Linear Optimal Power Flow Model for Modern Distribution Systems: Management of Normally Closed Loop Grids and On-Load Tap Changers

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***Abstract*—Modern distribution systems face new challenges due to the rise of small-scale renewable energy resources and the electrification of energy demand. Additionally, the complexity of network management increases with the exchange of flexibility services between distribution and higher voltage levels. The system operator's role involves optimizing the planning and operation of the distribution network. In current scenarios, this includes managing flexibility resources (such as generators, demand response, and on-load tap changers) and implementing new operating strategies (such as closed-loop topologies and dynamic reconfiguration). This paper proposes a reformulation of one of the most used linear approximations of the Optimal Power Flow model for distribution networks. The novelty of this reformulation lies in integrating equations to handle tap-changing transformers with closed-loop/meshed network operation, while preserving the original linear formulation and computational tractability.**

***Index Terms*—Closed loop networks, DistFlow, Linear Power Flow, OLTC, Optimal Power Flow.**

NOMENCLATURE

Indices and sets

Symbol	*Definition*
$i, j \in N$	Buses
$r \in R \subseteq N$	Reference buses
$(i, j) \in E \subseteq N \times N$	Branches
E_i, E'_i	Branches whose first (respectively, second) bus is bus i
$g \in G, G_i$	Generators, generators at bus i
$l \in L, L_i$	Loads, loads at bus i

Decision variables

Symbol	*Definition*
W_i	Squared voltage magnitude at bus i
$\Delta W_{ij}^{+}, \Delta W_{ij}^{-}$	Squared voltage magnitude variation induced by the OLTC of branch ij, positive and negative
θ_i	Voltage phase angle at bus i
P_{ij}, Q_{ij}	Active and reactive power flow in branch ij
P_l^{load}, Q_l^{load}	Active and reactive power absorption of load l
P_l^{curt}	Active power curtailment of load l
P_g^{gen}	Active power produced by generator g
Q_g^{gen+}, Q_g^{gen-}	Reactive power of generator g, positive and negative

Parameters

Symbol	*Domain*	*Definition*
V_{ij}^{nom}	$\mathbb{R}_{>0}$	Rated voltage of branch ij
$V_{ij}^{min}, V_{ij}^{max}$	$\mathbb{R}_{>0}$	Minimum/maximum voltage magnitude at bus i
τ_{ij}^{nom}	$\mathbb{R}_{>0}$	Nominal regulation of OLTC transformer of branch ij (at center tap)
$\tau_{ij}^{min}, \tau_{ij}^{max}$	$\mathbb{R}_{>0}$	Minimum/maximum regulation of OLTC transformer of branch ij
R_{ij}	$\mathbb{R}$	Resistance of branch ij
X_{ij}	$\mathbb{R}$	Reactance of branch ij
δ_{ij}	$(-\pi, +\pi)$	Phase shift of the OLTC transformer of branch ij
S_{ij}^{max}	$\mathbb{R}_{\geq 0}$	Maximum apparent power flowing in branch ij
P_l^{ref}	$\mathbb{R}_{\geq 0}$	Reference active power absorption of load l
φ_l	$(-\pi/2, +\pi/2)$	Power factor angle of load l
P_g^{min}, P_g^{max}	$\mathbb{R}$	Minimum/maximum active power produced by generator g
Q_g^{min}	$\mathbb{R}_{\leq 0}$	Minimum reactive power exchanged by generator g
Q_g^{max}	$\mathbb{R}_{\geq 0}$	Maximum reactive power exchanged by generator g
c_{ij}^{OLTC}	$\mathbb{R}_{\geq 0}$	Cost for using regulation of OLTC transformer of branch ij
c_l^{VOLL}	$\mathbb{R}$	Curtailment cost of load l
c_g^{Pgen}	$\mathbb{R}$	Cost for active power production of generator g
c_g^{Qgen}	$\mathbb{R}_{\geq 0}$	Cost for reactive power exchange of generator g

I. INTRODUCTION

Both the planning and operation of power systems require the adoption of optimization algorithms aimed at guaranteeing the minimization of operational and capital expenditures to achieve acceptable levels of adequacy, reliability, and quality of supply. More and more challenges need to be faced by electricity network operators, especially because of the increase in distributed generation and the electrification of energy demand (vehicles, heating systems, etc.). In particular, the lower voltage levels are the ones more affected by the energy transition and, for this reason, distribution system operators are called to face new regulated roles which have the objective of optimizing the planning and operation of their infrastructure, with the active participation of local resources flexibility [1].

To achieve any specific optimal objective of the power system, which implies the consideration of the physics behind its components, network operators and the scientific community rely on the well-known Optimal Power Flow (OPF) equations. The conventional formulation consists of a set of nonlinear constraints which, despite the recent progress of computer and mathematical sciences, still lead to implementation issues [2], especially when the optimization problem considers integer variables. For this reason, significant research effort and literature have been dedicated to the definition of approximations and relaxations of the OPF model, with the common aim of obtaining its numerical tractability without significantly weakening the accuracy of results [3], [4]. One of the most promising OPF linear approximations is represented by the *Simplified DistFlow*. Its first formulation dates back to 1989 [5] and it is derived from the (nonlinear) *DistFlow* model, which has been demonstrated to be a good compromise between simplicity and accuracy when: a) the network is operated radially; and b) electrical lines are characterized by high R/X ratios. Both these conditions are met in conventional distribution grids, leading to its practicality for medium and low voltage systems optimization.

