The tower surge impedance, on the other hand, was calculated considering the revised Jordan's Formula [13]. According to [13], it is possible to calculate the surge impedance by considering vertical conductors. The self impedance is obtained by (1). For evaluating the mutual surge impedance of conductors of same height one can use (2).

$$Z_{ii} = 60\left[\ln\frac{4h}{r} - 1\right] \tag{1}$$

$$Z_{ij} = 60\ln\frac{2h + \sqrt{4h^2 + d_{ij}^2}}{d_{ij}^2} + 30\frac{d_{ij}}{h} - 60\sqrt{1 + \frac{d_{ij}^2}{4h^2}} \tag{2}$$

where h is the height of the conductor, r is the conductor radius, d_{ij} is the distance between the center of the i^{th} conductor and the j^{th} conductor, Z_{ii} is the self impedance and Z_{ij} is the mutual impedance between the i^{th} conductor and the j^{th} conductor.

Using (1) and (2), one can write an equivalent representation, considering the whole multiconductor system as a single transmission line with equivalent surge impedance Z_{eq} given by (3).

$$Z_{eq} = \frac{\sum_{j=1}^{n} Z_{1j}}{n} \tag{3}$$

where n is the number of parallel conductors. If slants and crossbar are disregarded in modeling the tower, then only four vertical conductors are used to represent the tower ($n = 4$).

3) Grounding System Modeling: The grounding system is one of the main elements to reduce lightning overvoltage across insulator strings. According to [14], for first-return-stroke, impulse grounding impedance shows results very similar to those obtained under the physical representation of electrodes. In this sense, the grounding system modeling used in this paper is based on the impulse grounding impedance concept, i.e., the ratio of the grounding potential rise and the impressed current peaks. This model is represented as a resistance on the ATP.

4) Insulator Strings Modeling: The modeling of insulator strings used in this work is based on the Voltage-Time Curve. However, as it is approached differently in each method, additional details about this modeling are presented in the Subsections II-C and II-D.

B. Lightning Current

According to Oliveira et al. [15], it is possible to obtain a representative current waveform of negative discharges based on the sum of seven Heidler's functions (represented in (4)). Based on [16], Oliveira proposed a set of equations that are capable of reconstructing the waveforms of typical lightning current measured at Morro do Cachimbo Station, by knowing only the current first peak (I_{p1}), the time interval between the 10% and 90% of current first peak (T_{30}) and the current second peak (I_{p2}). The methodology is based on applying adequate multipliers, i.e., applying the values of the parameters presented in (5), (6), (7), and (8).

$$i(t) = \sum_{k=1}^{7} \left(I_{0k}/\eta_k\right) \exp\left(-t/\tau_{2k}\right) \left\{\left(t/\tau_{1k}\right)^{n_k} / \left[1 + \left(t/\tau_{1k}\right)^{n_k}\right]\right\} \tag{4}$$

$$I_0 = \alpha \left[\begin{array}{ccccccc} 6 & 5 & 5 & 8 & 16.5 & 17 & 12\times\delta \end{array}\right] \text{kA} \tag{5}$$

$$\tau_1 = \beta \left[\begin{array}{ccccccc} 3 & 3.5 & 4.8 & 6 & 7 & 70 & 12 \end{array}\right] \mu\text{s} \tag{6}$$

$$\tau_2 = \beta \left[\begin{array}{ccccccc} 76 & 10 & 30 & 26 & 23.2 & 200 & 26 \end{array}\right] \mu\text{s} \tag{7}$$

$$n = \left[\begin{array}{ccccccc} 2 & 3 & 5 & 9 & 30 & 2 & 14 \end{array}\right] \tag{8}$$

where I_{0k}, τ_{1k}, τ_{2k} and n_k are, respectively, the k^{th} element of I_0, τ_1, τ_2, n, $\eta_k = \exp\left[-\left(\tau_{1k}/\tau_{2k}\right)\left(n_k\tau_{2k}/\tau_{1k}\right)^{1/n_k}\right]$, $\alpha = 0.02475 I_{p1}$, $\delta = 3.6537(I_{p2}/I_{p1} - 0.8568)$, and $\beta = 0.3328 T_{30}$.

It is important to highlight that this procedure is valid for the interval:

- $0.2198 \leq \alpha \leq 4.3955$;
- $0.47 \leq \beta \leq 4.22$;
- $0.7 \leq \delta \leq 1.3$.