Nevertheless, the current challenges of distribution network management are driving system operators to consider the closed-loop configuration [6]. In this case, the interconnection of neighboring feeders by means of switches defines a non-radial (meshed) topology, and simulations/real field tests demonstrate its benefits in terms of reliability and maintenance costs [6], system restoration improvement [7], hosting capacity increase [8], and energy losses reduction [9]. Nevertheless, closed-loop as normal operation practice is still an engineering challenge, since the definition of the optimal topology is not always straightforward [9][10], and optimization tools capable of correctly modelling this operation mode are mandatory. In fact, literature also demonstrates that the closed-loop configuration, in certain circumstances, can lead to lower performance with respect to the radial operation [8], [11]. Another reason to use optimization (i.e. OPF) for the planning and operation of closed-loop distribution systems is represented by the possibility of experiencing violations of voltage/loading constraints [11]. Therefore, in addition to the optimal network configuration, OPF should consider the flexibility of other network elements such as tap-changing transformers [8], [11], reactive power control [8], [12], and static compensators [13].

One of the most important assets in a distribution network, which plays a fundamental role in voltage regulation, is represented by the On-Load Tap Changer (OLTC). Its working principle is based on the exclusion/inclusion of distribution transformer windings such that the voltage transformation ratio can be dynamically adjusted without interrupting the power supply. Literature counts numerous local control strategies [14]; however, the increasing complexity of modern power systems often requires the definition of OLTC setpoints in coordination with control actions addressed to other voltage regulating devices (reactive power of distributed generation, static compensators, etc.). According to this, the planning and the operation of OLTC actions need to be integrated into the OPF formulation.

The most rigorous representation of tap-changing transformers introduces nonconvex constraints within the optimization model, which requires a nonlinear mixed-integer solver with all the related complexities [15]. A possible relaxation practice considers the transformer voltage ratio as a continuous variable (i.e., neglecting the OLTC discrete steps), which has been proven to be effective even for large systems [16]. The same nonconvexity issue is experienced when the OLTC model is integrated within linear OPF formulations (*Simplified DistFlow* included). However, thanks to mixed-integer modelling of the tap changer, the linearity of the problem can be preserved [17]. Furthermore, OLTC can be modelled by relaxing the voltage constraints of the reference bus. This case is representative of a common situation of distribution networks, in which the voltage of the substation busbar is optimized thanks to the direct action of the tap-changing transformer [18]. This strategy, which is applicable when only one OLTC is simulated, results in being particularly convenient for OPF engines that do not integrate variable transformers (e.g., MATPOWER [19]).

The analysis highlights the importance of including both closed-loop operation and OLTC in the OPF model for optimizing modern power systems. To achieve this, the paper proposes enhancing the *Simplified DistFlow* model by incorporating additional variables and constraints. This integration allows for the consideration of the physics associated with both aspects, while preserving linearity and avoiding the need for integer variables.

The remainder of the paper is organized as follows. Section II extends the *Simplified DistFlow* power flow model to incorporate closed-loop operation and OLTC. A complete OPF problem based on the proposed power flow model is then built in Section III. In Section IV, the proposed model is validated by experimental comparison with the nonlinear model and its numerical accuracy is analyzed. Finally, Section V summarizes the conclusions of this paper.

II. Extended Linear DistFlow

A. Branch flow model derivation

The *DistFlow* model can be derived from the simplification of power flow equations in various ways. For example, by considering the single-phase equivalent branch scheme depicted in Fig. 1, its mathematical representation can be written as:

$$\overline{V}_i - \overline{V}_j = \left(R_{ij} + \mathbf{j}X_{ij}\right)\overline{I}_{ij} \qquad (1)$$

where $\overline{V}_i$ and $\overline{V}_j$ are the voltage phasors of the terminals *i* and *j* respectively, R_{ij} and X_{ij} are the series resistance and reactance of the branch *ij* (**j** is the imaginary unit), and $\overline{I}_{ij}$ is the phasor of the current flowing from *i* to *j*.

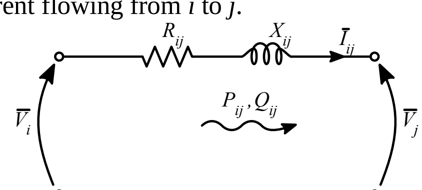

Figure 1. Electrical model of a line.

To express the voltage drop as a function of the branch active and reactive power (P_{ij} and Q_{ij}, respectively), one approach is to multiply the average voltage phasor by the complex conjugate $\underline{I}_{ij}$ of the branch current:

$$P_{ij} + jQ_{ij} = \frac{\overline{V}_i + \overline{V}_j}{2} \underline{I}_{ij} . \quad (2)$$

Substituting (2) into (1) leads to:

$$\overline{V}_i - \overline{V}_j = 2\left(R_{ij} + \mathbf{j}X_{ij}\right)\frac{P_{ij} - jQ_{ij}}{\underline{V}_i + \underline{V}_j} . \quad (3)$$

With some mathematical manipulations, it can be shown that the above expression is equivalent to the following branch equations:

$$V_i^2 - V_j^2 = 2R_{ij}P_{ij} + 2x_{ij}Q_{ij} \quad (4)$$

$$V_iV_j \sin\left(\theta_i - \theta_j\right) = X_{ij}P_{ij} - R_{ij}Q_{ij} \quad (5)$$

where V_i and V_j are the magnitudes of $\overline{V}_i$ and $\overline{V}_j$, respectively, while θ_i and θ_j represent the corresponding phase angles.

These equations are exact, assuming that power is referred to the center of the branch, specifically to the point where the voltage is $\left(\overline{V}_i + \overline{V}_j\right)/2$. However, the power at the terminals differs due to the active and reactive losses induced by the branch resistance and reactance. Therefore, an approximation is introduced by assuming that these losses are negligible, such that P_{ij} and Q_{ij} flow through the branch without alterations. Literature [20] also proposes a similar formulation capable of considering the effects of losses without increasing the model complexity. However, it is based on variables that are not linearly related to the active/reactive power exchanges, which makes the equations impractical to be integrated within an optimal power dispatch problem.

B. On-Load Tap Changer transformer

The mathematical modelling of an OLTC is derived from the circuital representation shown in Fig. 2.

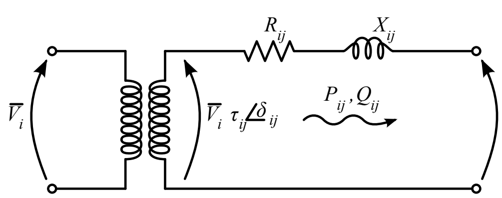

Figure 2. Electrical model of the generic power flow branch.

In a power flow context, where the tap setting is an input parameter, an OLTC is modeled as a conventional transformer with voltage ratio τ_{ij} and phase shift δ_{ij}. To include an OLTC in the branch model, the branch equations (4) and (5) become:

$$\tau_{ij}^2 V_i^2 - V_j^2 = 2R_{ij}P_{ij} + 2x_{ij}Q_{ij} \quad (6)$$

$$\tau_{ij}V_iV_j \sin\left(\theta_i - \theta_j + \delta_{ij}\right) = X_{ij}P_{ij} - R_{ij}Q_{ij} . \quad (7)$$

C. Linearized branch flow model

In the *Simplified DistFlow* model, linearization is obtained through a change of variables. A variable W for the squared voltage magnitude is used instead of the voltage magnitude V, so that (6) becomes:

$$\tau_{ij}^2 W_i - W_j = 2R_{ij}P_{ij} + 2x_{ij}Q_{ij} . \quad (8)$$

Extending the *Simplified DistFlow* model requires linearizing (7) as well. We accomplish this by performing two approximations to its left-hand side. Considering that P_{ij} and Q_{ij} are referenced to the center of the branch, as noted at the end of Section II.A, the first approximation is:

$$\tau_{ij}V_iV_j \approx \left(\frac{\tau_{ij}V_i + V_j}{2}\right)^2 \approx \left(V_{ij}^{nom}\right)^2 \quad (9)$$

where V_{ij}^{nom} represents the rated voltage value at which the branch operates. The second is the small angle approximation:

$$\sin\left(\theta_i - \theta_j + \delta_{ij}\right) \approx \theta_i - \theta_j + \delta_{ij} . \quad (10)$$

Therefore, (7) is approximated as:

$$\theta_i - \theta_j + \delta_{ij} = \frac{X_{ij}}{\left(V_{ij}^{nom}\right)^2}P_{ij} - \frac{R_{ij}}{\left(V_{ij}^{nom}\right)^2}Q_{ij} . \quad (11)$$

To the best of the authors' knowledge, the above equation has not previously been suggested within a linear branch flow model. Nevertheless, [7] approached this by proposing a comparable equation, albeit without incorporating the transformer model, within the framework of an optimal restoration problem.

D. Closed-loop operation of distribution network

Equation (8) is the fundamental equation of the *Simplified DistFlow* model [5], [17]. Its practicality stems from the fact that it directly returns the (squared) voltage magnitude drop by linearly combining the branch active and reactive powers. Moreover, this single equation is sufficient to approximate the entire physics of the branch if the network is operated radially. In fact, a power flow model for a connected power system composed of N buses and E branches requires:

- N squared bus voltage magnitude variables (W_i);
- E branch active power variables (P_{ij});
- E branch reactive power variables (Q_{ij});

and:

- $N - 1$ nodal active power balance equations;
- $N - 1$ nodal reactive power balance equations;
- E squared voltage drop equations (i.e., (8));
- 1 squared bus voltage magnitude reference equation.