C. Monte Carlo Method

The MCM is a statistical technique based on the Strong Law of Large Numbers, i.e., it estimates the average of the results considering a large number of trials. When applied to lightning performance of transmission lines, the ATP tool is called for each new sample generated, and both maximum voltage and insulator breakdown are evaluated.

In this case, the evaluation of insulator breakdown is straightforward, being considered in each simulation. In this paper, the Voltage-Time Curve ($v \times t$), as shown in (9), is used [17]. According to [17], if the value of the overvoltage across the insulator string is higher than the $v \times t$, it is deemed a breakdown.

$$v \times t = \left(400 + \frac{710}{t^{0.75}}\right) \times l_s \tag{9}$$

where t is the time in microseconds, and l_s is the insulator string length in meters.

According to [4], it is possible to estimate the BFOR by considering the probability of shutdown occurrence, which is the number of breakdown obtained according to the MCM divided by the total number of simulations ($\frac{N_{break}}{N_{tot}}$). With this in mind, the BFOR can be estimated by applying (10).

$$BFOR = 0.6 \cdot N_g \cdot W_a \cdot \frac{N_{break}}{N_{tot}} (100/1000) \tag{10}$$

where N_g is the ground flash density, and W_a is the lateral width of attraction calculated using (11).

$$W_a = b + 2 \cdot R_a \tag{11}$$

The parameter W_a incorporates the mean equivalent radius of attraction R_a, the formulation of which was proposed by [18] and is obtained using (12).

$$R_a = 14 H^{0.6} \tag{12}$$

In (11), b is the distance between the shielding wires, and according to the tower configuration of this study, b is equal zero. In (12), H is the average height of the most exposed conductor (generally the shielding wire), influenced by the terrain profile. In this study, a flat terrain is considered, according to (13), where Y_g is the height of the shielding wire in the tower, Y_{gms} is the height of the shielding wire in the middle of the spans, and $(Y_g - Y_{gms})$ is the sag of the shielding wire.

$$H = Y_g - \frac{2}{3}\left(Y_g - Y_{gms}\right) \tag{13}$$

D. Unscented Transform Method

The use of UTM for lightning performance of transmission lines is similar to that of MCM, with the difference that a very reduced set of samples is generated using specific equations. To understand the UTM, let $X = [I_p, \theta]$ be the vector of random input variables of the problem under study. This vector

X has mean and covariance in accordance with (14) and (15), respectively.

$$\bar{X} = [\bar{I_p}, \bar{\theta}] \tag{14}$$

$$P_{xx} = \begin{bmatrix} \sigma_{I_p}^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} \tag{15}$$

Suppose $Z = g(X)$ is the model used to compute the maximum overvoltage across the insulator string, the statistical parameters of maximum lightning overvoltages, mean, and standard deviation are estimated using (16) and (17), respectively.

$$\bar{Z} = \sum_{i=1}^{m} w_i \cdot g(S_i) \tag{16}$$

$$\sigma_Z = \sqrt{\sum_{i=1}^{m} w_i \cdot \left(g(S_i) - \bar{Z} \right)^2} \tag{17}$$

where w are the weights, S are the sigma points, and m is the total number of points.

The sigma points and weights are calculated based on the strategy presented in [19]. Equations (18) to (22) define this strategy.

$$S_1 = \bar{X} \tag{18}$$

$$S_i = \bar{X} + \sqrt{(N_{\text{RV}} + \kappa) \cdot P_{\text{xx}}} \quad \text{for } i = 2, \ldots, N_{RV} + 1 \tag{19}$$

$$S_{i+N_{\text{RV}}} = \bar{X} - \sqrt{(N_{\text{RV}} + \kappa) \cdot P_{\text{xx}}} \quad \text{for } i = 2, \ldots, N_{RV} + 1 \tag{20}$$

$$w_1 = \frac{\kappa}{N_{\text{RV}} + \kappa} \tag{21}$$

$$w_i = w_{i+N_{\text{RV}}} = \frac{1}{2 \cdot (N_{\text{RV}} + \kappa)} \quad \text{for } i = 2, \ldots, N_{RV} + 1 \tag{22}$$

where N_{RV} is the number of random variables, which in this work is equal to 2, and $\kappa \in \Re$ is used to reduce prediction errors since it provides a degree of freedom for better adjustment of the moments. In this study, the parameter κ was adjusted empirically and set equal -0.15. More details concerning the development and formulation of the UTM can be found in [19].