This amounts to $N + 2E$ variables in $2N + E - 1$ independent linear equations. Thus, the linear system has a unique solution when $E = N - 1$, a condition that is only verified for radial networks.

When the distribution system is operated with closed loops, i.e., $E > N - 1$, the model above becomes an underdetermined linear system, and no longer has a unique solution. To restore uniqueness, (11) needs to be added to the model. Incorporating phase angles requires adding:

- N bus phase angle variables (θ_i);

and:

- E bus phase angle difference equations (i.e., (11));
- 1 bus phase angle reference equation;

totaling $2N + 2E$ variables and $2N + 2E$ independent linear equations. Since the number of variables and equations is equal regardless of N and E, the solution uniqueness of the resulting linear system holds for any network topology. Thus, (8) and (11) together are sufficient to model branches with OLTC in a meshed network.

III. Optimization of Distribution Systems

A model for the Optimal Power Flow (OPF) problem is developed in this section based on the equations discussed in the previous section. Some of the input parameters of the Power Flow problem – namely: generation, demand, and OLTC tap setting – become variables of the OPF problem, leading to infinite solutions. Constraints are introduced to ensure that network components' specifications are met, and the additional variables are appropriately weighted in the objective function to select the best among the feasible solutions.

The proposed model is linear. It uses squared voltage magnitude (W) and voltage phase angle (θ) as bus variables, whereas the branch variables are active and reactive power (P and Q, respectively), and the squared voltage magnitude variation induced by OLTC ($\Delta W^+ - \Delta W^-$, defined later).

A. *Voltage magnitude constraints*

The voltage magnitude is a key variable to control in maintaining the quality of supply, and the network operator must ensure that the voltage magnitude is within specified minimum and maximum limits. Since the *Simplified DistFlow* model uses squared voltage magnitudes, this constraint is defined as:

$$\left(V_i^{min}\right)^2 \le W_i \le \left(V_i^{max}\right)^2 \tag{12}$$

Voltage phase angle is unbounded and defined up to a constant. Therefore, fixing it to zero (or any other value) eliminates the solution degeneracy:

$$\theta_r = 0 \qquad \forall r \in R. \tag{13}$$

B. *Linear branch flow equations and OLTC flexibility*

In the OPF problem, with the OLTC transformation ratio τ_{ij} being a variable, (8) would be nonlinear. The literature offers various approaches based on the adoption of integer variables. However, a discrete modelling of the OLTC has a significant impact on the model complexity, and a continuous formulation, similar to the one adopted in [8], is preferred. The proposed method involves adding a pure voltage generator $\overline{V}_{ij}$ to a fixed-ratio transformer with the aim of modelling the voltage variation due to OLTC actions (Fig. 3).

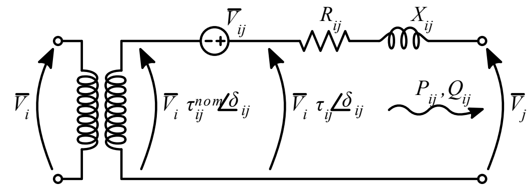

Figure 3. Proposed electrical model of a branch including a OLTC.

With this approach, the linearized branch flow equations are:

$$\left(\tau_{ij}^{nom}\right)^2 W_i - W_j + \Delta W_{ij}^+ - \Delta W_{ij}^- = 2R_{ij}P_{ij} + 2X_{ij}Q_{ij} \qquad \forall (i,j) \in E, \tag{14}$$

$$\theta_i - \theta_j + \delta_{ij} = \frac{X_{ij}}{\left(V_{ij}^{nom}\right)^2}P_{ij} - \frac{R_{ij}}{\left(V_{ij}^{nom}\right)^2}Q_{ij} \qquad \forall (i,j) \in E. \tag{15}$$

In (14), τ_{ij}^{nom} is the nominal voltage ratio of the transformer (at center tap) and $\Delta W_{ij}^+ - \Delta W_{ij}^-$ represents the squared voltage magnitude variation induced by the OLTC (i.e., the effect of selecting a specific tap). The latter is split into its positive and negative parts, which are weighted with a penalty term c_{ij}^{OLTC} in the objective function (23). This would not be necessary in a nonlinear model. In fact, when the network supplies voltage-dependent loads, the voltage variations have an impact on the power flows, energy losses, and therefore on the economic dispatch of generators [15]. Since the proposed model does not consider the influence of voltage on the power variables, except in the case of voltage congestions, the penalty term in the objective function ensures that the OLTC is only used when needed.

Constraints on the OLTC tap range $\tau_{ij}^{min} \le \tau_{ij} \le \tau_{ij}^{max}$ are formulated as:

$$\begin{cases} 0 \le \Delta W_{ij}^+ \le \left[\left(\tau_{ij}^{max}\right)^2 - \left(\tau_{ij}^{nom}\right)^2\right] \mathrm{W}_i \\ 0 \le \Delta W_{ij}^- \le \left[\left(\tau_{ij}^{nom}\right)^2 - \left(\tau_{ij}^{min}\right)^2\right] \mathrm{W}_i \end{cases} \qquad \forall (i,j) \in E \tag{16}$$

where τ_{ij}^{min} and τ_{ij}^{max} are the minimum and maximum transformation ratios, respectively, such that $\tau_{ij}^{min} \le \tau_{ij}^{nom} \le \tau_{ij}^{max}$ The actual transformation ratio can be computed in post-processing as:

$$\tau_{ij} = \sqrt{\left(\tau_{ij}^{nom}\right)^2 + \frac{\Delta W_{ij}^+ - \Delta W_{ij}^-}{W_i}} \tag{17}$$

Even though distribution systems normally do not foresee the operation of phase shifting transformers, it might be expected that closed-loop configuration could require this typology of asset/flexibility to optimally control the power flow. In these circumstances, as it can be noticed from the analysis of (15), the parameter δ_{ij} can be promoted to optimization variable (with an associated variation cost) without undermining the linearity of the model.

C. Branch flow limit

The distribution system optimization must prevent branch overloading. As a proxy for current limits, a branch flow limit ensures that the apparent power does not exceed the rated value of the branch: $P_{ij}^2 + Q_{ij}^2 \le \left(S_{ij}^{max}\right)^2$. This second-order cone constraint can be linearized by relaxing the circular shape of the feasible region in the plane spanned by P_{ij} and Q_{ij} into a regular polygon, such as the octagon depicted in Fig. 4. The corresponding linear formulation is:

$$\begin{cases} -S_{ij}^{max} \le P_{ij} \le S_{ij}^{max} \\ -S_{ij}^{max} \le Q_{ij} \le S_{ij}^{max} \\ -\sqrt{2}S_{ij}^{max} \le P_{ij} + Q_{ij} \le \sqrt{2}S_{ij}^{max} \\ -\sqrt{2}S_{ij}^{max} \le P_{ij} - Q_{ij} \le \sqrt{2}S_{ij}^{max} \end{cases} \quad \forall (i,j) \in E. \tag{18}$$

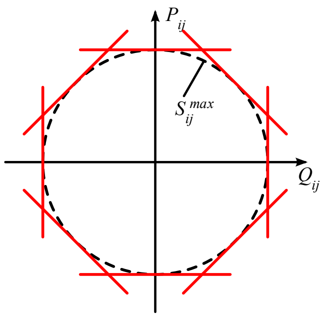

Figure 4. Linearization of the branch flow limit constraint with a regular octagon.

D. Curtailable loads

The main purpose of distribution systems is to supply energy to end users. OPF models typically represent them as power absorption units connected to network buses, with power demand as a model parameter. However, allowing for load curtailment is helpful to ensure optimization convergence even in the case of infeasibility due to, e.g., branch flow limits or bus voltage limits. By assigning a high cost c_l^{VOLL} to active power curtailment P_l^{curt} in the objective function (23), the active and reactive power absorption of loads can be modeled as:

$$\begin{cases} P_l^{load} = P_l^{ref} - P_l^{curt} \\ Q_l^{load} = P_l^{load} \tan\left(\varphi_l\right) \end{cases} \quad \forall l \in L \tag{19}$$

where P_l^{ref} is the reference active power absorption, the load power factor angle φ_l is assumed constant, P_l^{curt} is subject to:

$$0 \le P_l^{curt} \le P_l^{ref} \qquad \forall l \in L. \tag{20}$$

E. Generators

Small-scale generators connected to distribution networks are mostly based on renewable energy sources. Since the technology often does not have a significant impact from the modeling perspective, all generators are represented as a source of active power P_g^{gen} and reactive power $Q_g^{gen+} - Q_g^{gen-}$ subject to rectangular capability constraints:

$$\begin{cases} P_g^{min} \le P_g^{gen} \le P_g^{max} \\ 0 \le Q_g^{gen+} \le Q_g^{max} \\ 0 \le Q_g^{gen-} \le -Q_g^{min} \end{cases} \qquad \forall g \in G \tag{21}$$

where P_g^{min}, P_g^{max}, Q_g^{min} and Q_g^{max} define the rectangular capability of the generator g. As for classical OPF formulations [19], the same element can be adopted for modelling the connection with the transmission network.

The utilization of generators is driven by two cost terms in the objective function (23). A cost c_g^{Pgen}, where the term $c_g^{Pgen} P_g^{gen}$ is proportional to the active power, is used to set the dispatching priority of local generation, or to model the price of the energy imported from the transmission network. As for reactive power, since it can be provided with no expense by the generators, most of the OPF models do not foresee additional dispatching costs. Nevertheless, exact OPF models intrinsically minimize its utilization since reactive power exchange increases network losses. To achieve a similar behavior, the objective function counts a second term $c_g^{Qgen}\left(Q_g^{gen+} + Q_g^{gen-}\right)$, where a positive cost c_g^{Qgen} penalizes deviations from zero.