Applying the strategy as (18) to (22), the number of pairs of sigma points and weights is given by (23), i.e. $m = 5$.

$$m = (2 \cdot N_{RV}) + 1 \tag{23}$$

For each sample of the UTM, the electromagnetic transient is evaluated, and the maximum overvoltage observed across the insulator string of the 3 phases and the time when it occurs are stored in V_{max} and t_{max}, respectively. To evaluate the lightning performance, it is estimated the value of the maximum insulator string supported overvoltage V_{UT} by verifying the point where the curves containing the (V_{max}) and (t_{max}) cross the $v \times t$ curve. Since the UTM uses only 5 points to estimate the maximum overvoltages, it is necessary to fit a curve that relates the V_{max} and t_{max} points. The curve that best fits the V_{max} and t_{max} relation is found using the exponential regression model presented in (24). Then, the coordinates where the curves defined by (9) and (24) intersect has coordinates (V_{UT}, t_{UT}).

$$f(x) = A \cdot e^{\zeta x} \tag{24}$$

where A and ζ are coefficients to determine.

After determining coordinates (V_{UT}, t_{UT}), it is checked the probability of the maximum overvoltage being greater than V_{UT}. This estimation is done by using the Probability Density Function (PDF) of the maximum overvoltage obtained with the aid of the UTM. This value is the probability of lightning striking the tower top and generating a shutdown. It is also analogous to $\frac{N_{break}}{N_{tot}}$ from MCM, then, to compute BFOR, (10) is applied. It is important to comment that, to reconstruct the PDF the mean and standard deviation estimated by (16) and (17) are used assuming a Lognormal fit for the maximum overvoltage. To illustrate the proposed methodology, the steps are summarized in the flowchart shown in Fig. 1.

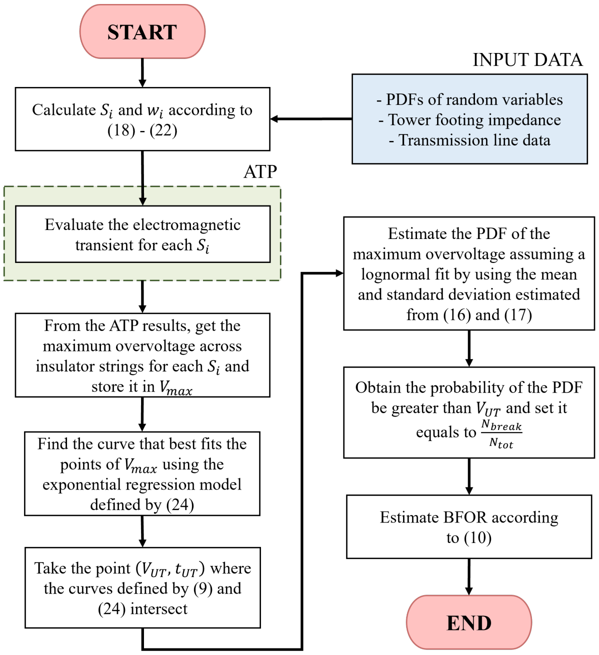

Fig. 1. Flowchart of the proposed methodology: lightning performance using UTM.

It is important to emphasize that the evaluation of the electromagnetic transient carried out by the ATP is the step that consumes the most computational time in the lightning performance calculation. In this sense, the main gain of UTM in computational terms is the estimation of BFOR using a very small number of samples, e.g. 5, while in MCM, in general, the evaluation of thousands of samples is necessary to converge the estimates.

III. RESULTS AND ANALYSES

The base case corresponds to a real 138-kV line in Brazil. Its data are summarized in Table I. The transmission towers used on this line have a typical horizontal silhouette.

TABLE I
SPECIFICATIONS OF THE 138-KV TRANSMISSION LINE

Operating voltage:	138 kV	Height of phase C wire:	18.55 m
Tower Span:	333 m	Sag of phase:	11.54 m
Number of conductors per phase:	1	Sag of shield wire:	8.57 m
Number of shield wires:	1	Horizontal distance between phases:	5.5 m
Phase conductor code:	Penguin	Shield wire code:	5/16” HS
Height of phase A wire:	22.15 m	Height of shield wire:	26.55 m
Height of phase B wire:	20.35 m	Length of insulator string:	1.504 m

In the ATP simulation, the transmission system is represented by a configuration of seven towers as represented in Fig. 2. This configuration includes the tower that is most susceptible to lightning strikes, as well as the three neighbouring towers on each side. Fig. 3 shows the tower silhouette and data. Each individual tower is characterized as a lossless line, with its surge impedance determined using the Jordan modified model [13]. Additionally, these towers exhibit an electromagnetic wave velocity that is 80% of the speed of light in free space, as documented in [1] and [20]. The impedance of each tower segment is represented in Fig. 4.