F. Bus power balance

The bus power balance equations link the power transferred along branches leaving from or arriving to a bus i (E_i and E_i', respectively) with the injected power from generators (G_i) and absorbed power by loads (L_i) connected to the same bus:

$$\begin{cases} \sum_{g \in G_i} P_g^{gen} - \sum_{l \in L_i} P_l^{load} - \sum_{(i,j) \in E_i} P_{ij} + \sum_{(i,j) \in E_i'} P_{ij} = 0 \\ \sum_{g \in G_i} \left(Q_g^{gen+} - Q_g^{gen-}\right) - \sum_{l \in L_i} Q_l^{load} + \\ \qquad - \sum_{(i,j) \in E_i} Q_{ij} + \sum_{(i,j) \in E_i'} Q_{ij} = 0 \end{cases} \quad \forall i \in N. \tag{22}$$

G. OPF problem formulation

Compared to conventional OPF, the proposed model requires three additional terms in the objective function for minimizing the utilization of generators' reactive power (Section III.E), OLTC control actions (Section III.B), and load curtailment (Section III.D). It is formulated as:

$$\begin{aligned} \text{minimize} \quad & \sum_{g \in G} c_g^{Pgen} P_g^{gen} + \sum_{g \in G} c_g^{Qgen}\left(Q_g^{gen+} + Q_g^{gen-}\right) + \\ & + \sum_{(i,j) \in E} c_{ij}^{OLTC}\left(\Delta W_{ij}^{+} + \Delta W_{ij}^{-}\right) + \\ & + \sum_{l \in L} c_l^{VOLL} P_l^{curt} \end{aligned} \tag{23}$$

subject to:	(12)	bus voltage magnitude bounds
	(13)	reference bus phase angle
	(14),(15)	branch flow equations
	(16)	OLTC bounds
	(18)	branch flow limits
	(19)	load power
	(20)	load curtailment bounds
	(21)	generator power bounds
	(22)	bus power balance

IV. Experimental Validation

The previous sections illustrate an extension of the *Simplified DistFlow* equations, and their manipulations aimed at defining a complete OPF model. The main advantage of the proposed formulation consists of being completely linear, which guarantees numerical stability and high performances in terms of computational burden. Furthermore, the introduced novelties extend the domain of application of the *Simplified DistFlow*, making it capable of managing the most recent challenges of distribution system optimization. Considering that the scientific community has already validated the effectiveness of the different approximations and relaxations of the power flow equations [3], [4], this section solely focuses on evaluating the accuracy of the proposed model in representing closed-loop distribution systems and optimizing OLTC operation. It also provides an analysis of its computational performance compared to other formulations.

A. Modelling accuracy of closed-loop distribution systems

The first objective of the proposed study was to adapt the *Simplified DistFlow* model for managing meshed topologies. For this reason, over 1300 medium-voltage distribution networks (synthetically generated with the procedure described in [21]) were randomly reconfigured to obtain closed-loops. Realistic demand and generation scenarios were applied to the considered systems and then simulated with the proposed model. The evaluation of its accuracy was performed by using the results provided by MATPOWER [19] as a reference. For these tests, to avoid interferences, voltage and loading constraints were relaxed, and the OLTC of the distribution transformer was disabled.

First of all, the voltage magnitude and phase angle errors were evaluated by computing the maximum mismatch experienced with respect to MATPOWER results for each network. Fig. 5 reports the distribution of these errors, from which it is possible to quantify the model uncertainty. Although the model exhibits a good performance in predicting the voltage phasors, the neglection of losses and the small-angle assumption might drive to significant deviations in power flow predictions. Fig. 6 illustrates the distributions of the maximum errors observed in the considered networks, represented separately for lines and transformers.

From the analysis of the simulation results, it can be noticed that the model uncertainty depends on the specific network element and power component being considered. The average active power mismatch is about 1.0÷1.3% and shows a similar distribution for both lines and transformers. Regarding reactive power, transformers exhibit larger errors (average of 5.1%) compared to those seen in lines (1.7%) which have a distribution comparable to that of active power. Since part of the observed error is due to the neglection of losses, it must be observed that their reactive portion is relevant for distribution transformers: their typical reactance is much greater compared to the impedance of other network branches and this can be considered one of the most noticeable contributions to the model uncertainty.

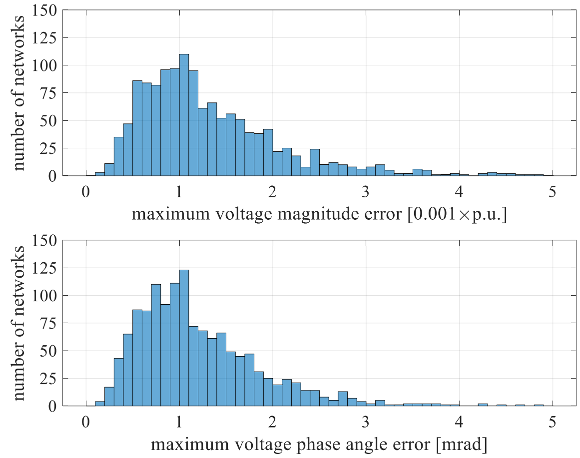

Figure 5. Voltage magnitude and phase angle uncertainty of the proposed model.