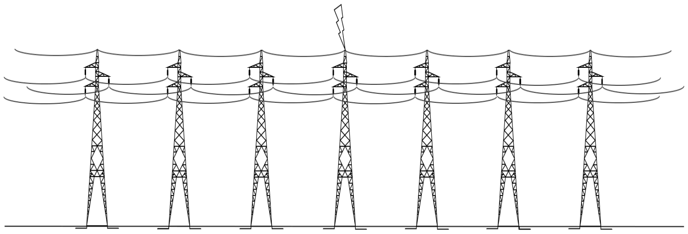

Fig. 2. Base system for the lightning performance assessment.

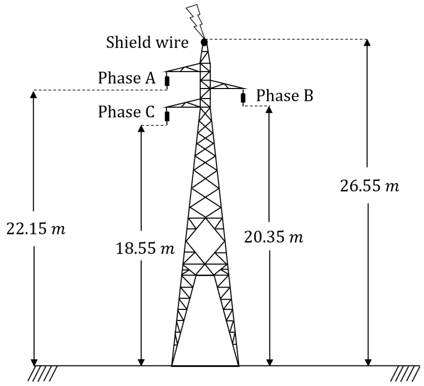

Fig. 3. Tower silhouette.

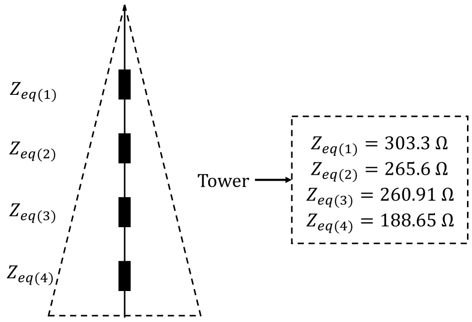

Fig. 4. Tower model.

To assess the reliability of the proposed methodology, tests were conducted with various impulsive impedance values, ranging from 5 to 100 Ω. Both the UTM and MCM tests employ the same ATP model. These computations were executed on a desktop computer running Windows 11 and powered by an Intel Core I7-12700 12th Gen processor, clocked at 2.1 GHz, with 16 GB of RAM.

The percentage deviations between the results obtained via MCM and UTM for mean and standard deviation of the overvoltage are presented in Fig. 5 and 6, respectively. The results in Fig. 5 show that comparing both methods, differences lower than 2% for all impulsive impedance analysed were found. For the standard deviation, shown in Fig. 6, similar results were found. In this case, differences are lower than 5%. Comparing the variations of these two moments in the Z_p range evaluated, while the differences for the mean decrease as Z_p increases, the differences for the standard deviation have different behavior, it increases as Z_p increases. Despite this fact, the proposed methodology used to estimate BFOR was not affected by this variation observed. It is noteworthy that UTM took just 5 samples to estimate the moments, while MCM took 3×10^5 samples for all cases. In this regard, it is important to note that a reduced number of samples evaluated using MCM would be sufficient to converge the estimate of the mean and standard deviation of the maximum overvoltage in the insulator strings. However, to effectively compare the estimated PDF reconstructed using UTM and the estimated PDF of MCM, a high number of samples in MCM is necessary.

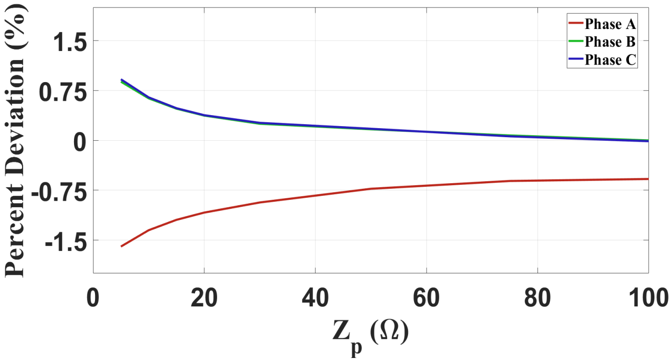

Fig. 5. Percentage difference between the mean of the maximum overvoltage across the insulator strings obtained using the UTM and MCM as a function of the grounding impedance.

To estimate the BFOR it is necessary to calculate the coefficients A and ζ of the exponential regression model as stated in Subsection II-D. The values of these coefficients for

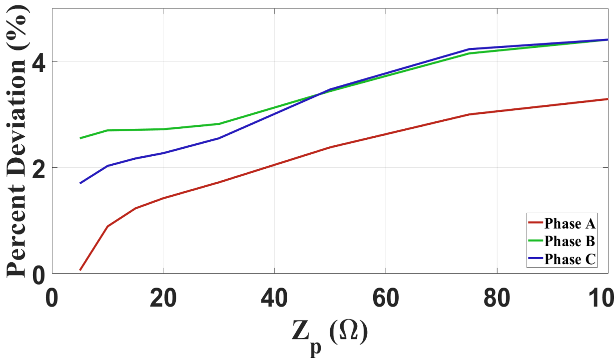

Fig. 6. Percentage difference between the standard deviation of the maximum overvoltage across the insulator strings obtained using the UTM and MCM as a function of the grounding impedance.

each Z_p are presented in Table II and the associated curves to find the coordinates (V_{UT}, t_{UT}) are shown in Fig. 7.

TABLE II
COEFFICIENTS A AND ζ FOR EACH GROUNDING IMPEDANCE, AND THE TIME WHERE THE CURVES INTERSECT

Z_p (Ω)	A	ζ	Time (μs)
5	388094	0.0563	12.1
10	408978	0.0740	9.2
15	430804	0.0867	7.6
20	457771	0.0943	6.7
30	507417	0.1052	5.5
50	603664	0.1154	4.1
75	709480	0.1205	3.2
100	798905	0.1233	2.7

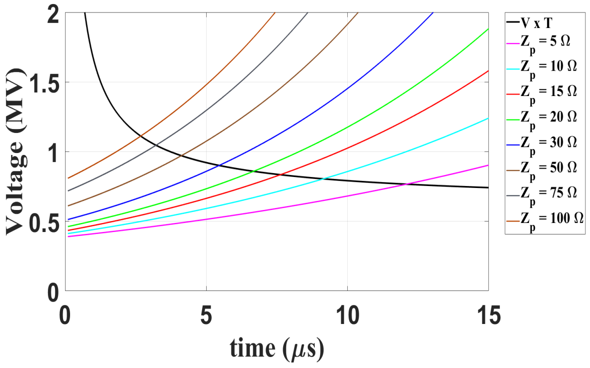

Fig. 7. Regression curves and their intersection with $v \times t$ curve.

The BFOR estimated using MCM and UTM are presented in Table III. According to these results, the percentage differences were lower than 17% for all cases. Considering that the UTM is based on the reconstruction of the PDF with only 5 points, there will be points in which the PDF due to the UTM will slightly diverge from the MCM PDF, leading to this percentage differences. In this case, the worst case scenario for the lightning performance estimation occurred around $Z_p = 10\Omega$. However, it is noteworthy to comment that in most cases the percentage errors were lower than 10%.

TABLE III
BACKFLASHOVER RATES ESTIMATED WITH MCM AND UT

Z_p (Ω)	MCM (fl/yr/100km)	UTM (fl/yr/100km)	Differences (%)
5	0.6588	0.6606	0.28
10	6.1017	7.3133	16.57
15	14.2870	16.0258	10.85
20	21.7016	23.2742	6.76
30	31.5380	31.9044	1.15
50	38.8990	38.2133	-1.79
75	41.1435	40.4965	-1.60
100	41.7798	41.3055	-1.15

Some important remarks should be made on the computational effort. Both methodologies provide similar results in terms of mean and standard deviation of the overvoltage, as well as the BFOR rates. But, while the MCM requires 1230 min to run 3×10^5 samples, the proposed methodology only requires 2.18 s to run 5 samples. In this time required by UTM is already included the necessary time to calculate the weights and sigma points, which corresponds to 0.95 s.

IV. CONCLUSIONS

This paper presents an alternative approach to estimating the lightning performance of transmission lines. The proposed methodology is based on the Unscented Transform method (UTM) combined with the Alternative Transient Program (ATP), instead of considering the traditional approach which is based on the Monte Carlo method (MCM). Two random variables were considered in the problem to illustrate the potential of the proposed methodology, the peak current and phase voltage.

According to the results, considering the MCM as a reference, the percentage differences obtained for the UTM, for the range of impulsive impedance evaluated, are lower than 5% for overvoltage mean and standard deviation. For the lightning performance, the percentage differences are lower than 17% with a speedup of around 3×10^4 times.

Although the results in this paper are promising, in future works it is expected to reduce the maximum differences by applying techniques that consider more sigma points.

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