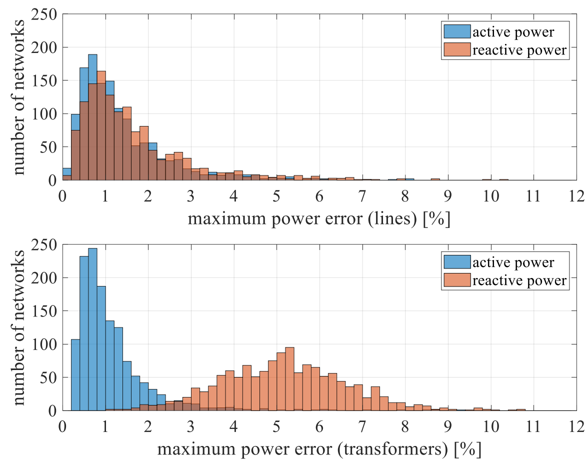

Figure 6. Percentage lines/transformers power error of the proposed model

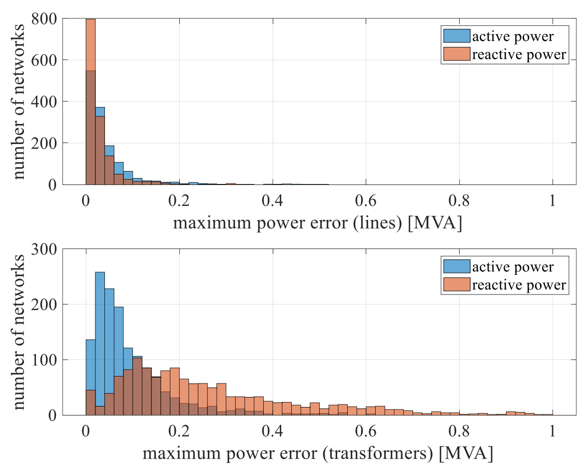

Figure 7. Absolute lines/transformers power error of the proposed model

Even though the apparently noticeable uncertainty of the transformer model, it must be noticed that reactive power transfer is normally smaller compared to the active power. This means that, in terms of absolute error, the two power components may have comparable uncertainties (as confirmed by the data reported in Fig. 7). Based on this, it can be concluded that the adopted approximations do not significantly impact on the returned electrical quantities in the majority of optimal power flow applications.

B. *Modelling accuracy of OLTC control*

The last element to be tested for the proposed model consists of the optimization of OLTC actions on distribution transformers. Therefore, a reference distribution system is selected to evaluate the impact of the adopted approximations: the IEEE 33-bus distribution benchmark [22]. With the aim of obtaining an interesting operating scenario (in which OLTC requires optimization), the following modifications have been implemented:

- The network does not include the distribution transformer; therefore, it is added to the model by modifying the transformation ratio of the first branch.
- The generation capacity is not sufficient to determine voltage issues and, consequently, OLTC actions. Therefore, the peak power of existing generators is increased from 200 kW to 1.6 MW each.
- Loads absorb the power profiles suggested by [22]. The source also contains photovoltaic production patterns, which are applied to the generators.

Again, the accuracy of the model is evaluated by comparing the obtained results with those returned by the nonlinear model (MATPOWER). Since this last tool does not include OLTC optimization, the optimal voltage of the reference bus (which is free to move between 0.95 and 1.05 p.u.) is assumed to be the optimal OLTC setpoint. The optimization is performed individually for 96 time steps, representing a sunny day with a 15-minute resolution.

Looking at the results reported in Fig. 8, it is immediate to recognize that the two optimization algorithms lead to significantly different results. MATPOWER maximizes the voltage of the reference bus to 1.05 p.u. (OLTC setpoint) with the objective of reducing the system losses. In fact, high voltages mean low currents to meet the same power demand. Nevertheless, during the mid-day hours, the presence of photovoltaic generation requires OLTC adjustments to prevent overvoltage issues. On the other hand, the proposed model does not consider losses and, as expected, it does not alter the transformation ratio during the first and last hours of the day. However, when photovoltaic production significantly affects the network voltage, the linear OPF reduces the OLTC setpoint and the resulting time evolution matches that of the reference model. According to the curves plotted in Fig. 8, the OLTC actions prevent violations for all the tested nominal voltage ratios $\tau^{nom} = \{1.00, 1.02, 1.05\}$ p.u., and the same behavior as that of MATPOWER can be accurately replicated by setting $\tau^{nom} = 1.05$ p.u.

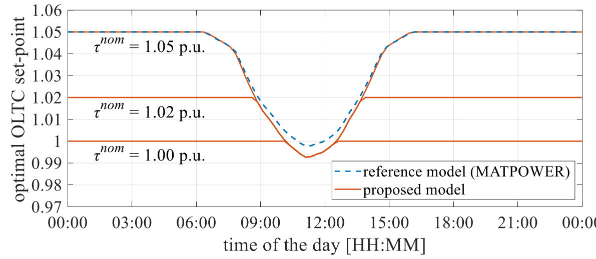

Figure 8. Optimal OLTC setpoint returned by the proposed and reference model

It can be concluded that, even though the proposed linear model could significantly deviate from the optimal solution when losses reduction drives the optimization objective, the obtained results clearly demonstrate its accuracy when actions to contain possible network constraints violations are requested. Furthermore, experience shows that controlling OLTC deviations with respect to its nominal position (as implemented within the proposed model) aligns more with the control strategies adopted by network operators.

C. *Computational performance*

The main advantage deriving from adopting a linear model for simulation and optimization of electrical systems is represented by the computational efficiency. In fact, it is well established that linear OPF formulations should be preferred over the exact one, insofar the introduced approximations are considered acceptable.

To evaluate the performance of the proposed model, its computational time has been compared with that of the exact nonlinear OPF formulation. The single time-step 141-bus distribution system documented in [23] was selected as the benchmark and optimized repeatedly with 100 different random reconfigurations (to introduce closed-loop topologies). The entire process was carried out in the MATLAB (R2002a) environment on a Windows 10 PC, mounting a 3.70 GHz Intel(R) Xeon(R) W-2145 CPU and 128 GB of RAM, and returned the times reported in Tab. I.

TABLE I. EXACT VERSUS PROPOSED LINEAR MODEL PEFORMANCE (LARGE DISTRIBUTION NETWORK)

OPF/optimization solver	**model**	**time [ms]**
MATPOWER (v6.0)	nonlinear OPF (exact)	131 ± 31
MATLAB Optimization Toolbox (v9.3 – R2022a)	proposed model	14.6 ± 2.0

Considering that MATPOWER was set to use the same optimization toolbox selected for the proposed model, the results clearly demonstrate the advantages of the linear formulation, which is on average 9 times faster. From the analysis of the returned variables, the voltage phasor and power transit errors are within the intervals discussed in section IV.A.

A further experiment, aimed at identifying the performance of the proposed model with respect to the classical linear formulation (*Simplified DistFlow*) was conducted, and the results are reported in Tab. II. The IEEE 33-bus distribution benchmark has been used with the 4-day profiles reported in [22], featuring a 15-minute time resolution (i.e. 384 power samples per load and per generator). Although one of the added values of the proposed formulation consists of managing meshed topologies, its performance comparison with the *Simplified DistFlow* imposes the adoption of a radial configuration for the selected distribution network. Nevertheless, this exercise allows for evaluating the impact of adding constraints (14)-(16) to the classical linear OPF.

TABLE II. EXACT VERSUS SIMPLIFIED DISTFLOW VERSUS PROPOSED LINEAR MODEL PERFORMANCE (LARGE TIME HORIZON)

OPF/optimization solver	model	time [s]
MATPOWER (v6.0)	nonlinear OPF (exact)	15.41 ± 0.09
MATLAB Optimization Toolbox (v9.3 – R2022a)	simplified DistFlow	0.94 ± 0.01
	proposed model	1.15 ± 0.01

From the timing analysis reported in Tab. II, all linear models further confirm their superiority with respect to the exact OPF, being 13÷16 times faster. Intuitively, the proposed formulation results to be systematically more demanding compared to the *Simplified DistFlow*, as the optimization time increases by +22%. Of course, pre-solving algorithms can be implemented in order to identify unnecessary constraints and reduce the model complexity: this enhancement can be performed during model construction by a) limiting the definition of (15) to closed-loops that can be identified from the network graph analysis; and b) using (8) instead of (14) and (16) when modelling lines and fixed-ratio transformers.

V. CONCLUSIONS

The definition of numerically tractable and efficient OPF models is still an engineering challenge, as confirmed by the vast related literature. From the analysis of it, a linear method for optimally managing distribution grids was selected and reformulated by the authors to integrate some modern operational practices currently undergoing testing in the real field. The work presented in this paper has led to the derivation of a general linear OPF formulation that can be applied to distribution networks of any topology and voltage level. Furthermore, upon examining its equations, it can be noticed that it consists of a generalization of the well-known DC-OPF, which expands its applicability to high-voltage systems.

In addition to the modelling, the manuscript analyzes the uncertainty driven by the adopted simplifications. Specifically, the model proved to be accurate in predicting voltage magnitudes, phase angles and power flows. Furthermore, the optimization outcomes are consistent with those returned by the nonlinear model, except when the optimal solution is driven by the reduction of energy losses. In fact, linearity is obtained by neglecting the active/reactive power leakage experienced in network branches.

Finally, the demonstrated performance of the model opens the way to further development: time-domain optimization and inter-temporal constraints, integer variables to select optimal planning investments, etc. are just some of the possible future integrations.

ACKNOWLEDGMENT

This work has been financed by the Research Fund for the Italian Electrical System under the Three-Year Research Plan 2022-2024 (DM MITE n. 337, 15.09.2022), in compliance with the Decree of April 16th, 2018.

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